BMS

INSTITUTE OF TECHNOLOGY AND MANAGEMENT

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DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

DIGITAL COMMUNICATION

17EC61

STUDY MATERIAL

VI SEMESTER

Module-1

Bandpass Signal to Equivalent Lowpass

Content:

Part A

- Hilbert Transform
    - ❖ Definition (Tansform and its Inverse transform)
    - ❖ Impulse and Frequency response of HT
    - ❖ Properties of HT
    - ❖ Applications of HT
    - ❖ Problems
- Pre-envelopes
    - ❖ Definition and Representation
    - ❖ Methods of computation (Time domain and Frequency domain approach)
    - ❖ Pre-envelopes of Low pass signal.
    - ❖ Problems
- Complex envelopes of Bandpass Signals
    - ❖ Representation
    - ❖ Problems
- Canonical Representation of Bandpass signals
    - ❖ Representation
    - ❖ Polar representation of Band pass signals
    - ❖ Relationship between Cartesian and Polar representation of Band pass Signals.
- Complex Low-Pass Representation of Band pass System
- Complex representation of band pass signals and systems
    - ❖ Time domain Procedure
    - ❖ Frequency domain Procedure.

(Text 1: 2.8, 2.9, 2.10, 2.11, 2.12, 2.13).

Part B

Line codes:

- ❖ Unipolar
- ❖ Polar
- ❖ Bipolar (AMI)
- ❖ Manchester code and their power spectral densities. (Text : Ch 6.10).

Overview of HDB3, B3ZS, B6ZS

Text Book:

Simon Haykin, —Digital Communication System, John Wiley & sons, First Edition, 2014, ISBN 978-0-471-64735-5.

Topic1 • Hilbert Transform

1. Define Hilbert transform. List the properties of the Hilbert transform **(04Marks) [June/July 2018]. [Jan 2020].**

2. Define Hilbert transform. What are its applications. Prove that a signal $g(t)$ and its Hilbert transform $\tilde{g}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$. (5Marks) **[Dec 2018/Jan 2019]**

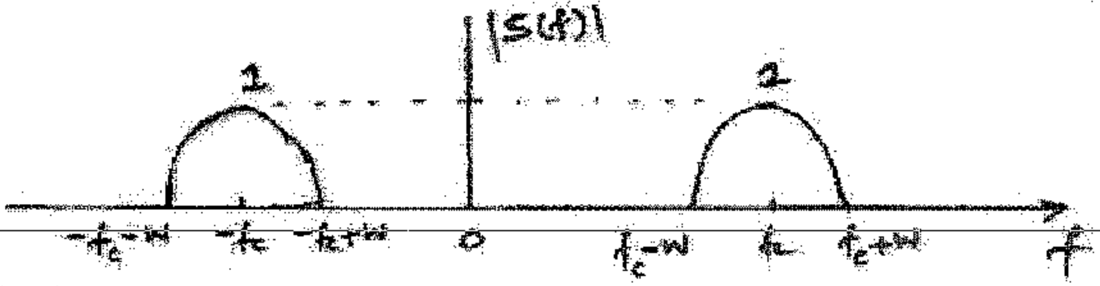
3. Determine the Hilbert transform of the Signal $g(t) = sinc(t)$. **(4Marks) [June/July 2019].**

4. Obtain the Hilbert transform of **(4Marks) [Dec2019/Jan 2020]**
   - i. $x(t) = \cos(2\pi f t) + sin(2\pi f t)$
   - ii. $x(t) = e^{-j2\pi f t}$

Topic2 . Pre-envelopes and Complex Envelopes

1. Define Pre-envelop of a real valued signal. Given a Bandpass signal $s(t)$. sketch the amplitude spectra of signal $s(t)$, pre-envelope $s_+(t)$ and complex envelope $\tilde{s}(t)$. **(4 Marks) [June/July 2018].**

2. Determine the pre-envelope and complex envelope of the signal shown in Figure. **(6 Marks) [June /July 2019]**



3. Determine the pre-envelope and complex envelope of the RF pulse defined by $x(t) = Arec\left(\frac{t}{T}\right)\cos(2\pi f_c t)$. **(6 Marks) [Dec 2018/Jan 2019]**

Topic 3. Canonical Representation of Bandpass signals

1. Obtain the canonical representation of Bandpass Signals. **(6 Marks) [June/July 2018].**

2. Explain canonical representation of Bandpass Signals. **(8 Marks) [Dec 2019/Jan 2020].**

3. Consider a bandpass signal $s(t)$ which is represented in terms of in-phase and quadrature components. Suggest a suitable scheme for:
    I. Extracting the in-phase and quadrature components from the bandpass signal.
    II. Reconstructing the band pass signal from in-phase and quadrature components. **(6Marks)[June/July 2019].**

4. Express bandpass signal $s(t)$ in canonical form. Also explain the scheme for deriving the inphase and quadrature components of bandpas signal $s(t)$. **(6 Marks) [Dec 2018/Jan 2019]**

Topic 4.

Complex Low-Pass Representation of Band pass System

1. Explain with relevant expressions the procedure for computational analysis of a bandpass system driven by a bandpass signal. **(6 Marks) [Dec 2018/Jan 2019]**

1. Explain the time-domain procedure for the complex representation of bandpass signals and systems. **(6 Marks) [June/July 2019]** (Topic 5.)

Topic 5.

Complex representation of band pass signals and systems

2. Derive the expression for the complex lowpass representation of Bandpass Systems. (8 Marks) **[June/July 2018, Dec 2019/Jan 2020]** (Topic 4.)

Part B Topic 6.

Line codes:

- Unipolar
- Polar
- Bipolar (AMI)
- Manchester code and their power spectral densities. (Text : Ch 6.10).
- **Overview of HDB3, B3ZS, B6ZS**

1. Compare the power spectra of various line codes in terms of bandwidth, dc component, noise immunity and synchronization capability with neat sketches. **(5Marks) [Dec 2018/Jan 2019].**

2. For a binary sequence **0100000001011** construct
    i. RZ Bipolar
    ii. Machenster
    iii. B3ZS and B6ZS formats. **(7 Marks) [June/July 2019].**
    iv. HDB3 format.

3. Draw the power spectra
   i. NRZ polar signal
   ii. R ZAMI Signal
   iii. NRZ Unipolar **(3M/4M) [June July 2019 & Dec 2019/Jan 2020].**

4. what is line code coding ? for the binary stream **011010** sketch the following line codes
   i. Unipolar NRZ
   ii. Polar NRZ
   iii. Unipolar RZ
   iv. Bipolar RZ
   v. Manchester **(6Marks) [June July 2018]**

5. For the binary stream **11011100.** Sketch the following line codes
   i. Unipolar NRZ
   ii. Polar NRZ
   iii. Unipolar RZ
   iv. Bipolar NRZ
   v. Manchester **(4Marks) [Dec 2019/ Jan2020]**

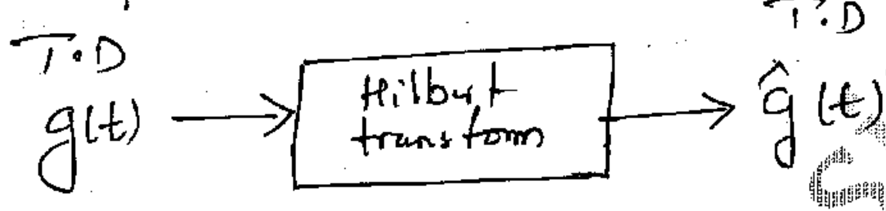
6. Write a note on HDB_N Signalling. **(4 Marks) (June July 2018).**

7. What is the advantage of HDB3 code over conventional alternate mark inversion (AMI) code. Code the pattern "**10100000110000011000000**" using HDB3 Encoding and AMI Encoding. **(4Marks) [Dec 2018/ Jan2019]**

Topic 1. Hilbert Transform

** ← J/J 2018 Jan 2020

Definition :- Hilbert transform of a signal g(t) is defined as the transform in which phase angle of all components of the signal is shifted by ±90°.



$$g(t) \xleftrightarrow{H.T} \hat{g}(t)$$

$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{(t-\tau)} \cdot d\tau$$

$$\boxed{\hat{g}(t) = g(t) * 1/\pi t}$$

Impulse Response of H.T

I.R of H.T is $\boxed{h(t) = \frac{1}{\pi t}}$

Frequency Response :-

$$\underset{I.R}{h(t)} \xleftrightarrow{F.T} \underset{F.R}{H(f)}$$

W.k.t.

$$\text{Sgn}(t) \xleftrightarrow{F.T} \frac{2}{j\omega} = \frac{1}{j\pi f}$$

using duality property of F.T

i.e $x(t) \xleftrightarrow{F.T} X(f)$

then $X(t) \xleftrightarrow{F.T} x(-f)$

Note:-
* Sgn(f) is an odd fn
* i.e $\text{Sgn}(-f) = -\text{sgn}(f)$

$$\frac{1}{j\pi t} \xleftrightarrow{F.T} \text{Sgn}(-f) = -\text{Sgn}(f)$$

$$\boxed{\frac{1}{\pi t} \xleftrightarrow{F.T} -j\,\text{sgn}(f)}$$

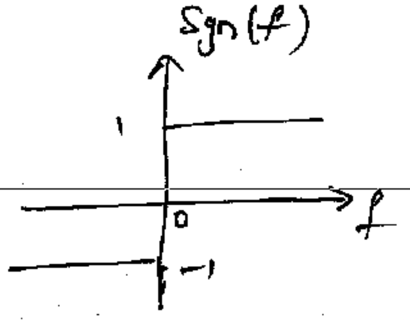
FR:-

$$\therefore H(f) = -j\,\text{sgn}(f)$$

$$|H(f)| = |-j\,\text{sgn}(f)| = 1$$

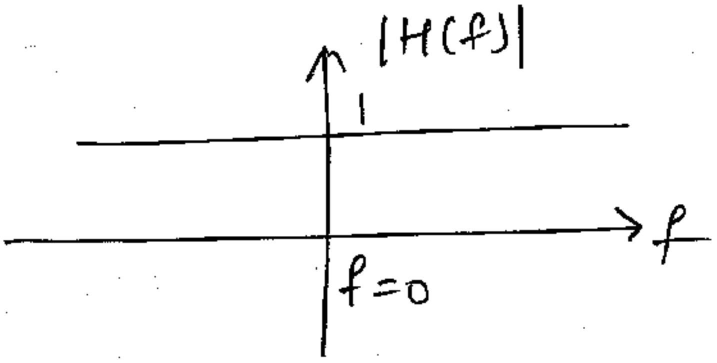
Note:-

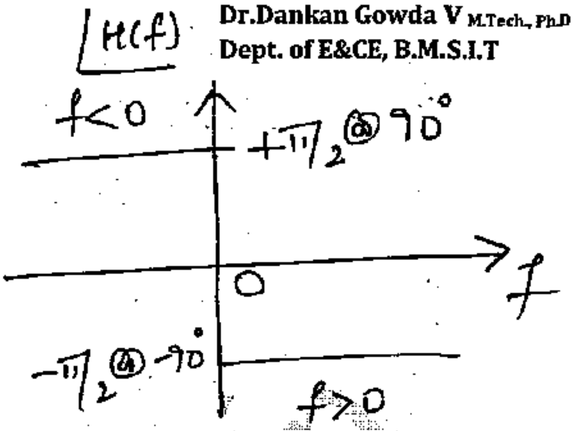
$$\text{Sgn}(f) = \begin{cases} 1 & ; f > 0 \\ 0 & ; f = 0 \\ -1 & ; f < 0 \end{cases}$$

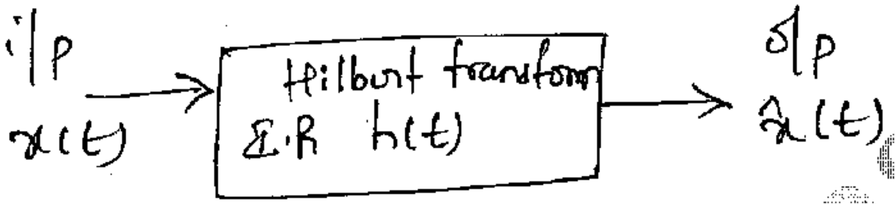


$$\therefore -j\,\text{sgn}(f) = \begin{cases} -j & ; f > 0 \\ +j & ; f < 0 \end{cases}$$

$$\therefore \angle H(f) = \angle -j\,\text{sgn}(f) = \begin{cases} -\pi/2 & ; f > 0 \\ +\pi/2 & ; f < 0 \end{cases}$$







$$\hat{x}(t) = x(t) * h(t)$$

Convolution property

$$x(t) * h(t) \overset{F.T}{\longleftrightarrow} X(f)H(f)$$

$$\boxed{\hat{x}(t) = x(t) * \frac{1}{\pi t}}$$

F.T

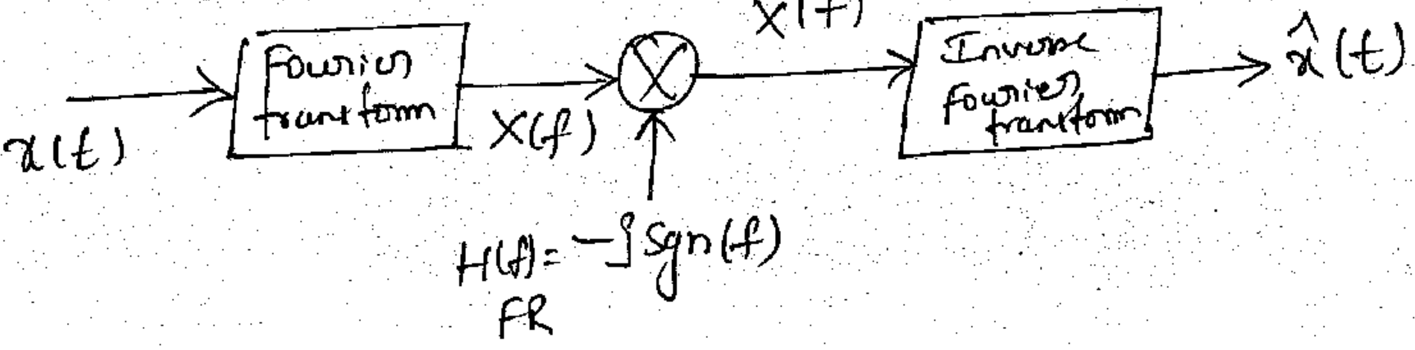
$$\hat{X}(f) = X(f) \cdot H(f)$$

$$\boxed{\hat{X}(f) = X(f) \cdot [-j\,\mathrm{sgn}(f)]}$$

IFT

$$\boxed{\hat{x}(t) = IFT\,[\underbrace{X(f) \cdot H(f)}_{\hat{X}(f)}]}$$

H.T



i> Find the H.T of $x(t) = \cos(2\pi f_c t)$

Note:- $\cos 2\pi f_c t \xleftrightarrow{F.T} \frac{1}{2}[\overset{f>0}{\delta(f-f_c)} + \overset{f<0}{\delta(f+f_c)}] = X(f)$

$$\sin 2\pi f_c t \xleftrightarrow{F.T} \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$$

Method 1:- using F.D approach.

$$x(t) \longrightarrow \boxed{\underset{HT}{h(t)} = \frac{1}{\pi t}} \longrightarrow \hat{x}(t)$$

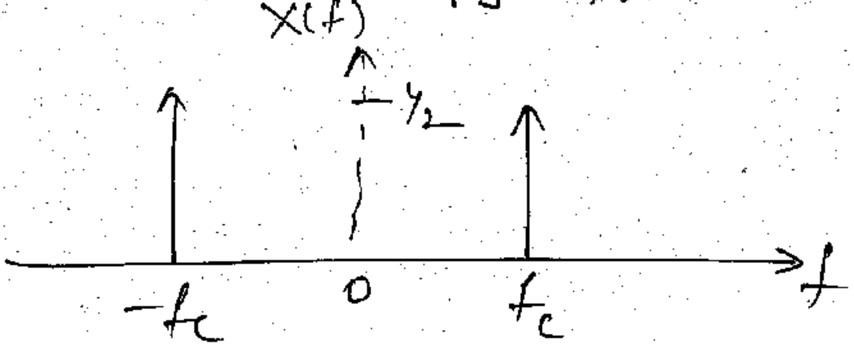
$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\Big\updownarrow F.T$$

$$\hat{X}(f) = X(f)[-j\,\mathrm{sgn}(f)]$$

$$= -j\,X(f)\,\mathrm{sgn}(f)$$

$$\hat{X}(f) = \begin{cases} -j\,X(f) & ; f>0 \\ 0 & ; f=0 \\ +j\,X(f) & ; f<0 \end{cases}$$



$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$= \cos(2\pi f_c t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \cos\left[2\pi f_c t - \pi/2\right]$$

$\cos(\theta - 90^\circ) = \sin\theta$

$$\therefore \boxed{\hat{x}(t) = \sin(2\pi f_c t)}$$

Note:- i. H.T of Even function results in an odd function and vice-versa.

ie $\overset{\text{even fn TD}}{\cos 2\pi f_c t} \xleftrightarrow{H.T} \overset{\text{T.D odd fn}}{\sin 2\pi f_c t}$.

ii. unlike F.T, Hilbert transform does not alter the domain of the signal.

ii) Find H.T of

$$x(t) = \cos(2\pi f_c t) + \sin(2\pi f_c t).$$

Jan 2020

Note:- $\sin(\theta - 90^\circ) = -\cos\theta.$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \left[\cos(2\pi f_c t) + \sin(2\pi f_c t)\right] * \frac{1}{\pi t}$$

$$= \left\{\cos(2\pi f_c t) * \frac{1}{\pi t}\right\} + \left\{\sin 2\pi f_c t * \frac{1}{\pi t}\right\}$$

$$= \cos(2\pi f_c t - \pi/2) + \sin(2\pi f_c t - \pi/2)$$

$$\boxed{\hat{x}(t) = \sin(2\pi f_c t) - \cos(2\pi f_c t)}$$

$$\hat{X}(f) = \frac{1}{2}\left[-j\,\delta(f-f_c) + j\,\delta(f+f_c)\right]$$

$$= \frac{-j}{2}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

Note:-

$j = \frac{-1}{j}$

$-j = \frac{1}{j}$

$$\hat{X}(f) = \frac{+1}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right]$$

$\Big\updownarrow$ I.F.T

$$\boxed{\hat{x}(t) = \sin(2\pi f_c t)}$$

Method-2 :-

Time domain approach.

$$x(t) \longrightarrow \boxed{\begin{matrix}H.T\\ h(t)\end{matrix}} \longrightarrow \overset{o/p}{y(t)} = \hat{x}(t) = x(t) * h(t)$$

$$\hat{X}(f) = [X(f)]\cdot[-j\,\mathrm{sgn}(f)]$$

$$|\hat{X}(f)| = |X(f)|\cdot|\overset{1}{-j\,\mathrm{sgn}(f)}|$$

$$|\hat{X}(f)| = |X(f)|$$

and $\angle\hat{X}(f) = \angle X(f) - 90^\circ$

$\therefore$ o/p $y(t) = \hat{x}(t) = x(t-90^\circ)$ i.e -90° phase -shifter.

(iii) $x(t) = e^{-j2\pi f t}$ Jun 2020

soln: $x(t) = \cos 2\pi f t - j\sin 2\pi f t$ | $e^{-j\theta} = \cos\theta - j\sin\theta$

$\Updownarrow$ H.T

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \left[\cos 2\pi f t - j\sin 2\pi f t\right] * \frac{1}{\pi t}$$

$$= \left\{\cos(2\pi f t) * \frac{1}{\pi t}\right\} - \left\{j\sin 2\pi f t * \frac{1}{\pi t}\right\}$$

$$\hat{x}(t) = \cos(2\pi f t - 90^\circ) - j\sin(2\pi f t - 90^\circ)$$

$$\hat{x}(t) = \sin 2\pi f t + j\cos 2\pi f t$$

$$\hat{x}(t) = j\left[\cos 2\pi f t - j\sin 2\pi f t\right]$$

$$\boxed{\hat{x}(t) = je^{-j2\pi f t}}$$

(iv) $x(t) = \mathrm{Sinc}(t)$ June| July 2019.

soln: $\mathrm{Sinc}(t) = \frac{\sin \pi t}{\pi t}$

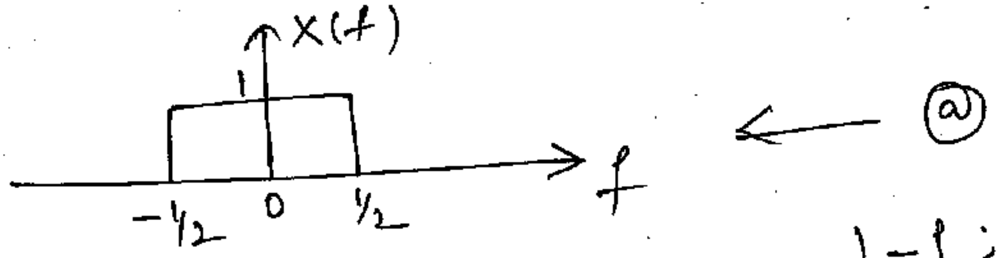
$$x(t) \longrightarrow \boxed{H\cdot T\left(\frac{1}{\pi t}\right)} \longrightarrow y(t) = \hat{x}(t) = ?$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

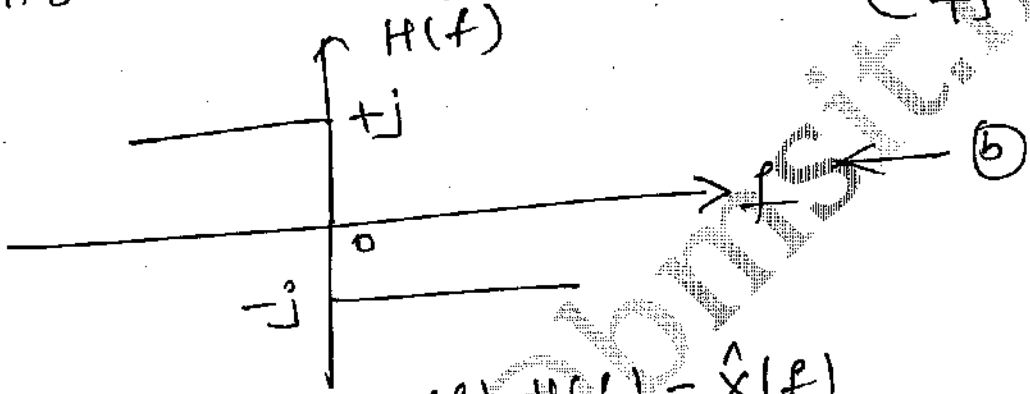
$\Updownarrow$ FT

$$\hat{X}(f) = X(f)\cdot H(f)$$

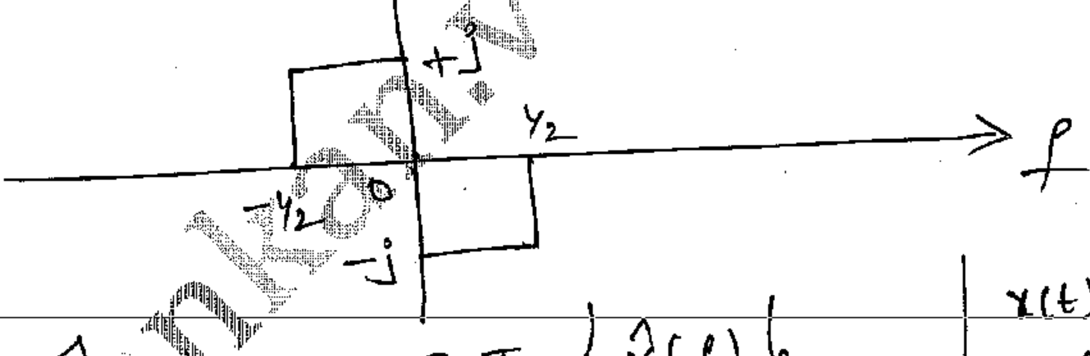
$x(t) = \text{Sinc}(t) \xleftrightarrow{F.T} \text{rect}(f) = X(f)$



$\frac{1}{\pi t} \xleftrightarrow{F.T} -j\,\text{sgn}(f) = H(f) = \begin{cases} -j \,;\, f>0 \\ 0 \,;\, f=0 \\ +j \,;\, f<0 \end{cases}$



$X(f) \cdot H(f) = \hat{X}(f)$



$\hat{x}(t) = \text{I.F.T}\left\{\hat{X}(f)\right\}$

$= \int_{-\infty}^{\infty} \hat{X}(f) \cdot e^{j2\pi ft}\, df$

$= \int_{-1/2}^{0} j\, e^{j2\pi ft}\, df + \int_{0}^{1/2} (-j)\, e^{j2\pi ft}\, df$

$x(t) \xleftrightarrow{F.T} X(f)$

$X(f) \xleftrightarrow{IFT} x(t)$

$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$

$$\hat{x}(t) = \frac{j}{j2\pi t} e^{j2\pi f t}\Big|_{-1/2}^{0} - \frac{j}{j2\pi t} e^{j2\pi f t}\Big|_{0}^{1/2}$$

$$= \frac{1}{2\pi t}\left[1 - e^{-j\pi t}\right] - \frac{1}{2\pi t}\left[e^{j\pi t} - 1\right]$$

$$= \frac{1}{2\pi t}\left[1 - e^{-j\pi t} - e^{+j\pi t} + 1\right]$$

$$\hat{x}(t) = \frac{1}{2\pi t}\left[2 - (e^{j\pi t} + e^{-j\pi t})\right]$$

$$\hat{x}(t) = \frac{1}{\pi t}\left[1 - \frac{1}{2}(e^{j\pi t} + e^{-j\pi t})\right]$$

$$\boxed{\hat{x}(t) = \frac{1}{\pi t}\left[1 - \cos\pi t\right]}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$1 - \cos\theta = 2\sin^2\theta/2$$

$$\boxed{\text{sinc}\, t = \frac{\sin\pi t}{\pi t} \xleftrightarrow{H\cdot T} \frac{1}{\pi t}(1 - \cos\pi t) = \sin\left(\frac{\pi t}{2}\right)\text{sinc}\left(\frac{t}{2}\right)}$$

Illy from above result $\frac{\sin t}{t} \xleftrightarrow{H\cdot T} \frac{1 - \cos t}{t}$

further

$$= \frac{1}{\pi t}(1 - \cos\pi t) = \frac{1}{\pi t}\cdot 2\sin^2(\pi t/2)$$

$$= \frac{\sin^2(\pi t/2)}{(\pi t/2)} = \sin(\pi t/2)\,\frac{\sin(\pi t/2)}{(\pi t/2)}$$

$$\boxed{\hat{x}(t) = \sin(\pi t/2)\cdot\text{sinc}(t/2)}$$

✗ properties of Hilbert transform J|J 2018 Jan 2020

i. Hilbert transform is applicable to any signals that is fourier transformable.

ii. Hilbert transform differs from Fourier transform that it operates exclusively in time domain. (i.e it doesnot changes the domain of the signal).

iii. Since the domain doesnot change ∴ $\hat{x}(t)$ is not an equivalent representation of $x(t)$ i.e both are not Equivalent (completely different), rather it is entirely different signal.

In T.D

iv. Hilbert transformer is a −90° phase shifter without changing its amplitude.

v. In F.D if we X^{ℓ} +ve spectrum of $x(t)$ [i.e +ve frequency component of $X(f)$] by $-j$ and negative spectrum of $x(t)$ (i.e negative frequency of $X(f)$) by $+j$, we get Hilbert transform of $x(t)$ [i.e $\hat{x}(t)$] in frequency domain [i.e $F.T\{\hat{x}(t)\}$] take inverse Fourier transform to get $\hat{x}(t)$.

i.e
$$\underset{\downarrow \text{take IFT } \hat{x}(t) \checkmark}{\hat{X}(f)} = X(f)\cdot[-j\,\mathrm{sgn}(f)] = \begin{cases} -j\,X(f) & : f>0 \\ +j\,X(f) & : f<0 \end{cases}$$

vi. Ho Signal $x(t)$ and H.T $\hat{x}(t)$ have the Same amplitude Spectrum. ie $|X(f)| = |\hat{X}(f)|$

vi. Hilbert transform of an odd Signal is Even and H.T of Even signal is odd.

vii. if $\hat{x}(t)$ is the H.T of $x(t)$, then H.T of $\hat{x}(t)$ is $-x(t)$.

$$\text{i.e}\quad x(t) \overset{\text{H.T}}{\longleftrightarrow} \overset{-90^\circ \phi \text{ shift}}{\hat{x}(t)} \overset{\text{H.T}}{\longleftrightarrow} \overset{-180^\circ \phi \text{ shift}}{-x(t)}$$

$$\text{i.e}\quad \boxed{\hat{\hat{x}}(t) = -x(t)}$$

viii. $x(t)$ and $\hat{x}(t)$ are orthogonal.

$$\text{i.e}\quad \int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0.$$

ix. Energy/power Contained in any Signal $x(t)$ and E/P in $\hat{x}(t)$ are Same.
[i.e Autocorrelation fn, Energy Spectral density (ESD), power Spectral density PSD of Signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are Same].

x. the Hilbert transform of derivative of any Signal is equal to derivative of Hilbert transform of that Signal.

$$\text{i.e}\quad HT\left\{\frac{dx(t)}{dt}\right\} = \frac{d}{dt}\left\{HT[x(t)]\right\}.$$

Note:- [In Exam property 4, 6, 7, 8, 9, 10 are preferred]

10

Questions:- prove that a signal $x(t)$ and its Hilbert transform $\hat{x}(t)$ are orthogonal over the entire time interval $(-\infty, \infty)$. Dec/Jan 2018.

proof:- of orthogonal property (property 8).

i.e $\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0.$

LHS :- using parseval's energy theorem [i.e $E_{TD} = E_{F.D}$]

$$\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = \int_{-\infty}^{\infty} X(f)\,\hat{X}^{*}(f)\,df \leftarrow ⓐ$$

$$\hat{X}^{*}(f) = \hat{X}(-f)$$; for real valued signal.

W.k.t $\hat{X}(f) = -j\,\mathrm{sgn}(f)\cdot X(f)$ | $\mathrm{sgn}(-f) = -\mathrm{sgn}(f)$

$$\hat{X}(-f) = -j\,\mathrm{sgn}(-f)\cdot X(-f)$$
$$= +j\,\mathrm{sgn}(f)\,X(-f)$$

∴ eqⁿ ⓐ RHS

$$= \int_{-\infty}^{\infty} X(f)\,\hat{X}(-f)$$

$$= \int_{-\infty}^{\infty} X(f)\,[j\,\mathrm{sgn}(f)\,X(-f)]\,df$$

$$= j\int_{-\infty}^{\infty} X(f)\,X(-f)\,\mathrm{sgn}(f)\,df$$

Note:- $X(-f) = X^{*}(f)$

and $X(f)\,X^{*}(f) = |X(f)|^2$

$$= j\int_{-\infty}^{\infty} X(f)\,X^*(f)\,\mathrm{Sgn}(f)\,df$$

$$= j\int_{-\infty}^{\infty} \underbrace{|X(f)|^2}_{\text{even } f^n}\,\underbrace{\mathrm{sgn}(f)}_{\text{odd } f^n}\cdot df$$

Area under odd f^n is Zero.

$e \times o = 0$

$\int_{-\infty}^{\infty} \text{odd } f^n = 0$

$$= j\int_{-\infty}^{\infty} (e\times o)\,df = 0$$

$$= j\int_{-\infty}^{\infty} (\text{odd } f^n)\,df = j(0) = \underline{0}.$$

∴ $\boxed{\int_{-\infty}^{\infty} x(t)\,\hat{x}(t)\,dt = 0}$ proved.

Q. prove that Energy of $\hat{x}(t)$ and $x(t)$ are Same.

Soln:- property 9. proof:-

$x(t) \xleftrightarrow{H.T} \hat{x}(t)$ using parseval's Energy theorem.

if $E_{x(t)} = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df \leftarrow ①$

W.t $E_{\hat{x}(t)} = \int_{-\infty}^{\infty} |\hat{x}(t)|^2\,dt = \int_{-\infty}^{\infty} \{F.T[\hat{x}(t)]\}^2\,df$

$$= \int_{-\infty}^{\infty} |\hat{X}(f)|^2 \, df$$

$$= \int_{-\infty}^{\infty} |-j\,\mathrm{Sgn}(f) X(f)|^2 \, df$$

$$= \int_{-\infty}^{\infty} |X(f)\,\mathrm{Sgn}(f)|^2 \, df$$

$$|\mathrm{Sgn}(f)|^2 = 1$$
$$|-j|^2 = +1$$
$$|-j\,\mathrm{Sgn}(f)|^2 = +1$$

$$= \int_{-\infty}^{\infty} |X(f)|^2 \, df = \mathcal{E}_{x(t)} \quad (\text{from eq}^{n}\ ①)$$

i.e $\boxed{\mathcal{E}_{x(t)} = \mathcal{E}_{\hat{x}(t)}}$ Joules.

property 11.

let $c(t)$ – high frequency Signal
$m(t)$ – Low frequency Signal

then
$$H.T\{m(t)\,c(t)\} = m(t)\,H.T\{c(t)\}$$
$$= m(t)\,\hat{c}(t)$$

ie Hilbert transform doesnot act on Low frequency Signals.

Find H.T following $x(t)$.

i. $x(t) = t^3/_3 \cos\omega_c t$.

Sln: w.k.t $\underset{\uparrow_{LF}}{m(t)}\,\underset{\uparrow_{H.F}}{c(t)} \xleftrightarrow{H.T} m(t)\,\hat{c}(t)$.

ie Hilbert transform doesnot act on Low frequency Signals.

$$HT\left\{\underset{\uparrow_{L.F}}{\boxed{\frac{t^3}{3}}}\,\underset{\uparrow_{H.F}}{\cos\omega_c t}\right\} = \frac{t^3}{3}\,H.T\{\cos\omega_c t\}$$

$$= \frac{t^3}{3}\cos(\omega_c t - 90^\circ)$$

$$= \frac{t^3}{3}\sin\omega_c t$$

ii. $x(t) = \sin\omega t + \cos 2\omega t$.

$\updownarrow$ H.T

$\hat{x}(t) = \sin(\omega t - 90^\circ) + \cos(2\omega t - 90^\circ)$

$$\boxed{\hat{x}(t) = -\cos\omega t + \sin(2\omega t)}$$

Note:
$j = 1\angle 90^\circ = e^{j\pi/2}$
$-j = 1\angle -90^\circ = e^{-j\pi/2}$

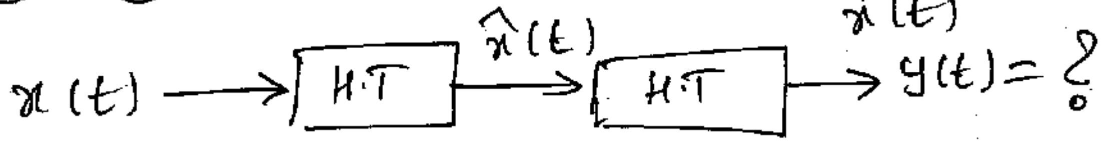
iii) $x(t) = \frac{4}{t}$

$$x(t) \longrightarrow \boxed{H.T} \longrightarrow \hat{x}(t) = x(t) \text{ with } -90^\circ \text{ phaseshift}$$

$= x(t - \pi/2)$

$$\therefore\ \hat{x}(t) = \frac{4}{t}\angle -9^0 = \frac{4}{t}[-j] = \frac{4}{t}e^{-j\pi/2}$$

iv. H.T of H.T ie find H.T of $\hat{x}(t)$.



$$\hat{x}(t) = x(t) * h(t)$$

$$y(t) = \hat{\hat{x}}(t) = \hat{x}(t) * h(t)$$

$\Updownarrow$ F.T $\qquad$ $\hat{X}(f) = X(f) \cdot H(f)$

$$\hat{\hat{X}}(f) = \hat{X}(f) \cdot H(f)$$

$$= [-j\, sgn(f) \cdot X(f)] \cdot [-j\, Sgn(f)]$$

$$\hat{\hat{X}}(f) = +j^2 \, \underset{1}{\cancel{Sgn^2}}(f)\, X(f) \qquad \left|\; \begin{array}{l} j^2 = -1 \\ Sgn^2(f) = 1 \end{array}\right.$$

$$\hat{\hat{X}}(f) = -X(f)$$

$\Updownarrow$ I.H.T

$$\boxed{\hat{\hat{x}}(t) = -x(t)}$$

Thus H.T of H.T gives −ve of original signal ie −180° phase shift.

v. $x(t) = \cancel{\frac{d}{}}\ \delta(t)$

$\delta(t) \xleftrightarrow{F.T} 1 = X(f)$

$$H.T\{\delta(t)\} = \delta(t) * \frac{1}{\pi t}$$

$$= \frac{1}{\pi t}$$

w.k.t

$\delta(t) * h(t) = h(t)$

ⓐ $\hat{X}(f) = X(f) \cdot [-j\,\mathrm{sgn}(f)]$

$\hat{X}(f) = 1 \cdot [-j\,\mathrm{sgn}(f)]$

$\updownarrow$ I.F.T

$\boxed{\hat{x}(t) = 1/\pi t}$

vi. $x(t) = \delta'(t)$.

using property $\hat{x}(t) = -x(t)$ property 10.

w.k.t $\overset{x(t)}{\delta(t)} \xleftrightarrow{H.T} \overset{\hat{x}(t)}{\frac{1}{\pi t}}$

$\frac{dx(t)}{dt} \xleftrightarrow{H.T} \frac{d}{dt}[H.T\ x(t)]$

$$\frac{d\delta(t)}{dt} = \delta'(t) \xleftrightarrow{H.T} \frac{d}{dt}\left[H.T\{\delta(t)\}\right]$$

$$= \frac{d}{dt}\left\{\frac{1}{\pi t}\right\} = \frac{-1}{\pi t^2}$$

$$\therefore \boxed{\delta'(t) \xleftrightarrow{H.T} \frac{-1}{\pi t^2}}$$

vii. H.T of $\frac{1}{\pi t^2}$

w.k.t $\overset{\text{let } x(t)}{\delta'(t)} \xleftrightarrow{H.T} \overset{\hat{x}(t)}{-1/\pi t^2}$

using $\hat{x}(t) \xleftrightarrow{H.T} -x(t)$.

$$\therefore \{-1/\pi t^2\} \xleftrightarrow{H.T} -\delta'(t)$$

$$\therefore \boxed{1/\pi t^2 \xleftrightarrow{H.T} \delta'(t)}$$

Note:- property Convolutions -

i. $y(t) = x(t) * h(t)$

ii. $y'(t) = x'(t) * h(t)$

iii. $y'(t) = x(t) * h'(t)$

iv. $y''(t) = x'(t) * h'(t) = x''(t) * h(t) = x(t) * h''(t)$

$x' = d/dt$
$x'' = d^2/dt^2$

viii. H.T of $\delta''(t)$

w.k.t $\delta(t) \xleftrightarrow{H.T} \frac{1}{\pi t}$

$\delta'(t) \xleftrightarrow{H.T} -1/\pi t^2$

$$H.T\{\delta''(t)\} = \frac{d}{dt}\left[H.T(\delta'(t))\right]$$

$$d/dt\left[-1/\pi t^2\right] = 2/\pi t^3$$

$$\boxed{\delta''(t) \xleftrightarrow{H.T} +\frac{2}{\pi t^3}}$$

ix. H.T of $\delta'''(t)$

$$\delta'''(t) = d/dt\left[\delta''(t)\right] \xleftrightarrow{H.T} \frac{d}{dt}\left[H.T\{\delta''(t)\}\right]$$

$$= d/dt\left[2/\pi t^3\right] = -6/\pi t^4$$

$$\boxed{\delta'''(t) \xleftrightarrow{H.T} -6/\pi t^4}$$

Note:- H.T of Some Standard functions

T.D		T.D
$x(t)$	$\xleftrightarrow{H.T}$	$\hat{x}(t)$
i. $\sin(2\pi f_c t)$		$-\cos(2\pi f_c t)$
ii. $\cos(2\pi f_c t)$		$+\sin(2\pi f_c t)$
iii. $e^{-j2\pi ft}$		$j e^{-j2\pi ft}$
iv. $\mathrm{Sinc}(t) = \frac{\sin \pi t}{\pi t}$		$\frac{1}{\pi t}\left[1-\cos \pi t\right] = \mathrm{Sinc}(t/2)\sin(\frac{\pi t}{2})$
v. $e^{+j2\pi ft}$		$j e^{+j2\pi ft}$
vi. $\cos 2\pi f_c t + \sin 2\pi f_c t$		$\sin(2\pi f_c t) - \cos(2\pi f_c t)$
Imp vii. $\frac{\sin t}{t}$		$\frac{1-\cos t}{t}$
viii. $\hat{x}(t)$		$-x(t)$
ix. $\delta(t)$		$\frac{1}{\pi t}$
x. $\frac{1}{\pi t}$		$-\delta(t)$
xi. $\delta'(t)$		$\frac{-1}{\pi t^2}$
xii. $\delta''(t)$		$2/\pi t^3$
xiii. $\delta'''(t)$		$-6/\pi t^4$
xiv. $\frac{1}{t}$		$-\pi \delta(t)$
xv. $\underset{LF}{m(t)}\ \underset{H.F}{c(t)}$		$m(t)\,\hat{c}(t)$.

10

Applications of Hilbert Transform Dec 2018/ Jan 2019.

i. it is used in the generation of single sideband (SSB) modulation to realize phase selectivity.

ii. it provides the mathematical basis for the representation of bandpass signals.

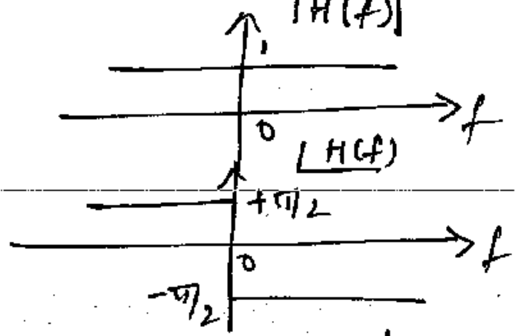
iii. used in Sampling of narrowband signals.

iv. used to represent Analytical signals.

v. Medical imaging.

Characteristics of H.T

$$h(t) = \frac{1}{\pi t} \xleftrightarrow{F.T} -j\,Sgn(f)$$



i. F.R is a All pass filter (APF).

ii. I.R is Linear.

iii. Non-causal

iv. Since h(t) is non-causal ∴ not physically realizable.

v. H.T doesnot alter the magnitude of i/p signal it changes only the phase component by -90°. ∴ it is also called -90° phaseshifter.

Xo. Topic 2.1:- Pre-Envelope

J|J 2018
J|J 2019.

* The pre-envelope of the signal $x(t)$ is defined as the Complex valued function

ie $x_+(t) = x(t) + j\hat{x}(t) \longleftarrow ①$

where $x(t)$ is a real valued signal and $\hat{x}(t)$ is the H.T of $x(t)$.

* The pre-envelope of a signal is useful in handling Bandpass signals and systems.

* the F.T of eqⁿ ①

$$X_+(f) = X(f) + j\hat{X}(f)$$

but $\hat{X}(f) = X(f)[-j\,\mathrm{sgn}(f)]$

$$X_+(f) = X(f) + [-j\,\mathrm{sgn}(f)\,X(f)]\,j$$

$j^2 = -1$
$-j^2 = +1$

$$\boxed{X_+(f) = X(f) + \mathrm{sgn}(f)\,X(f)} \longleftarrow ②$$

$$\mathrm{sgn}(f) = \begin{cases} 1 ; & f>0 \\ -1 ; & f<0 \\ 0 ; & f=0 \end{cases}$$

$$X_+(f) = \begin{cases} 2X(f) & ; f>0 \\ X(0) & ; f=0 \\ 0 & ; f<0 \end{cases} \longleftarrow ③$$

where $X(0)$ is the value of $X(f)$ at frequency $f=0$.

* eq^n ③ shows that the pre-envelope of a signal has "no frequency content" for all negative frequencies (i.e for $f<0$).

* Example :- fig a. shows the triangular amplitude spectrum of a Low pass signal.

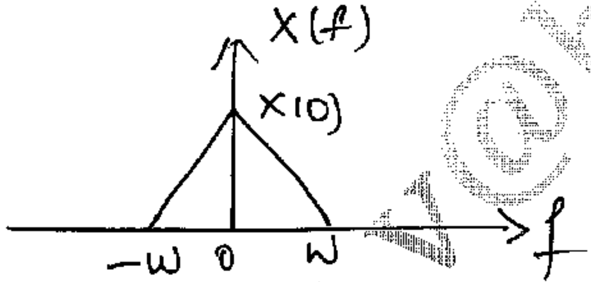


fig a.

* the pre-envelope of amplitude spectrum (fig a). is shown in fig b.

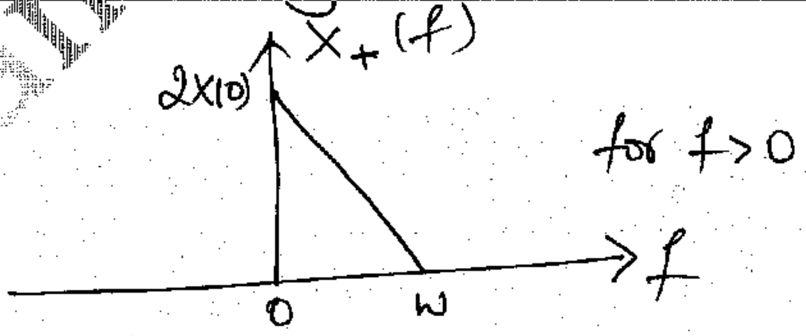


fig b:- pre-envelope of $X(f)$ i.e $X_+(f)$.

Method of obtaining pre-envelope of a signal $x(t)$:-

Method-1 T.D approach

given $x(t)$ its pre envelope is $x_+(t)$.

find H.T of $x(t)$ ie $\hat{x}(t)$.

pre-envelope $\boxed{x_+(t) = x(t) + j\hat{x}(t)}$

Method2 :- Frequency domain approach. (F.D).

given $x(t)$

Step1. find $X(f)$ i.e $x(t) \xleftrightarrow{F.T} X(f)$.

Step2. obtain

$$X_+(f) = \begin{cases} 2X(f) \; ; \text{for } f > 0. \end{cases}$$

Step3 Take I.F.T of $X_+(f)$ ie $x_+(t)$

$$x_+(t) = \int_0^\infty X_+(f)\, e^{j2\pi ft}\, df$$

$$\boxed{x_+(t) = 2\int_0^\infty X(f)\, e^{j2\pi ft}\, df}$$

Note:- i.e. pre-envelope $x_+(t)$ is defined for +ve frequencies [i.e $f>0$].

i.e. lly it is possible to define pre-envelope of negative frequencies as

$$x_-(t) = x(t) - j\hat{x}(t) \longleftarrow ⓐ$$

i.e complex conjugate of eq^n ①

take F.T of eq^n ⓐ

$$X_-(f) = X(f) - j\hat{X}(f)$$

$$= X(f) - j[-j\,sgn(f)\,X(f)]$$

$$\boxed{X_-(f) = X(f) - sgn(f)\,X(f)}$$

$$X_-(f) = \begin{cases} 0 \,; & f>0 \\ X(0) \,; & f=0 \\ 2X(f) \,; & f<0 \end{cases} \longleftarrow ⓑ$$

where $X(0)$ is the value of $X(f)$ at frequency $f=0$.

eq^n ⓑ shows that pre-envelope of a signal has no frequency content for all +ve frequencies (i.e $f>0$)

* the pre-envelopes $x_+(t)$ and $x_-(t)$ are complementary pairs of complex valued signals.

Complex Envelopes of Band pass Signals

Note1:-

* $x(t) \xleftrightarrow{F.T} X(f)$

→ if BW=2W (Band limited) Such as Signals are called narrow band Signals

→ if BW>>2W (Band unlimited) Such as signals are called wide band Signals.

Note2.

* Narrow band Means Signal Bandwidth is Less than (or) equal to bandwidth of Critical band.

Eg:- Audio, Speech Signals, FM, AM, SSB etc.

→ occupy much less frequency Spectrum, less transmitted power and Low data rate ∴ Low Speed of transmission.

Note3

* Wide band Signals:-

→ requires higher Bandwidth.

→ high data rate

→ ∴ high Speed of Communication.

Eg:- Direct Spread Spectrum (DSS), OFDM. etc.

Note4. Frequency shift property

if $x(t) \xleftrightarrow{F.T} X(f)$

then $x(t) e^{j2\pi f_c t} \xleftrightarrow{F.T} X(f-f_c)$ ← defined for +ve frequency $f>0$

$x(t) e^{-j2\pi f_c t} \xleftrightarrow{F.T} X(f+f_c)$ defined for -ve frequency $f<0$

(3)

Topic 2.2 Complex Envelope :-

* The pre-envelope of a narrow band signal $x(t)$ with its Fourier transform $X(f)$ Centered about frequency $\pm f_c$ is given by

$$x_+(t) = x'(t)\, e^{j2\pi f_c t} \longleftarrow ①$$

where $x'(t)$ is a complex envelope of the signal defined as

$$x'(t) = x_+(t)\, e^{-j2\pi f_c t} \longleftarrow ②$$

taking F.T $\quad x_+(t) \xleftrightarrow{F.T} X_+(f) = \begin{cases} 2X(f); f>0 \end{cases}$

and

$$x'(t) = x_+(t)\, e^{-j2\pi f_c t} \xleftrightarrow{F.T} X'(f) = X_+(f+f_c)$$

Example :-

Consider the amplitude Spectrum of a bandpass Signal $x(t)$ with Center frequency f_c.

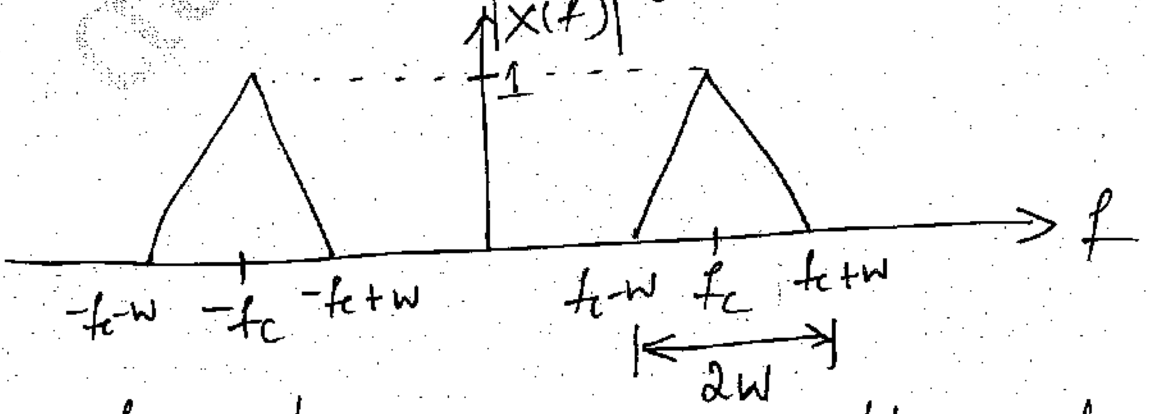


fig:- amplitude Spectrum of Band pass Signal $x(t)$

pre-envelope of $x(t)$ is $x_+(t)$

$$x_+(t) \xleftrightarrow{FT} X_+(f) = \begin{cases} 2X(f) & ; f>0 \\ 0 & ; ow \end{cases}$$

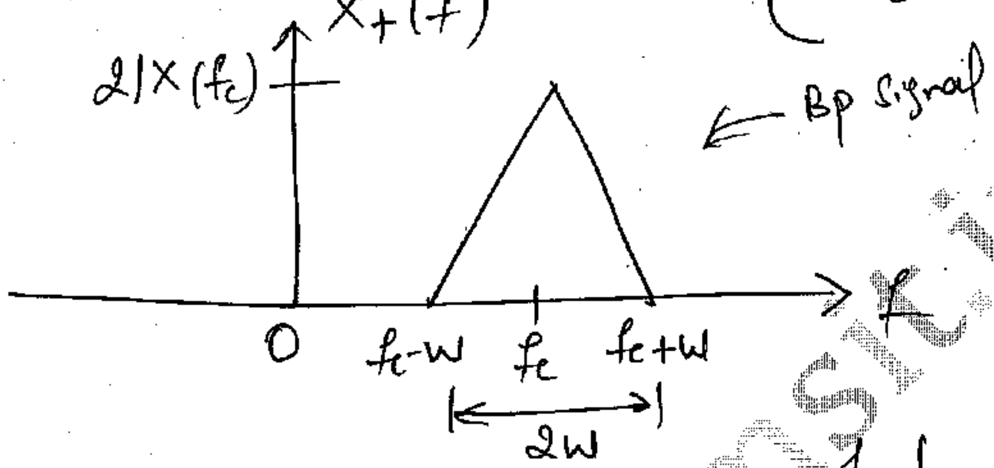


fig b:- amplitude spectrum of pre-envelope $x_+(t)$

* the ~~Comm~~amplitude spectrum of complex envelope $x'(t)$

$$x'(t) = x_+(t)\, e^{-j2\pi f_c t}$$

$$\Big\updownarrow FT$$

$$X'(f) = X_+(f+f_c)$$

i.e Left shift $X_+(f)$ by f_c unit.

$$|X'(f)| = X_+(f+f_c)$$

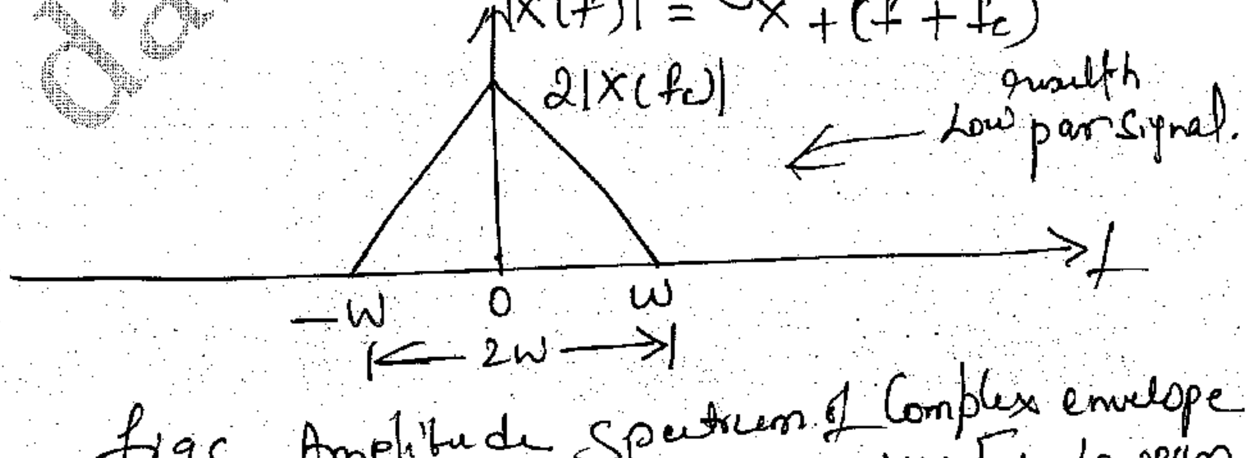
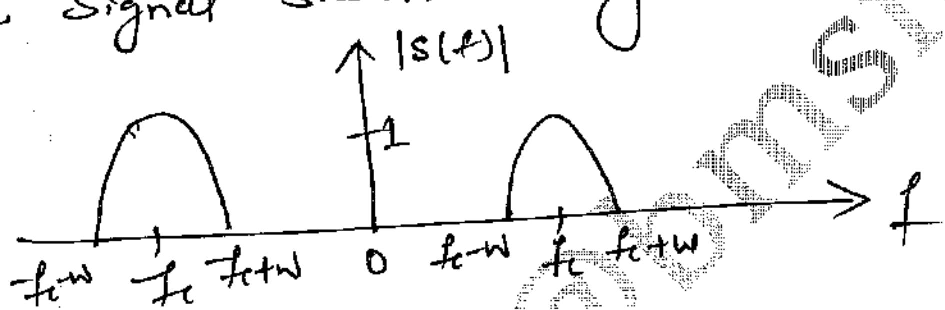


fig c. Amplitude Spectrum of Complex envelope $x'(t)$ [i.e Lowpass Signal]

<u>Note</u>:- Spectrum of the complex envelope $\tilde{x}(t)$ is Limited to the band $-W \leq f \leq W$ i.e. the Complex envelope of $\tilde{x}(t)$ of a bandpass Signal $x(t)$ is a <u>Lowpass Signal</u>.

<u>Question</u>. 1. Det. the pre-envelope and Complex envelope of the Signal shown in fig. J|J 2019.



2. Define pre-envelope of a real valued Signal given a BP Signal $S(t)$. Sketch the amplitude Spectrum of Signal $S(t)$ pre-envelope $S_+(t)$ and Complex envelope $\tilde{S}(t)$. J|J 2018.

<u>Soln</u>:- pre-envelope of a real valued Signal $S(t)$ is given by

$$S_+(t) = S(t) + j\hat{S}(t) \longleftarrow ①$$

Spectrum

$$S_+(f) = \begin{cases} 2S(f) & ; f > 0 \\ S(0) & ; f = 0 \\ 0 & ; f < 0 \end{cases}$$

given a Bandpass Signal S(f)

$$s(t) \xleftrightarrow{F.T} S(f)$$

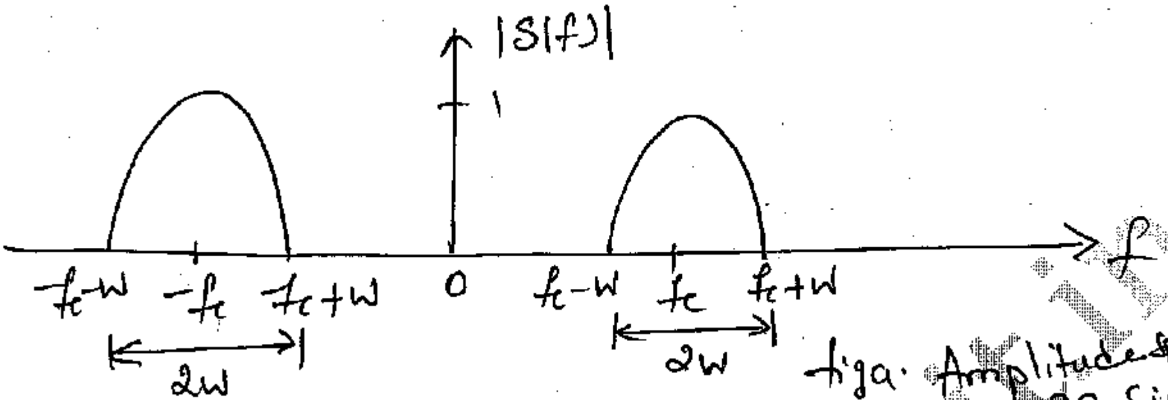


fig a. Amplitude spectrum of BP signal s(t)

* the pre-envelope of S(f) is

$$S_+(f) = \begin{cases} 2S(f) & f>0 \end{cases}$$

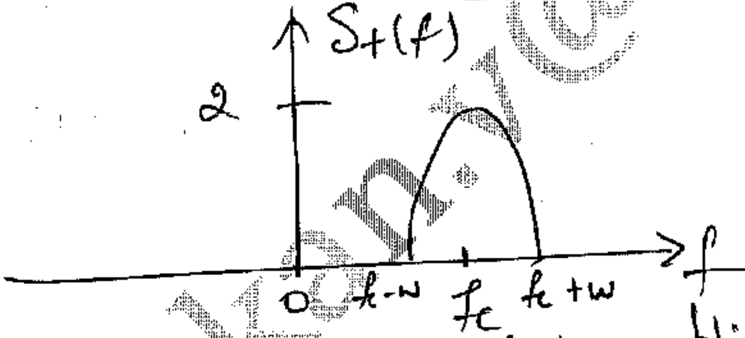


fig b. amplitude spectrum of preenvelope $S_+(f)$

~~* the pre-envelope~~

$$S_+(t) = s'(t)\, e^{j2\pi f_c t}$$

Complex envelope $s'(t) = e^{-j2\pi f_c t} S_+(t)$

$$\updownarrow F.T$$

$$\boxed{S'(f) = S_+(f+f_c)}$$

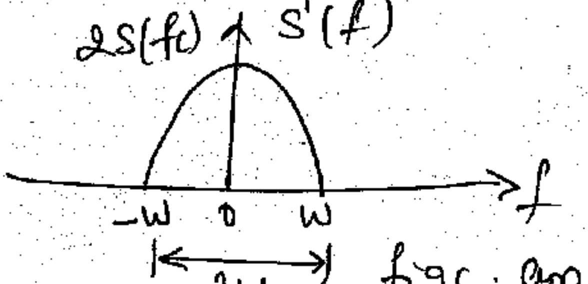


fig c. amplitude spectrum of Complex envelope $S'(f)$.

⑤

Question:

Determine the pre-envelope and complex envelope of the R·F (Radio frequency) pulse defined by ⑹m

$$x(t) = A\,\text{rect}\left(\frac{t}{T}\right) \cos 2\pi f_c t.$$

Jan 2019.

soln:

$$A\,\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{F.T} AT\,\text{sinc}(fT)$$

$$\cos(2\pi f_c t) = \frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}$$

$$A\,\text{rect}\left(\frac{t}{T}\right)\cos(2\pi f_c t) = \frac{A}{2} e^{j2\pi f_c t}\,\text{rect}\left(\frac{t}{2}\right) + \frac{A}{2} e^{-j2\pi f_c t}\,\text{rect}\left(\frac{t}{T}\right)$$

$\updownarrow$ F.T using frequency shift property

$$X(f) = \underbrace{\frac{AT}{2}\,\text{sinc}(f-f_c)T}_{+\text{ve frequency } (f>0)} + \underbrace{\frac{AT}{2}\,\text{sinc}(f+f_c)T}_{-\text{ve frequency } (f<0)}$$

$$X(f) = \begin{cases} \frac{AT}{2}\,\text{sinc}(f-f_c)T & : f>0 \\ 0 & : f=0 \\ \frac{AT}{2}\,\text{sinc}(f+f_c)T & ; f<0 \end{cases}$$

* the Spectrum of preenvelope $x_+(t)$ is given by

$$X_+(f) = \begin{cases} 2X(f) & ; f>0 \\ X(0) & ; f=0 \\ 0 & ; f<0 \end{cases}$$

$$X_+(f) = \begin{cases} AT\,\text{Sinc}(f-f_c)T & : f>0 \\ 0 & : f\le 0 \end{cases}$$

$$\updownarrow \text{I.F.T}$$

the pre-envelope $x_+(t)$ in T.D

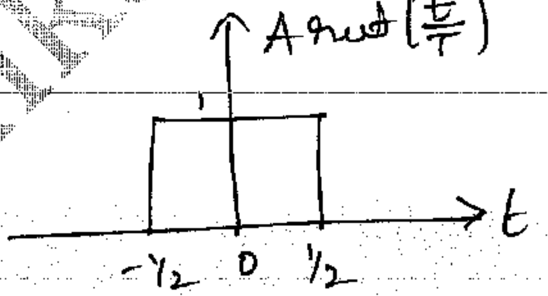
$$x_+(t) = A\,\text{rect}\left(\frac{t}{T}\right) e^{j2\pi f_c t} \longleftarrow ①$$

$$x_+(t) = x'(t)\, e^{j2\pi f_c t} \longleftarrow ②$$

Comparing eq^n ① and eq^n ②

the complex envelope is given by

$$\boxed{x'(t) = A\,\text{rect}\left(\frac{t}{T}\right)}$$



Note :-

1. Any Bandpass Signal can be represented by

$$\boxed{\underset{\text{Bandpass Signal}}{S(t)} = \underset{\text{in-phase Component}}{S_I(t)} \cos w_0 t - \underset{\text{Quadrature Component}}{S_Q(t)} \sin \underset{\text{reference (or) Carrier (or) Center frequency}}{w_0} t}$$

2. Complex envelope of S(t) in terms of inphase & Quadrature Component $S_{CE}(t) = \tilde{S}(t) = S_I(t) + jS_Q(t)$

3. Any bandpass signal can be represented in terms of Complex envelope

$$S(t) = Re\left[S_{CE}(t)\, e^{jw_0 t}\right] = Re\left[\tilde{s}(t)\, e^{jw_0 t}\right]$$

4. In phase and Quadrature refers to two Sinusoidals that have the same frequency and are 90° out of phase. By convention the 'I'-signal is a Cosine waveform and the Q-signal is a Sine waveforms.

Topic 3

Canonical Representation of Bandpass Signals

J|J 2018
Jan 2020 J|J 2019.
Jan 2019

* In general the Bandpass Signal $x(t)$ can be expressed interms of complex envelope $x'(t)$ as

$$x(t) = Re\left\{x'(t)\, \underbrace{e^{j2\pi f_c t}}_{r_1 e^{j\theta_1}}\right\} \longleftarrow ①$$

where $x'(t)$ is a complex-valued {complex envelope given by

$$x'(t) = x_I(t) + j\, \overset{= r_2 e^{j\theta_2}}{x_Q(t)} \longleftarrow ②$$

where $x_I(t)$ and $x_Q(t)$ are both real valued Lowpass functions.

using eq^n ② in eq^n ①

$$x(t) = Re\left\{[x_I(t) + j x_Q(t)][\cos 2\pi f_c t + j \sin 2\pi f_c t]\right\}$$

$$= Re\left\{[x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t] + j[x_Q(t)\cos 2\pi f_c t + x_I(t)\sin 2\pi f_c t]\right\}$$

$$\boxed{x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t} \longleftarrow ③$$

where $x_I(t)$ is a in-phase component and $x_Q(t)$ is a quadrature component of the Signal.

8. Extracting the inphase and Quadrature Components :-

* The Signal $x_I(t)$ and $x_Q(t)$ are the Lowpass signal with band limited to $-W \le f \le W$. These Signals can be derived from the band pass Signal $x(t)$ as shown in fig a.

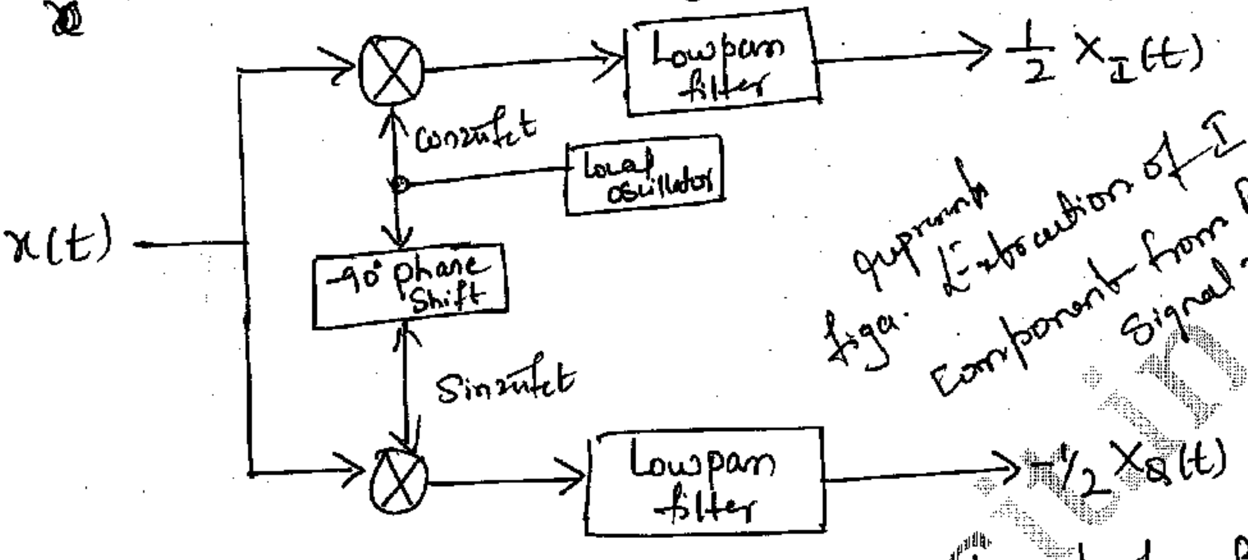
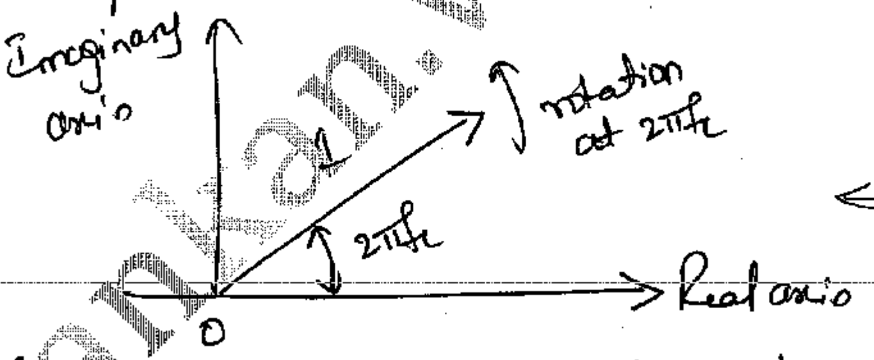


fig a. Extraction of I and Q Component from Band pass Signal x(t).

fig a:- Derivation of x_I and x_Q Component of a Bandpass Signal.

* fig b. shows the phasor representation of the complex exponential $e^{j2\pi f_c t} = 1 \angle 2\pi f_c$ | $re^{j\theta} = r\angle\theta$

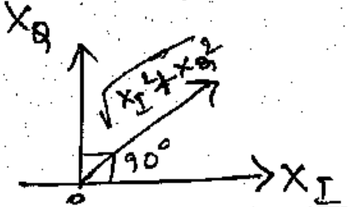


← ⓐ

fig b. phasor Representation of complex exponential.

* from eqⁿ ② $x'(t) = x_I(t) + jx_Q(t) = r\angle\theta$

where $r = \sqrt{x_I^2 + x_Q^2}$, $\theta = \tan^{-1}\left(\frac{x_Q}{x_I}\right)$.



← ⓑ

fig C. Inphase & Quadrature Component of the signal.

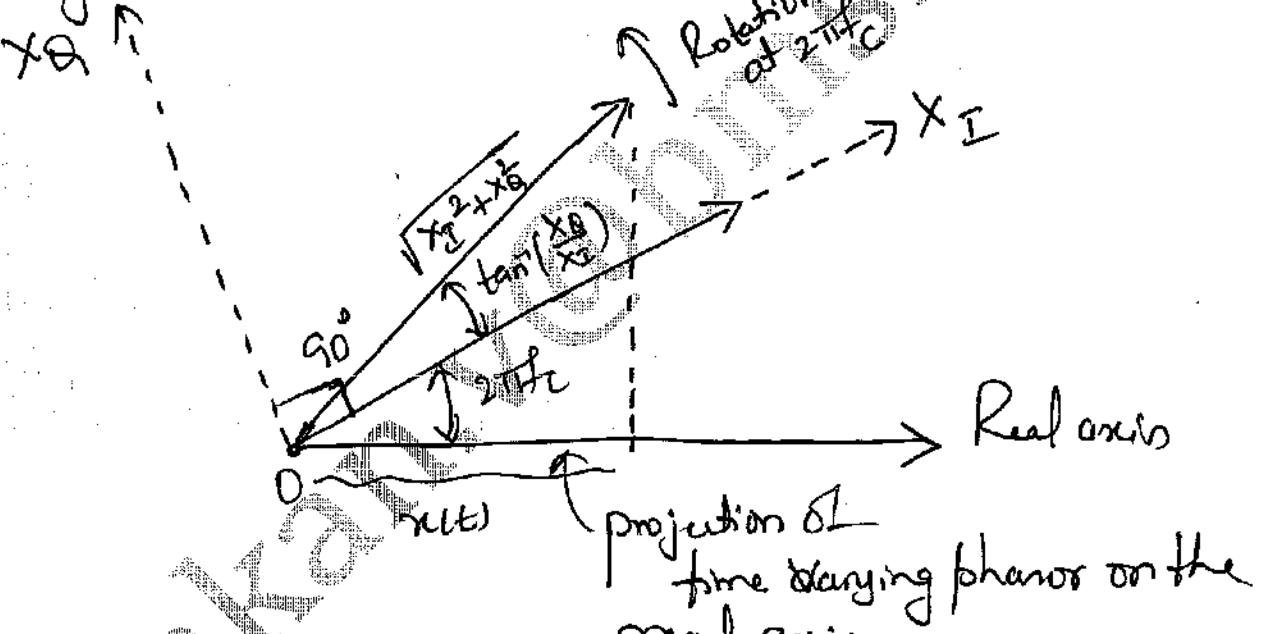
* from eqⁿ ① the term $e^{j2\pi f_c t} = r_1 \angle \theta_1$

and from eqⁿ ② the term $x'(t) = x_I(t) + j x_Q(t) = r_2 \angle \theta_2$

* the phase diagram of $x(t)$ $= \{x'(t)\, e^{j2\pi f_c t}\}$ results $(r_1 \cdot r_2) \angle \theta_1 + \theta_2$

ie the angles of these two phasors ⇒ added and their Lengths Multiplied.

ie figb and figc. are Combinedly shown in fig d.



figd:- Multiplication of complex envelope $x'(t)$ by Complex exponential $e^{j2\pi f_c t}$.

ii. Reconstruction of BP signal $x(t)$:-

* fig e. Shows that block schematic of the Circuit to reconstruction of $x(t)$ from its inphase $x_I(t)$ and Quadrature $X_Q(t)$ Components. the Method used is the Linear Modulation and mapping $x(t)$

from $x_I(t)$ and $x_Q(t)$ is known as panband modulation.

i.e $x(t) = x_I(t) \cos 2\pi f_c t - x_Q(t) \sin 2\pi f_c t$

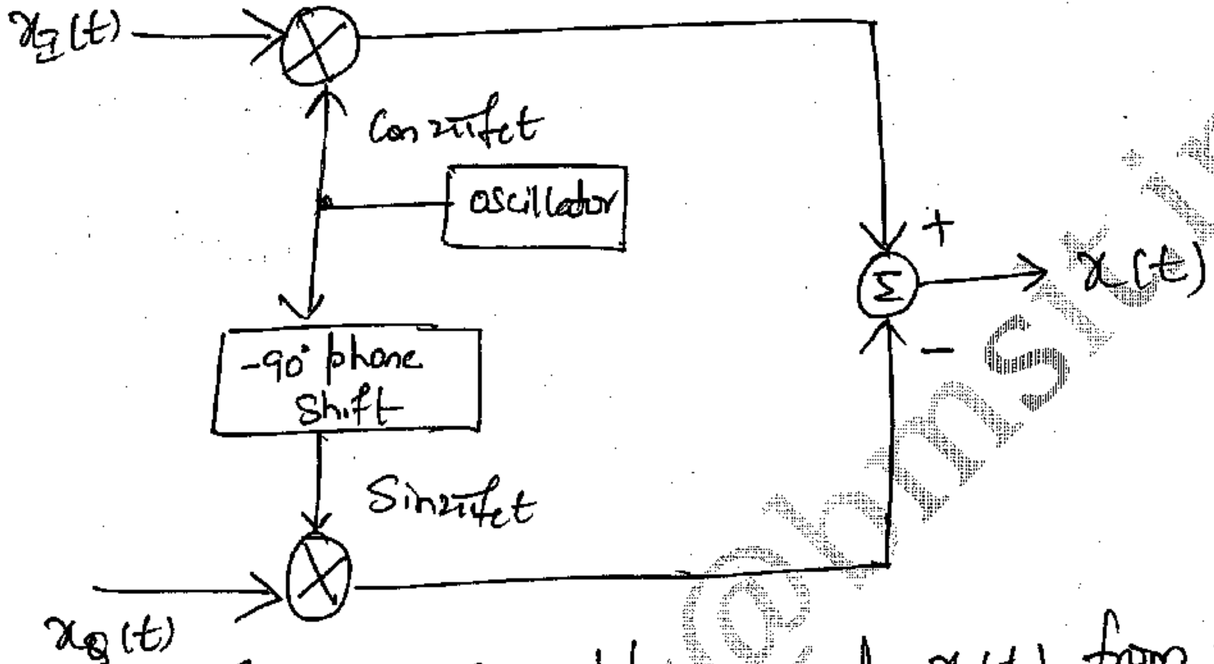


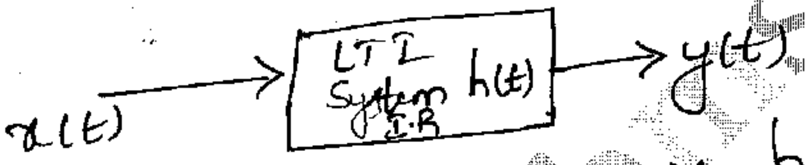
fig. A Bandpass Signal $x(t)$ from in-phase and Quadrature components.

Topic 4.

Complex Lowpass Representation of Bandpass System.

Dec/Jan 2019
J/J 2018
Dec/Jan 2020.

* Let the signal $x(t)$ be applied to Linear time invariant System with impulse response $h(t)$.



* The Bandpass impulse response $h(t)$ can be represented interms of two quadrature Low pass components $h_I(t)$ and $h_Q(t)$ as

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t) \longleftarrow ①$$

* The complex impulse response $h'(t)$ of the bandpass System is represented in terms of quadrature Components as

$$h'(t) = h_I(t) + jh_Q(t) \longleftarrow ②$$

* hence $h(t)$ can be represented interms of $h'(t)$ as

$$h(t) = Re\{h'(t)\ \underbrace{e^{j2\pi f_c t}}_{\text{for +ve frequency}}\} \longleftarrow ③$$

* Let $h'^*(t)$ be complex conjugate of $h'(t)$

$$h(t) = Re\{h'^*(t)\, \underbrace{e^{-j2\pi f_c t}}_{\text{for -ve frequency}}\} \leftarrow (4)$$

adding equations (3) and eq^n (4)

$$2h(t) = h'(t)\, e^{j2\pi f_c t} + h'^*(t)\, e^{-j2\pi f_c t} \leftarrow (5)$$

Note:- [imaginary part of eq^n (3) + eq^n (4) are cancelled due to addition]

$\downarrow$ F.T

Note:-
1. $e^{\pm j2\pi f_c t}\, x(t) \xleftrightarrow{F.T} X(f \mp f_c)$ frequency shift property
2. for real valued signals $H^*(f) = H(-f)$
3. $h(t) \leftarrow$ IR
$H(f) \leftarrow$ FR.

* taking F.T to eq^n (5)

$$2H(f) = H'(f - f_c) + H'(-f - f_c) \leftarrow (6)$$

the above equation for +ve frequency $f > 0$

$$\boxed{\underbrace{H'(f - f_c)}_{\text{Complex Lowpass frequency response}} = \underbrace{2H(f)}_{\text{Bandpass frequency response (FR).}}} \quad \text{for } f > 0 \leftarrow (7)$$

* Thus the Complex Lowpass frequency response $H'(f)$ of Linear time invariant System is obtained by taking bandpass frequency response $H(f)$ for

positive frequencies, shifting it to origin and scaling its amplitude by two.

* Complex Lowpass frequency response can be expressed in the inphase and Quadrature components as

$$H'(f) = H_I'(f) + j\,H_Q'(f) \longleftarrow ⑧$$

where $H_I'(f) = \frac{1}{2}\left[H'(f) + H'^*(-f)\right]$

and $H_Q'(f) = \frac{1}{2j}\left[H'(f) - jH'^*(-f)\right]$

* The Complex impulse response $h'(t)$ of the Bandpass System is obtained by taking I.F.T of eq^n ⑧

i.e $H'(f) \xleftrightarrow{IFT} h'(t)$

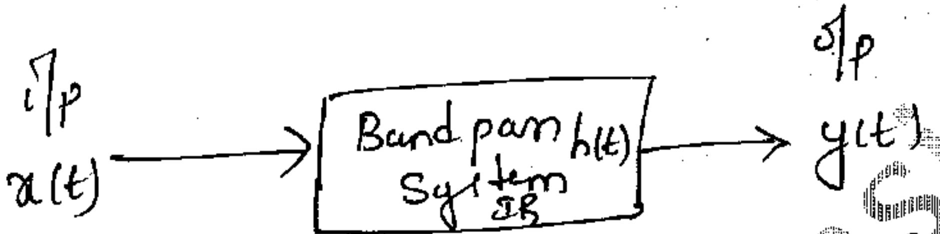
$$\boxed{h'(t) = \int_{-\infty}^{\infty} H'(f)\, e^{j2\pi ft}\, df} \longleftarrow (9)$$

Topic : 5

Complex Representation of Bandpass Signals and System

June/July 2019.

i. Time-domain procedure :-



* The output y(t) of the bandpass system to the input x(t) is related to its impulse response h(t) by a convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau \longleftarrow ①$$

* The above equation can be written in terms of pre-envelopes as follows:

$$y(t) = \int_{-\infty}^{\infty} Re\left[h_+(\tau)\right] \cdot Re\left[x_+(t-\tau)\right] d\tau$$

$$= \frac{1}{2} Re\left[\int_{-\infty}^{\infty} h_+(\tau)\, x_+(t-\tau)\, d\tau\right]$$

(using property of pre-envelope)

using def^n of pre-envelope

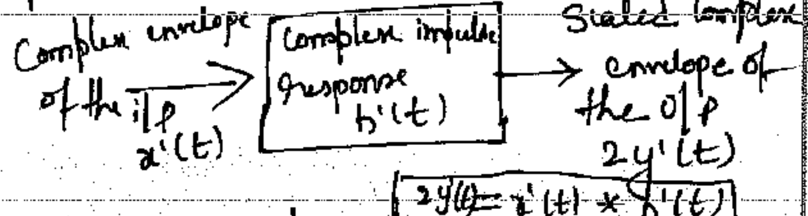
$$= \frac{1}{2} Re\left[\int_{\infty}^{\infty} h'(\tau) e^{j2\pi f_c \tau} x'(t-\tau) e^{j2\pi f_c (t-\tau)} d\tau\right]$$

$$y(t) = \frac{1}{2} Re\left[e^{j2\pi f_c t} \int_{-\infty}^{\infty} h'(\tau) x'(t-\tau) d\tau\right]$$

$$\text{let } y'(t) = \frac{1}{2}\int_{-\infty}^{\infty} h'(\tau) x'(t-\tau) d\tau \leftarrow ⓐ$$

$$\therefore y(t) = Re\{y'(t) e^{j2\pi f_c t}\} \leftarrow ⓑ$$

* The complex envelope $y'(t)$ of the output signal of a bandpass system is equal to Linear convolution of complex envelope of input signal $x'(t)$ and complex impulse response of Linear time invariant System $h'(t)$.



i.e from eq^n ⓐ

$$y'(t) = \frac{1}{2}\int_{-\infty}^{\infty} h'(\tau) x'(t-\tau) d\tau$$

$$2y'(t) = \int_{\infty}^{\infty} h'(\tau) x'(t-\tau) d\tau = x'(t) * h'(t)$$

$$2y'(t) = x'(t) * h'(t) \longleftarrow ⓒ$$

3

* Complex envelope of output signal $y'(t)$ is expressed interms of inphase and quadrature Components as follows.

$$y'(t) = y_I(t) + j\, y_Q(t)$$

by scaling 2

$$2y'(t) = 2y_I(t) + j\, 2y_Q(t) \longleftarrow ⓓ$$

from eqⁿ ⓒ

$$\text{i.e } 2y'(t) = x'(t) * h'(t) = h'(t) * x'(t)$$

$$= [h_I(t) + j\, h_Q(t)] * [x_I(t) + j\, x_Q(t)]$$

$$= h_I(t) * x_I(t) + j\, x_Q(t) * h_I(t) + j\, h_Q(t) * x_I(t) - h_Q(t) * x_Q(t)$$

$$2y'(t) = [h_I(t) * x_I(t) - h_Q(t) * x_Q(t)] + j\,[x_Q(t) * h_I(t) + h_Q(t) * x_I(t)] \longleftarrow ⓔ$$

Comparing eqⁿ ⓓ and eqⁿ ⓔ

$$\left.\begin{aligned} 2y_I(t) &= h_I(t) * x_I(t) - h_Q(t) * x_Q(t) \\ 2y_Q(t) &= x_Q(t) * h_I(t) + h_Q(t) * x_I(t) \end{aligned}\right\} \longleftarrow ⓕ$$

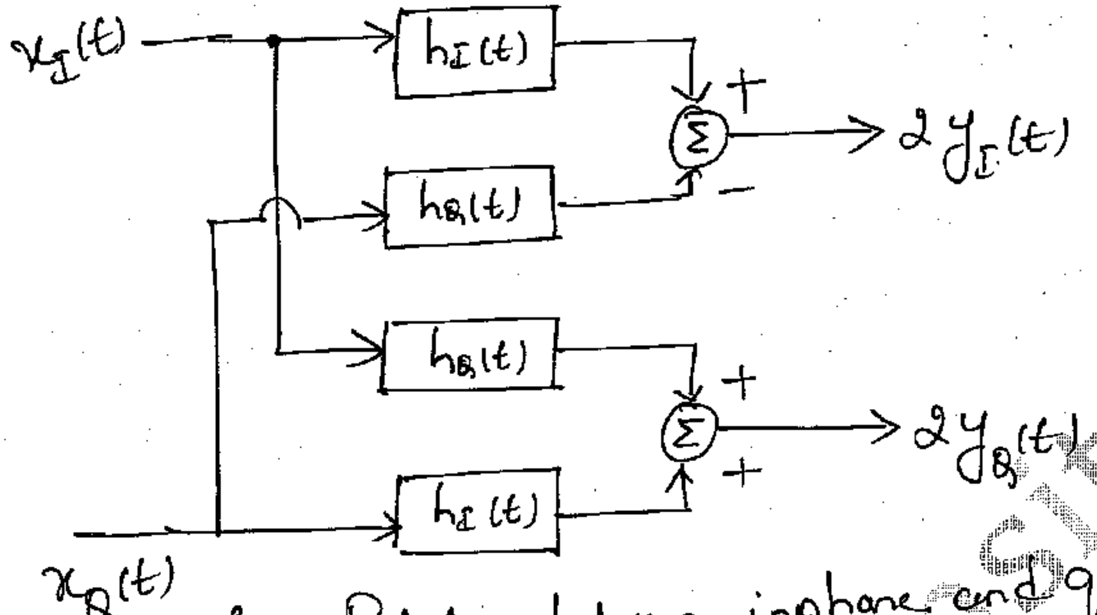


fig:- Relation between inphase and quadrature components of the response of a Bandpass filter. [eq^n ⓕ]

ii. Frequency Domain procedure:-

* Given the frequency response $H(f)$ of a Bandpass System, Computation of the output signal $y(t)$ of the System in response to an input bandpass signal $x(t)$ is summarized as

i.e given $x(t)$ & $H(f)$ find $y(t) = ?$.

step1. using equation $H'(f-f_c) = 2H(f)$; for $f>0$ determine $H'(f)$.

Step2.
Express the input Bandpass Signal $x(t)$ in the canonical form of
i.e $x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$
and Evaluate Complex envelope
ie $x'(t) = x_I(t) + j\, x_Q(t)$ where $x_I(t)$ is the inphase component of $x(t)$ and $x_Q(t)$ is the quadrature component.

$$x'(t) \xleftrightarrow{FT} X'(f)$$

i.e $X'(f) = FT\{x'(t)\}$

Step3. using eqn $2y'(t) = x'(t) * h'(t)$

$$\big\updownarrow FT$$

$$2Y'(f) = X'(f)\cdot H'(f) \leftarrow ⓐ$$

Step4.
from eqn ⓐ Evaluate $Y'(f)$ and find its IFT
i.e $y'(t)$. $\quad Y'(f) \xleftrightarrow{IFT} y'(t)$

Step5. the output signal $y(t)$ is obtained by using

$$y(t) = Re\{y'(t)\, e^{j2\pi f_c t}\}$$

to compute the desired output Signal.

Topic 6 :- Line Codes

Definition :- J/J 2018

The digital data [binary 1's or 0's] can be represented by different formats (or) waveforms. These waveforms are commonly known as digital data formats (or) their representation is called as ~~line~~ "Line coding".

* The Various Line coding techniques are

- → Unipolar NRZ
- → Unipolar RZ
- → polar NRZ
- → polar RZ
- → Bipolar NRZ (Alternate Mark Inversion AMI)
- → Bipolar RZ
- → Split phase Manchester

a. Unipolar NRZ :- Non Return to Zero

one polarity

* In Unipolar NRZ format, when Symbol '1' is to be transmitted, the signal has 'A' volts for full duration.

* when Symbol '0' is to be transmitted, the signal has zero volt's for complete Symbol duration.

i.e for symbol '1'

$$x(t) = A \quad \text{for} \quad 0 \le t \le T_b$$

for Symbol '0'

$$x(t) = 0 \quad \text{for} \quad 0 \le t \le T_b$$

* In NRZ, format the pulse donot return to zero on its own, if symbol '0' is to be transmitted, then pulse become zero.

b. Unipolar RZ :- Return to zero

for symbol '1'

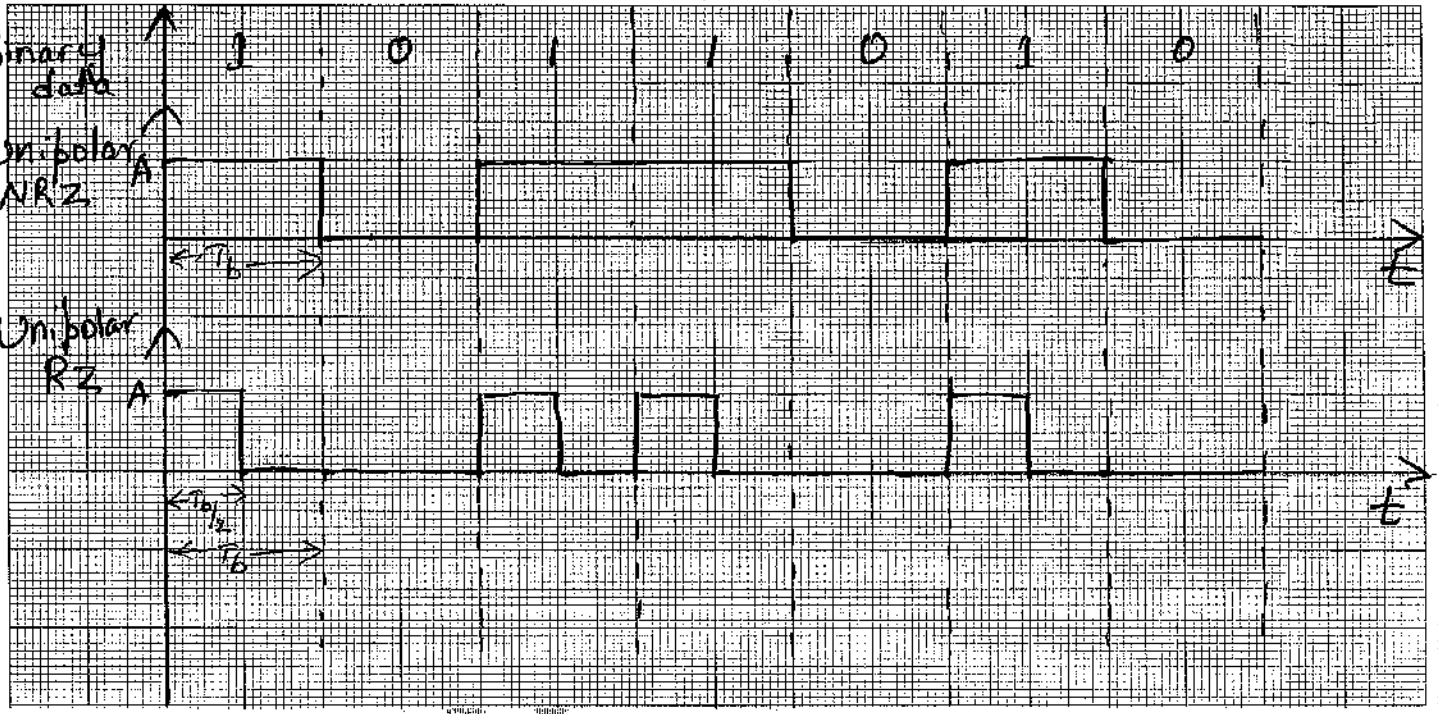
$$x(t) = \begin{cases} A; & 0 \le t \le T_b/2 \quad (\text{half interval}) \\ 0; & T_b/2 \le t \le T_b \end{cases}$$

for symbol '0'

$$x(t) = \{ 0; \quad \text{for } 0 \le t \le T_b \quad (\text{complete interval})$$

* In the Unipolar RZ format, the waveform has zero value when '0' is transmitted and waveform has 'A' volts when '1' is transmitted.

* In RZ form the 'A' volt's is present for $T_b/2$ period if Symbol '1' is transmitted and for remaining $T_b/2$ waveform returns to zero value.



c. polar NRZ :-

* In polar NRZ format the symbol '1' is represented by positive polarity and symbol '0' is represented by negative polarity. These polarities are maintained over a complete pulse duration.

$$x(t) = \{ A ; \quad \text{for} \quad 0 \le t \le T_b \Rightarrow \text{Symbol '1'}$$

$$x(t) = \{ -A ; \quad \text{for} \quad 0 \le t \le T_b \Rightarrow \text{Symbol '0'}$$

d. polar RZ :-

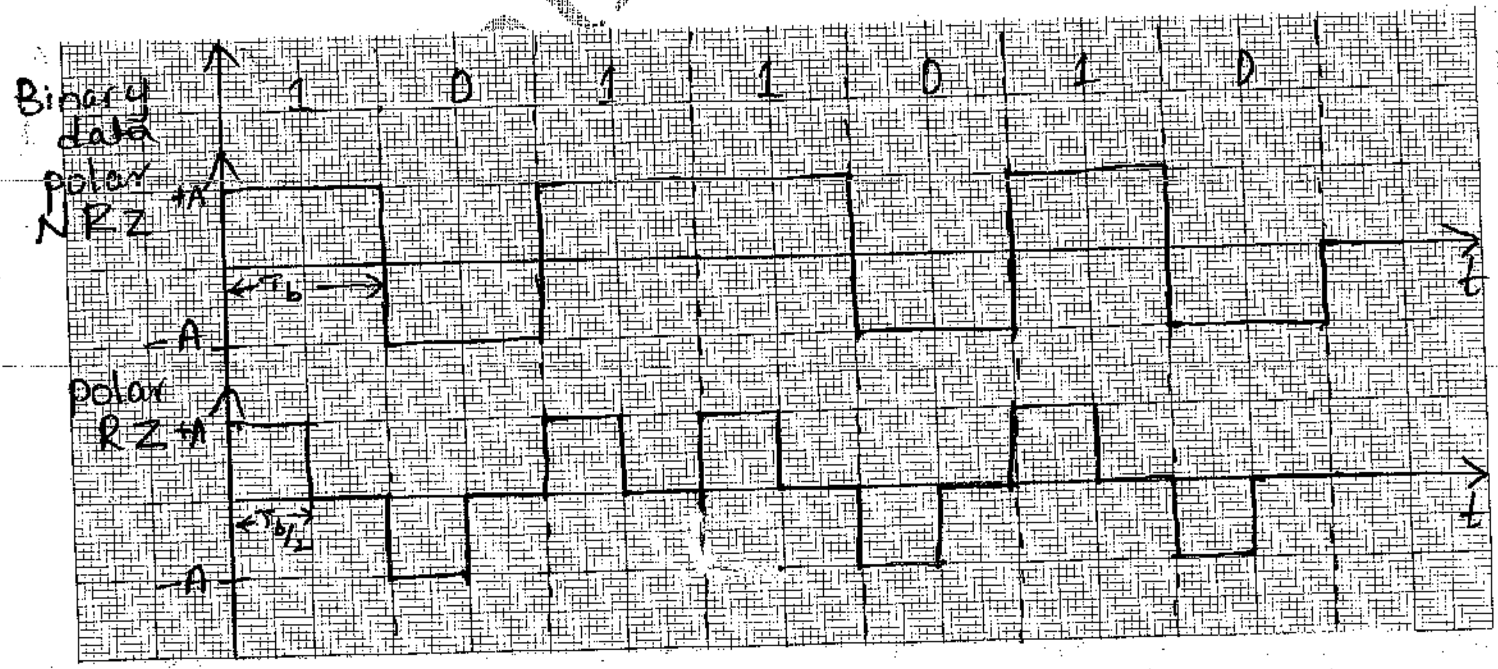
* In polar RZ format, symbol '1' is represented by positive voltage polarity and symbol '0' is represented by negative polarity. Since it is RZ format the pulse is transmitted only for half duration.

* for symbol '1'

i.e $$x(t) = \begin{cases} +A; & \text{for } 0 \le t \le T_b/2 \\ 0; & T_b/2 \le t \le T_b \end{cases}$$

for symbol '0'

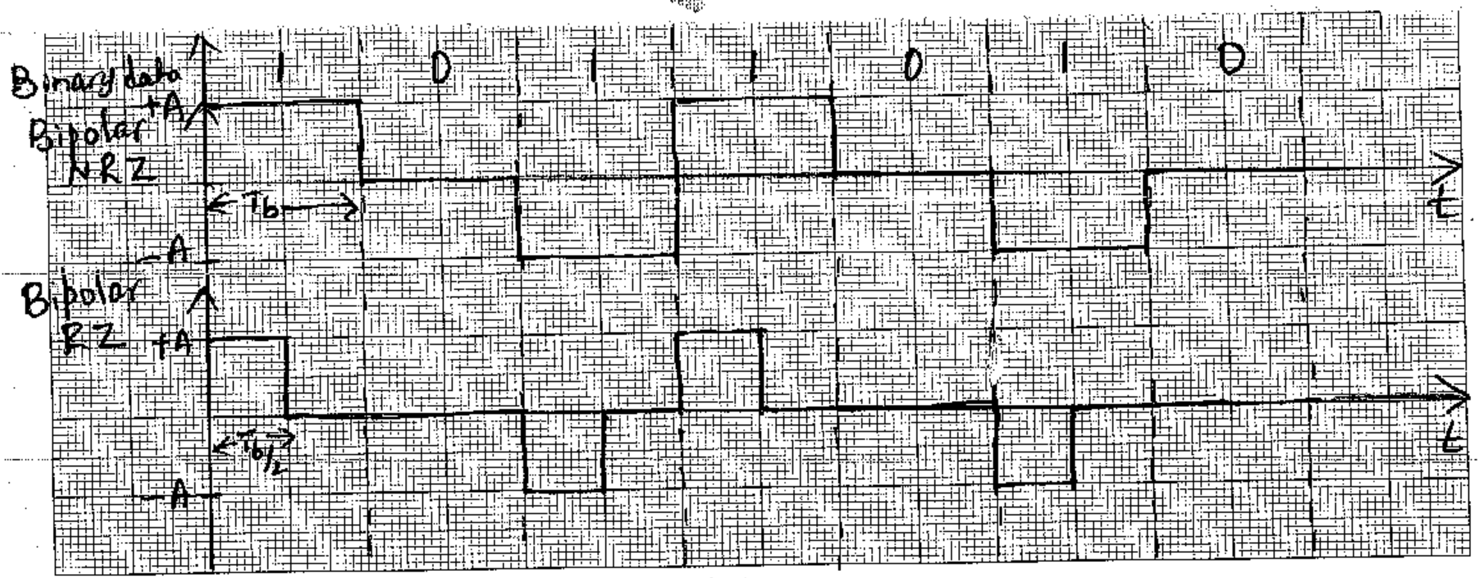
$$x(t) = \begin{cases} -A; & \text{for } 0 \le t \le T_b/2 \\ 0; & T_b/2 \le t \le T_b \end{cases}$$



e. f. Bipolar NRZ and Bipolar RZ

e. Bipolar NRZ :-

- * In this format successive 1's are represented by pulses with alternate polarity and 0's are represented by no pulses.
- * This method is also called "Alternate mark Inversion" (AMI)
- * if there are Even no of 1's the dc component of the waveform will be Zero.
- * The advantage of this format is that the ambiguities due to transmission sign inversion are Eliminated.



f. Bipolar RZ :- In this format successive 1's are represented by pulses with alternate polarity upto $0 \le t \le T_b/2$ and 0's are represented by no pulses.

g. Split Manchester :-

* if Symbol 1 is to be transmitted, then a positive half interval pulse is followed by a negative half interval pulse.

* if symbol '0' is to be transmitted. then a negative half interval pulse is followed by a positive half interval pulse.

* Thus for any symbol the pulse takes both +ve aswell as -ve values.

i.e Symbol '1'

$$x(t) = \begin{cases} A & : \text{for } 0 \leq t \leq T_b/2 \\ -A & ; T_b/2 \leq t \leq T_b \end{cases}$$

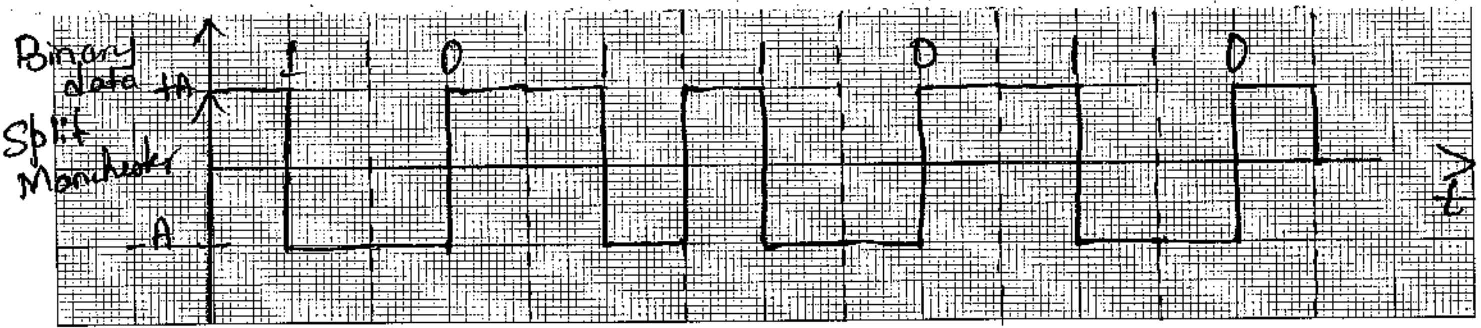
high to Low

Symbol '0'

$$x(t) = \begin{cases} -A & \text{for } 0 \leq t \leq T_b/2 \\ +A & : T_b/2 \leq t \leq T_b \end{cases}$$

Low to high.

* The main advantage of this format is that irrespective of the probability of occurence of symbols 1's and 0's, the waveform has Zero averge ⓡ dc value ∴ this method Saves more power.



Q4. the binary stream 011010 . Sketch

i. Unipolar NRZ

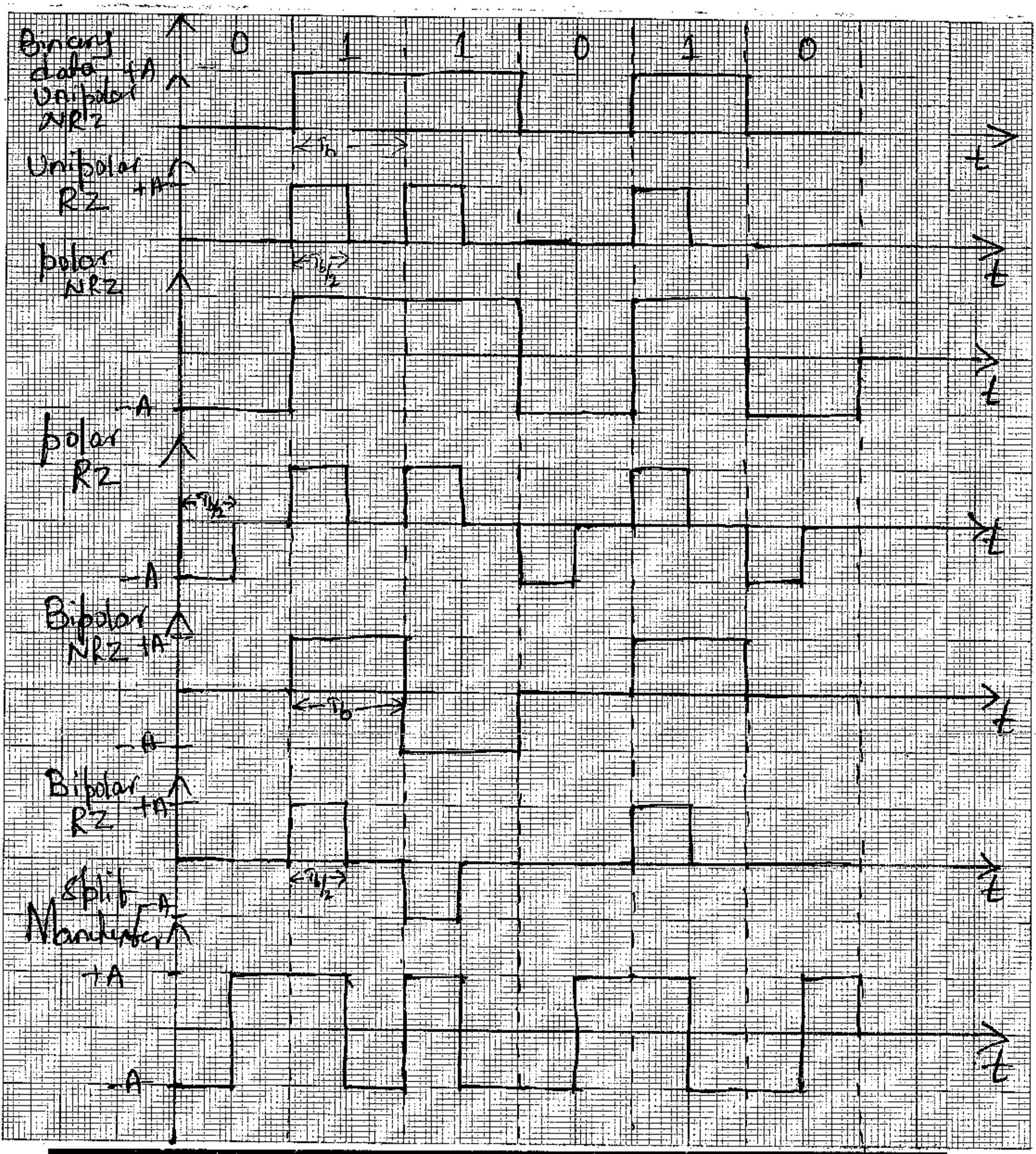
ii. polar NRZ

iii. Unipolar RZ

iv. Bipolar RZ

v. Manchester

J/J 2018
(6m)



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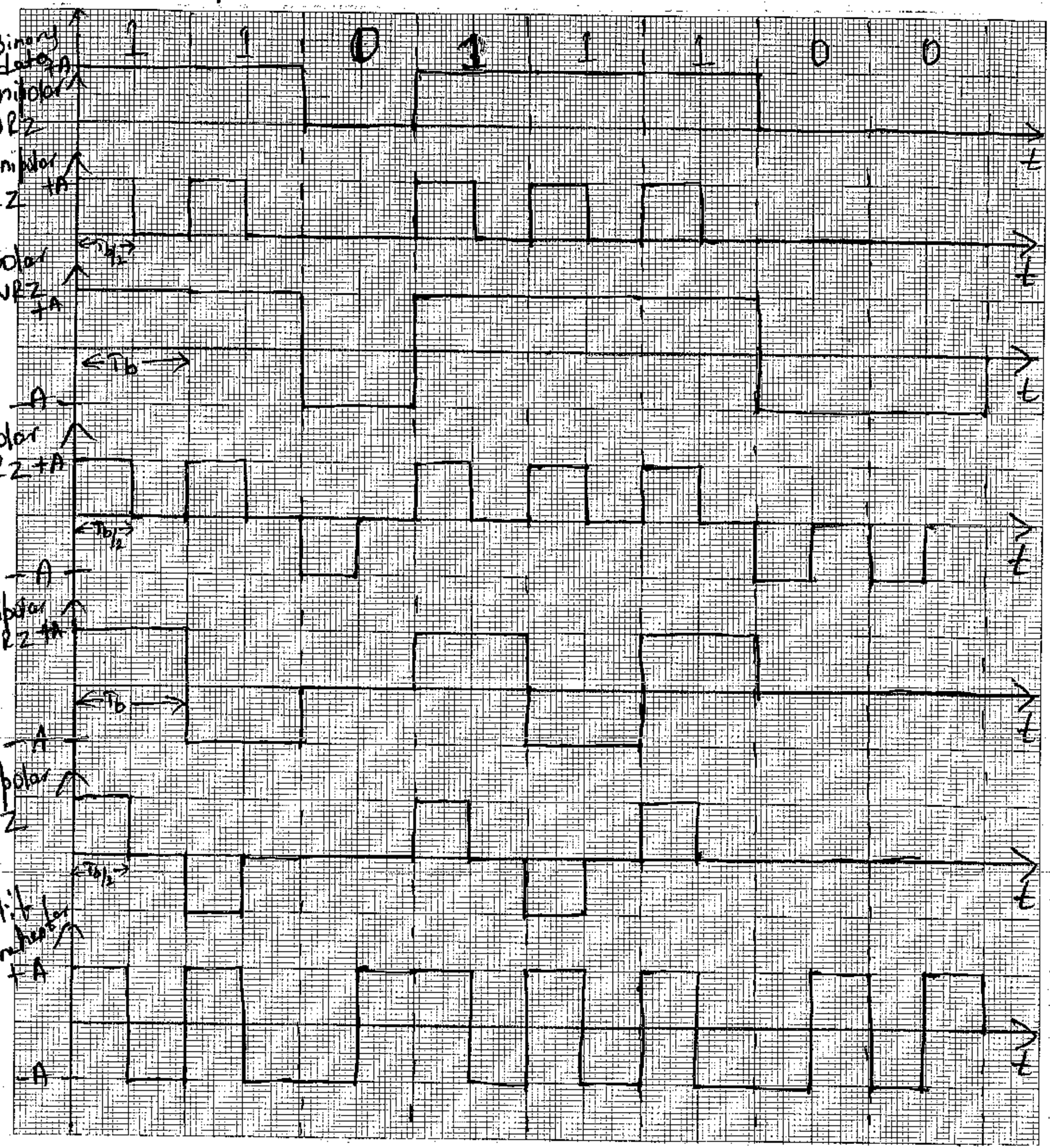
Q5. Binary Stream 11011100. Sketch

i. Unipolar NRZ
ii. polar NRZ
iii. Unipolar RZ
iv. Bipolar NRZ
v. Manchester

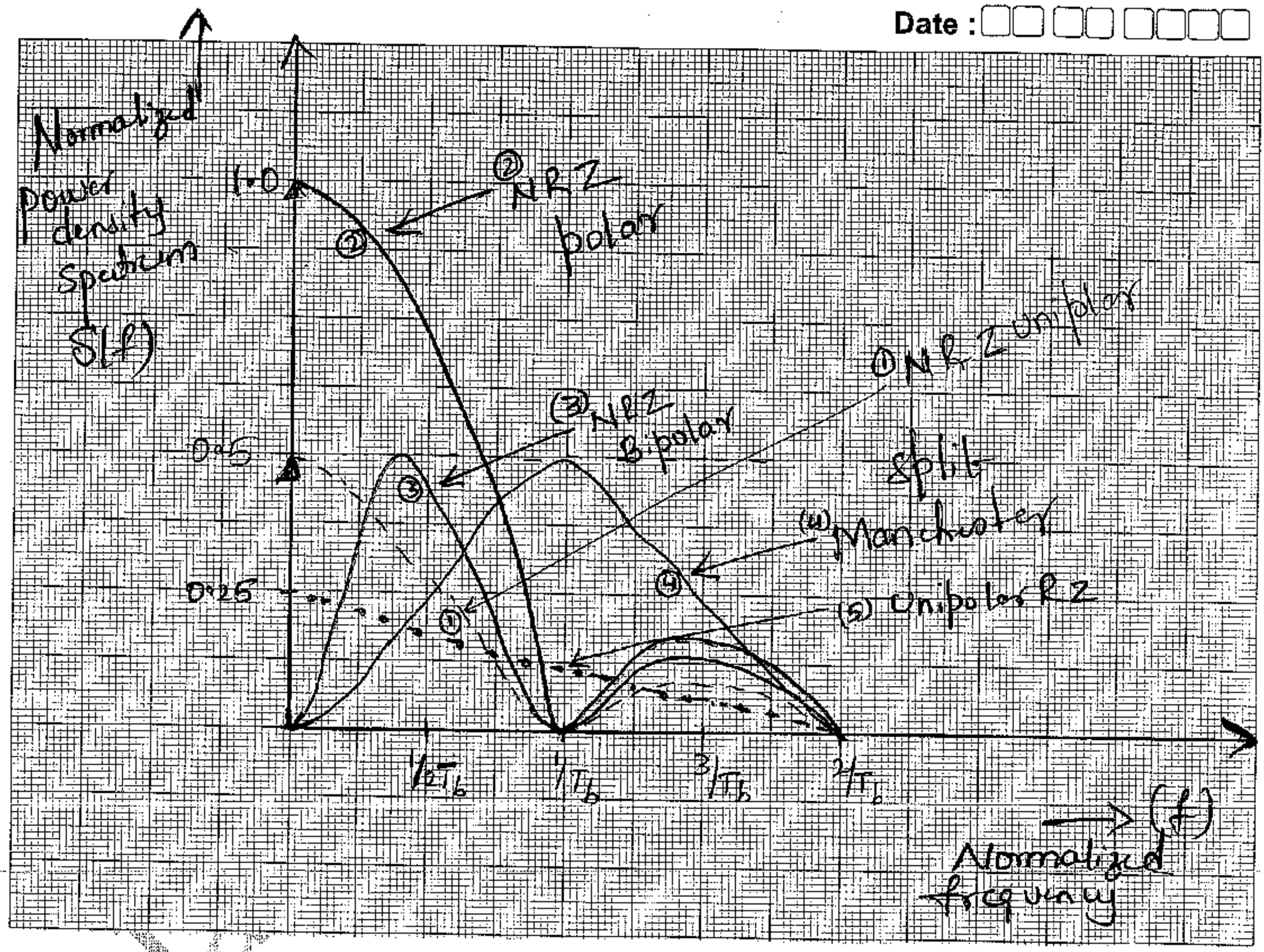
Dec 2019 | Jan 2020.



Power Spectra Of Line Codes

J|J 2019
Dec|Jun 2020.

Date : ☐☐ ☐☐ ☐☐☐☐



i. NRZ Unipolar:- $1 \rightarrow A$ 'V'
$0 \rightarrow$ '0' V

- Here the signal is unipolar. its amplitude can be +A (or) Zero. Hence the signal has some dc component.
- most of the power lies within the frequency $f = \frac{1}{T_b}$ Hz.
- bit rate $R_b = 1/T_b$ bits/sec.

* the power Spectra is Sinc Shaped and its main Lobe extends from dc to $1/T_b$. power contained in frequencies above bit rate is very small.

ii. NRZ polar format:- $1 \rightarrow$ 'A' Volts
$0 \rightarrow$ -'A' Volts

* In this format the wave form takes positive as well as negative amplitudes.

* if occurence of binary '1' or '0' is not equal then waveform has some dc value.

* In PSD the most of the power lies b/w dc to bitrate $1/T_b$.

* The main Lobe is Sinc pulse is from dc to bitrate frequency.

* the power contained in frequencies above the bitrate is very small.

iii. NRZ Bipolar (or) Alternate Mark Inversion (AMI)

$1 \rightarrow +A$ Volts, $\rightarrow -A$ Volts } for successive one's
$0 \rightarrow$ '0' Volts

* The Successive 1's are assigned pulses of alternating amplitudes. Hence wave form doesnot contain any dc component.

* power Lies inside the Bandwidth equal to bit rate ($1/T_b$). the power content in frequencies above bitrate is very small.

iv. Split Manchester format :-

* In Split Manchester format Every symbol is transmitted with +ve aswell as -ve amplitude. Hence there is no possibility of dc component in the Signal.

* most of the power Lies in the bandwidth twice of bit rate ($2/T_b$).

* peak of the spectra occurs near bit rate. width of the main pulse is twice of the other formats.

Perfomance parameter of Line codes :-

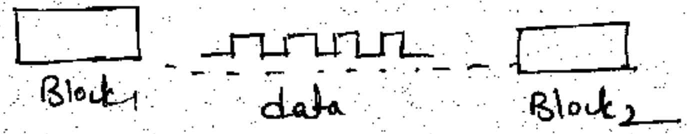
i. Transmision Bandwidth.

→ Line codes should have less Bandwidth

→ BW is one of the important parameter which defines cost of the Sytem.

ii. Transmision power Efficiency.

* Line codes are frequently used for Short range Communication.



Unipolar 0 0V, 1 aV] 0 to a

polar 0 −aV, 1 +aV] −a to a Volt.

iii. Low probability of Error

- code should have Low bit Error rate at Receiver.

$$BER = \frac{\text{Errored bit Rx}}{\text{Tx. bit}} \Rightarrow \text{must be Low as possible} \downarrow$$

iv. Self Synchronization.

* There should be enough timing information for building the code.

* Long series of 1's and 0's should not cause any problem in timing recovery.

v. Error detection and correction capability.

*Line coding should comfortably work with channel Encoder and detectors.

vi. Suitable Spectrum for bandlimited channel.

* The Symbol BW should be less than channel BW to avoid ISI.

vii. Dc Component

* it should not have dc component. (results simply power consuming)

* In transformers and Multi-stage amplifiers dc components adds distortion and Unwanted Energy Loss.

* DC Component = Mean amplitude (avg value)

$$\text{dc value} = \frac{N_0}{N}(A_0) + \frac{N_1}{N}(A_1)$$

A_0, A_1 - amplitude of bit Zero & bit 1.

N_0 - no of Zero's, N_1 - no of ones

N - Total no of bits

Q> Compare the power Spectrum of Various Line codes interms of BW, dc Component, Noise immunity and Synchronization Capability. (5m)

Jan 2019.

Soln:-	unipolar RZ	unipolar NRZ	polar RZ	polar NRZ	Bipolar NRZ (AMI)	Manchester
DC component	Present	Present	may be present	may be present	Absent	Absent
BW	f_b	$f_b/2$	f_b	$f_b/2$	$f_b/2$	f_b
Power requirement	Low	Low	High	high	high	Low (Comparatively)
Noise immunity	poor	fair	fair	fair	Good	Good
Synchronization	better	poor	Better	poor	Good	Good
polarity	two levels 0 — 0V 1 — AV	two levels 0 — 0V 1 — AV	~~three~~ (2) levels 0 — −A 1 — +A	~~three~~ (2) levels 0 → −A 1 → +A	three (3) levels 0 → 0V 1 → +A, −A	two levels 1 ¯|_ 0 _|¯ } '2' levels

Note:- Noise immunity is the ability of a system to perform its function when interference/noise is present.

Topic 6.2

Overview of HDB3, B3ZS and B6ZS

i. HDB3 | HDB$_N$ {High density Bipolar 3} Signaling

June/July 2018.

* In case of bipolar NRZ Signaling format there is no Signal transmitted during the period when binary '0' is present.

this Creates a problem in Synchronization when a Long Sequence of 0's present.

* This problem is Eliminated by adding pulses when no. of Consecutive 0's Exceeds 'N'. this type of coding is called "High density Bipolar (HDB)" Coding and is denoted by HDB$_N$.

here $N = 1, 2, 3, 4 \dots N \in Z^+$

if $N=3 \Rightarrow$ HDB$_3$ which is most widely used format.

* In message Sequence whenever (N+1) Zeros occurs, this group of Zero's is replaced by Special (N+1) binary digital Sequences.

* The Special Sequences is

	No. of 1's (previous)	previous bit	Special sequence
Case i.	Odd	+ve	000 +ve
Case ii.	Odd	-ve	000 -ve
Case iii.	Even	+ve	-ve 00 -ve
Case iv.	Even	-ve	+ve 00 +ve

Advantages of HDB3 over Conventional AMI [Bipolar NRZ]

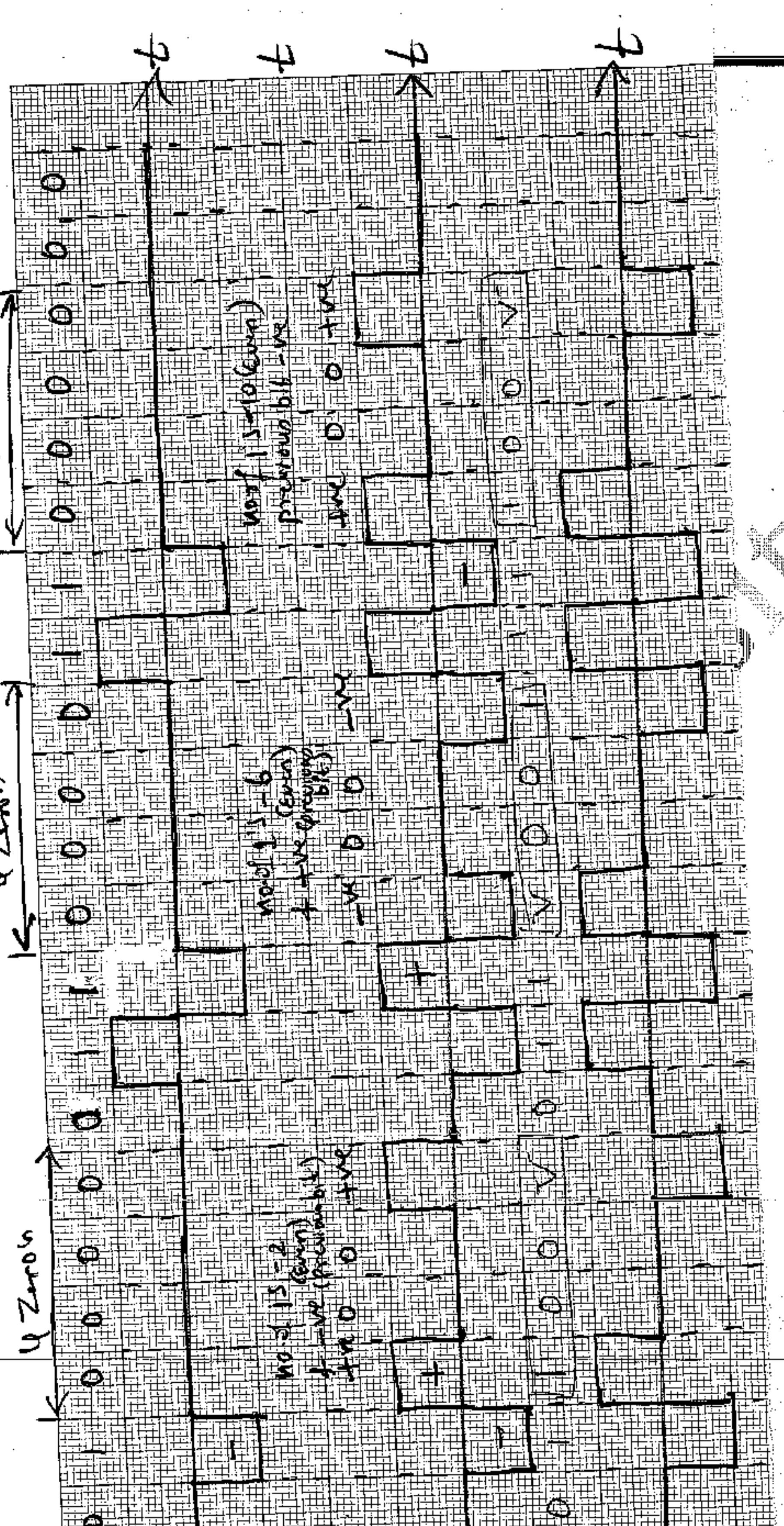
Dec 2018 | Jan 2019.

i. The problem of Synchronizing strings of consecutive 0's is solved.

ii. HDB3 introduces changes into the Bipolar AMI pattern every time four consecutive 0's are encountered.

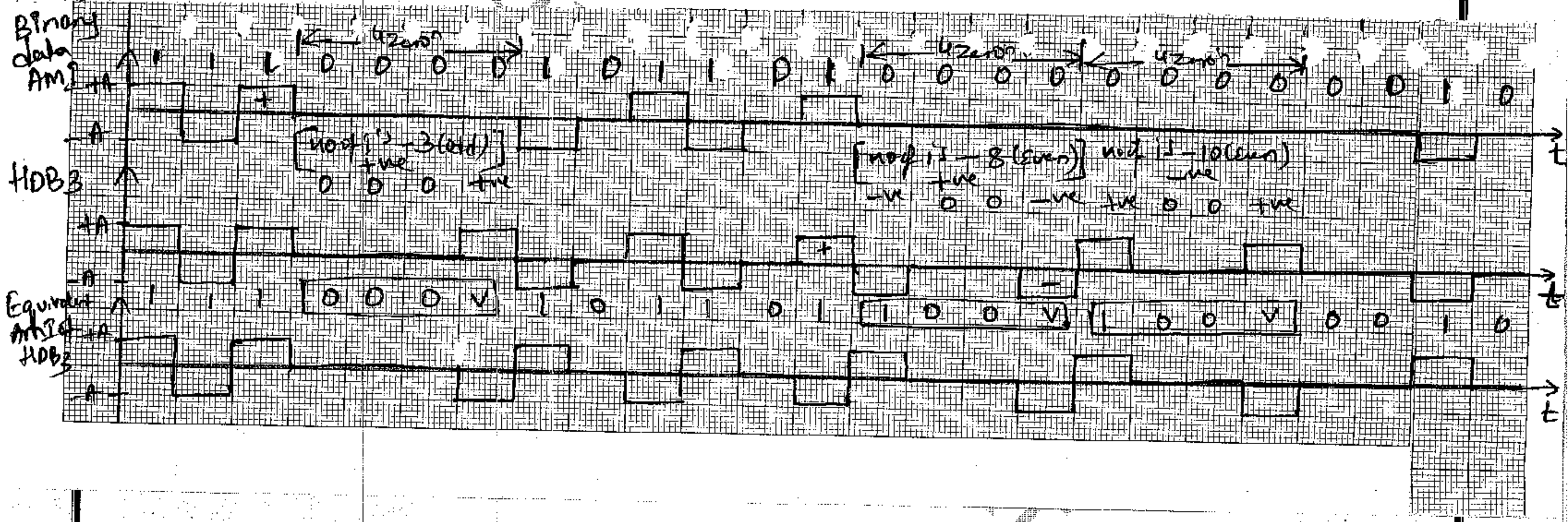
iii. HDB3 prevents more than four AMI "no change bits" from being set consecutively. this in turn prevents long runs of Zero's in the data stream.

Q) Code the pattern "1010000011000011000000" using HDB3 Encoding and AMI Encoding. Dec 2018 | Jan 2019.


Binary data
AMI
HDB3
-A
+A
Equivalent -A
AMI of HDB3
t
4 Zero's
no of 1's = 2 (even)
+ -ve (previous bit)
+ve 0 0 +ve
no of 1's = 6 (even)
+ -ve (previous bit)
-ve 0 0 -ve
no of 1's = 10 (even)
previous bit -ve
+ve 0 0 +ve
V
Date :

Q2. Encode using HDB3 linecoding technique

1110000 1011 01 0000 0000 00 10



9

B3ZS Signaling

* This coding is used in DS-3 Signals of digital telephone hierarchy.
* This code uses BOV to replace consecutive '3' (three) Zeros in the Sequence.

Eg:-

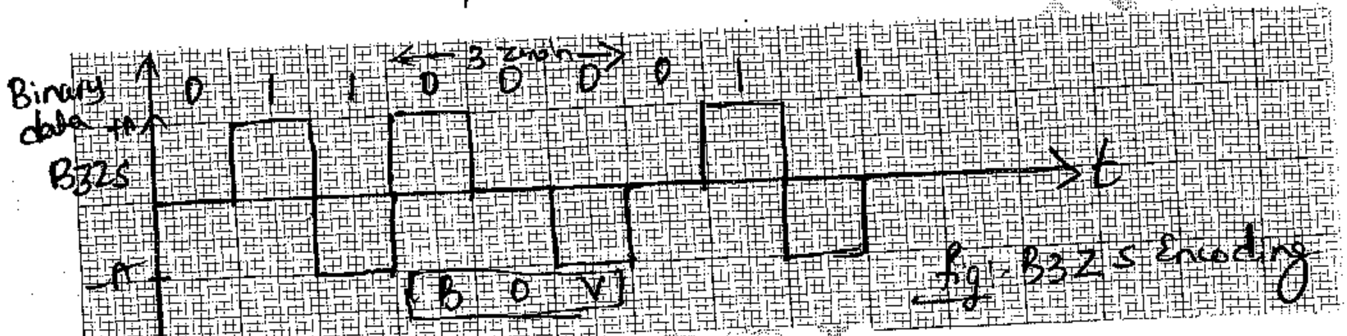


fig: B3ZS Encoding

B6ZS Signaling:-

* This coding is used in DS-2 Signals of digital telephone hierarchy.
* it replaces the String of six (6) Zeros with "OVBOVB".

Eg:-

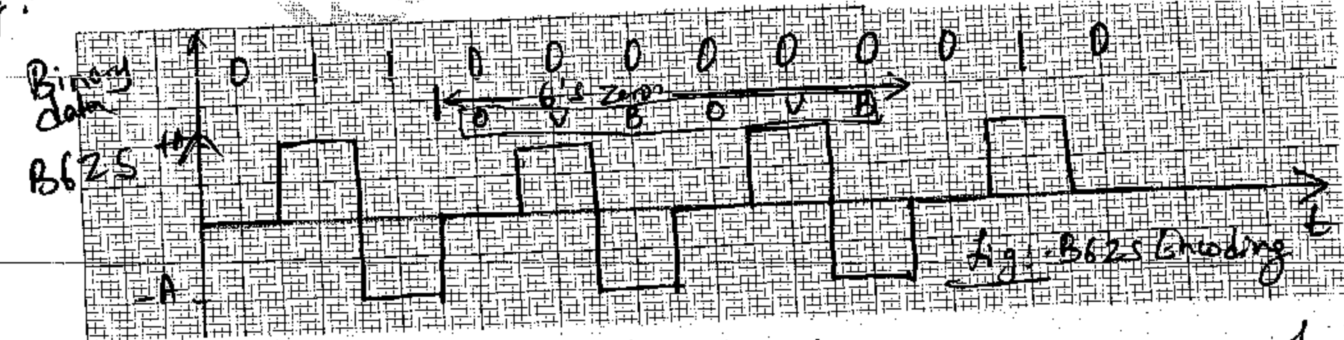


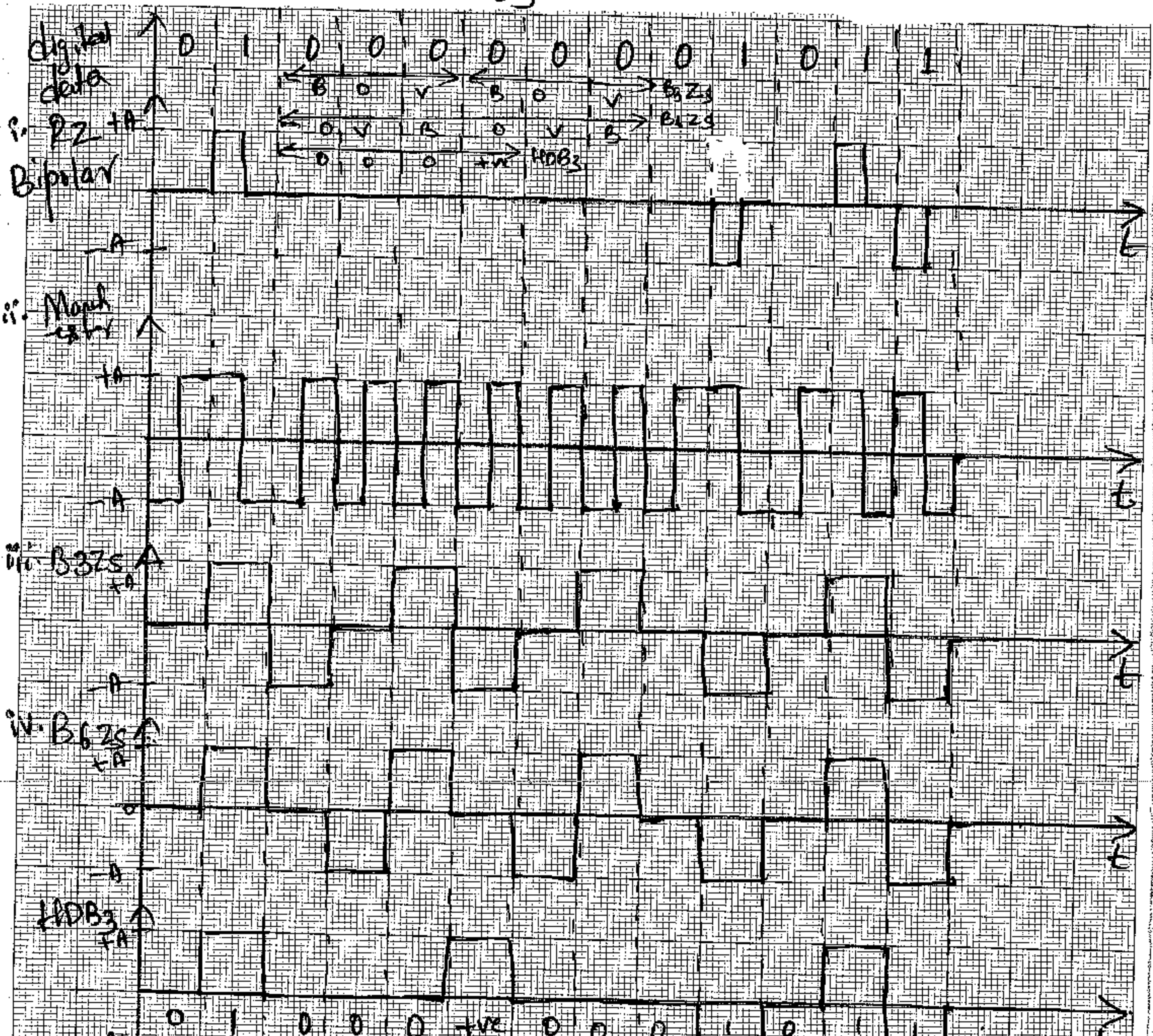
fig: B6ZS Encoding

Note:- DSx (Digital Signal X) is a term for the Series of digital transmission rates.

DS3 — 44mbps
DS2 — 6, 8, 34 mbps } data rates.

Q2. For a binary Sequence 0100000001011
Construct i. RZ Bipolar
ii. Manchester
iii. B3Zs and B6Zs format
iv. HDB3

(7m)
J|J 2019.



Module-2

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SIGNALING OVER AWGN CHANNELS

2.1 Introduction,
2.2 Geometric representation of signals,
  2.2.1 Gram-Schmidt Orthogonalization procedure,
2.3 Conversion of the continuous AWGN channel into a vector channel,
2.4 Optimum receivers using coherent detection
  2.4.1 ML Decoding
  2.4.2 Correlation receiver
  2.4.3 Matched filter receiver (Text 1: 7.1, 7.2, 7.3, 7.4).

Text Book:

Simon Haykin, —Digital Communication System, John Wiley & sons, First Edition, 2014, ISBN 978-0-471-64735-5.

2.2 Geometric representation of signals

1. Explain the geometric representation of set of M energy signals as linear combination of N orthogonal basis functions, illustrate for the case N=2 and M=3, with necessary diagrams and expressions. **(08 Marks) Dec 2018-Jan 2019.**
2. Explain the geometric representation of signals. Show that energy of the signal is equal to the squared length of the vector representing it. **(08 Marks) June-July 2018.**
3. Show that the energy of a signal is equal to squared length of the signal vector. **(08 Marks) Dec2019-Jan 2020**

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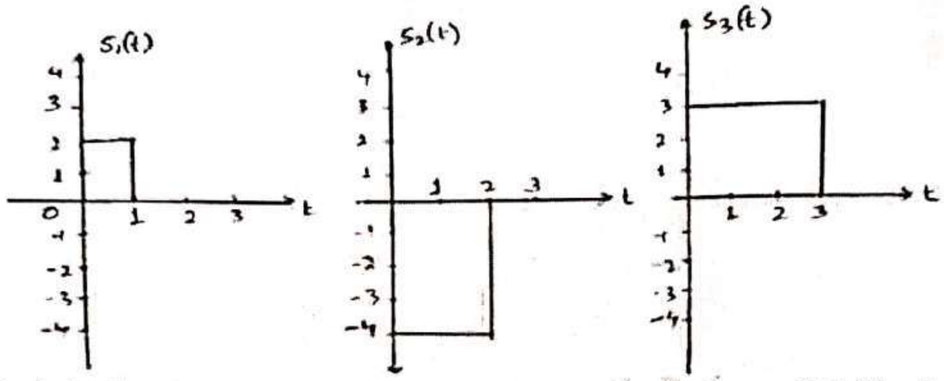
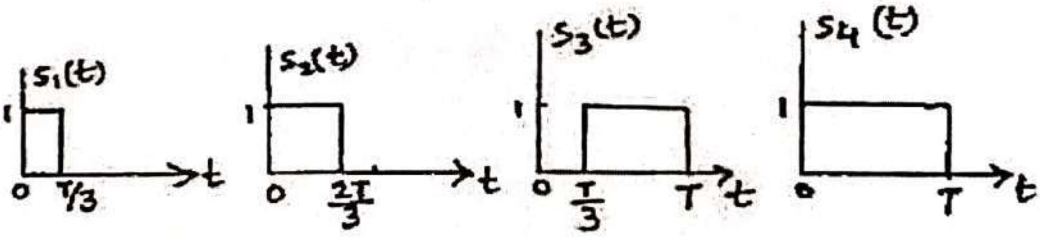
2.2.1 Gram-Schmidt Orthogonalization procedure

1. Using the Gram-Schmidt orthogonalization procedure, find a set of orthonormal basis functions to represent the three $s_1(t)$, $s_2(t)$ and $s_3(t)$ shown in Fig. Also express each of these signals in terms of the set of basis functions. **(08 Marks) Dec 2018-Jan 2019.**

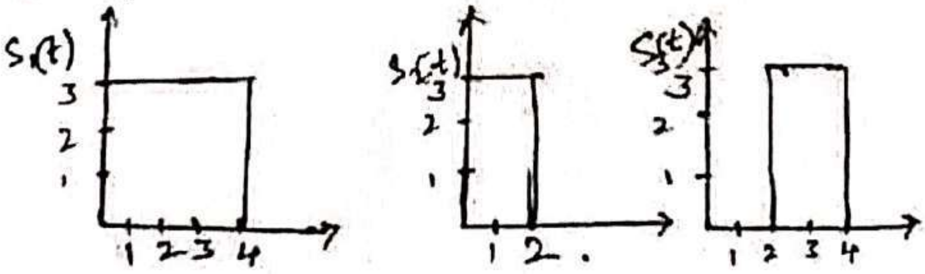


2. Explain the Gram-Schmidt othogonalization procedure. **(08 Marks) June-July 2018.**

3. For the signals $s_1(t), s_2(t)$, $s_3(t)$ and $s_4(t)$ shown in Fig, find a set of orthonormal basis functions using Gram-Schmidt orthogonalization procedure. **(08 Marks) June-July 2019.**



4. Three signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ shown in Fig. Apply Gram Schmidt procedure to obtain an orthonormal basis for the signals. Express signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ in terms of orthonormal basis functions. **(08 Marks) Dec 2018-Jan 2019.**



2.3 Conversion of the continuous AWGN channel into a vector channel

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1. Derive the expression for mean and variance of the correlator outputs. Also show that the correlator outputs are statistically independent. **(08 Marks) June-July 2018.**

2. Show that for a noisy input, the mean value of the j^{th} correlator output X_j depends only on S_{ij} and all the correlators outputs X_j, j=1,2,.......N, have a variance equal to the PSD $N_0/_2$ of the additive noise process w(t). **(08 Marks) June-July 2019.**

2.4 Optimum receivers using coherent detection

2.4.1 ML Decoding
2.4.2 Correlation receiver
2.4.3 Matched filter receiver

1. Obtain the Maximum Likelihood decision rule for the signal detection problem. **(10 Marks) June-July 2018.**

2. Explain the correlation receiver with neat diagrams and explanation of detector and the maximum-likelihood decoder blocks. **(08 Marks) Dec 2018-Jan 2019.**

3. Obtain the decision rule for maximum likelihood decoding and explain the correlation receiver. **(08 Marks) June-July 2019/ (08 Marks) Dec2019-Jan 2020**

4. Explain the correlation receiver using product integrator and matched filter. **(08 Marks) Dec2019-Jan 2020**

5. Explain the matched filter receiver. Obtain the expression for the impulse response of the matched filter. **(08 Marks) Dec 2018-Jan 2019**

6. Explain with neat diagram and necessary equations the matched filter receiver. **(07 Marks) June-July 2019.**

Detection and Estimation in Digital Commⁿ

- process of detection and Estimation in Digital Commⁿ system.

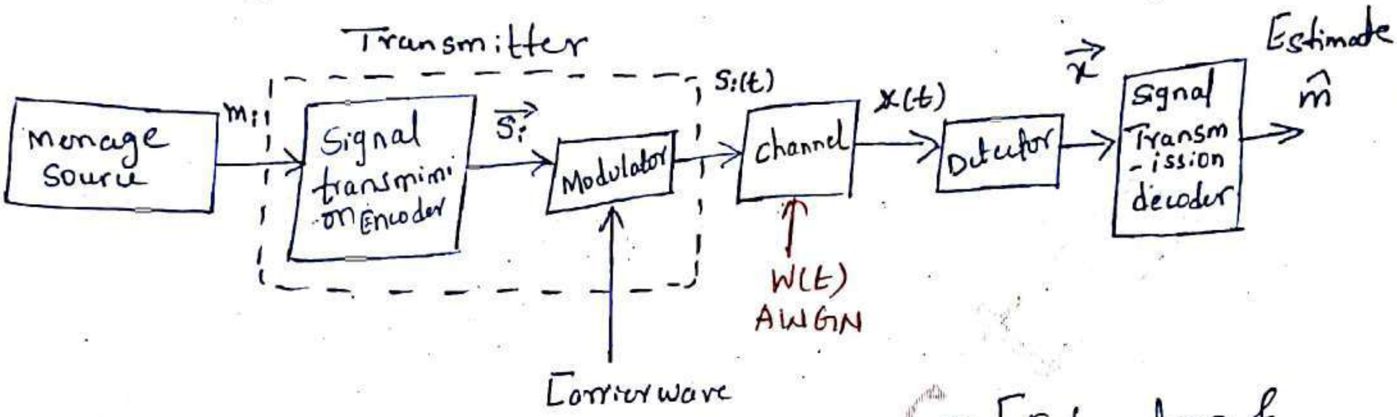


fig: model of Digital Commⁿ System [Estimation & Detection].

* Consider a block diagram of Digital communication system. message source in the first block which provides discrete messages of i's represented by $\{m_i\}$

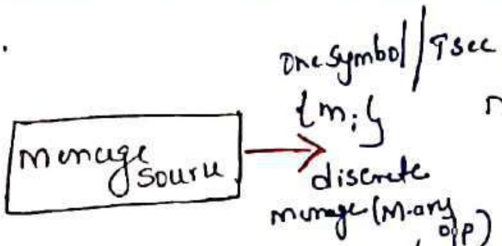
* Signal transmission Encoder converts m_i into sequence of set of real numbers in the form of symbol/vector $\vec{S_i}$.

* after Encoding $\vec{S_i}$ is modulated using carrier signal.

* The modulated signal is represented by $S_i(t)$ which is transmitted over a Linear channel having AWGN w(t).

* Then, the received signal x(t) is passed over a detector block to get observation vector $\vec{x}$. which is further decoded to get estimated signal $\hat{m}$.

Step 1.



Message Source contains M number of symbols emitt's one symbol per T seconds, ie $\{m_i\}$. These discrete message are called M-ary o/p.

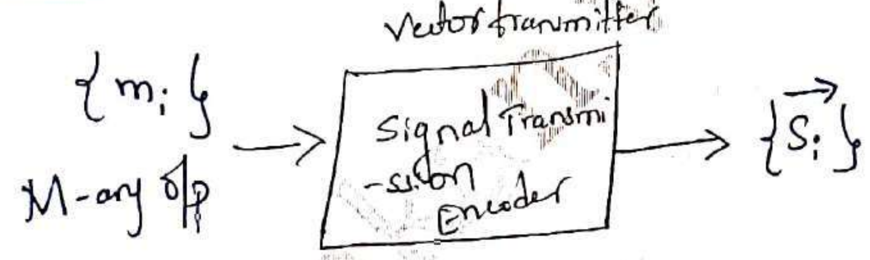
$$M = \{m_1, m_2, \dots m_M\}$$

Priori-probabilities
$p(m_1), p(m_2), \dots p(m_M)$
Specify message source o/p.

Each symbol have priori probabilities. If the prob of occurrence are equally likely then.

* when $m_1, m_2 \dots m_M$ are Equally likely then $\boxed{P_i = p(m_i) = \frac{1}{M}}$ for all i

Step 2.



M-ary o/p is passed through vector transmitter, which encodes m-ary into sequence of N-numbers.

(Set of N real nos. corresponding to each message $\{m_i\}$.)

(Dimension of $N \leq M$)

Eg:-

$\{m_i\}$ M-ary

$m_1 \longrightarrow$ Encode to set of N real no. $\longrightarrow \{\vec{S}_1\}$

$m_2 \longrightarrow$ " — " $\longrightarrow \{\vec{S}_2\}$

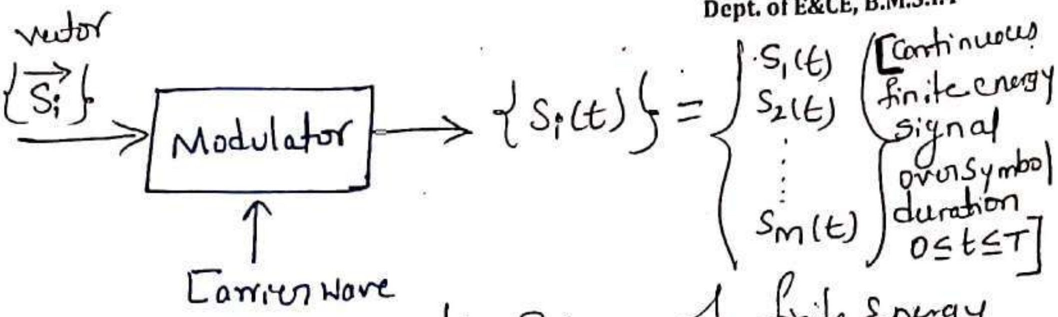
$\vdots$

$m_M \longrightarrow$ set of N real no. $\longrightarrow \{\vec{S}_M\}$

$\boxed{\{\vec{S}_i\} \simeq \text{Signal vector}}$ having dimension N. & which in $N \leq M$.

Step3

$\{\vec{S_1}, \vec{S_2} \ldots \vec{S_m}\}$
Each vector has 'N' dimension
$(N \leq M)$



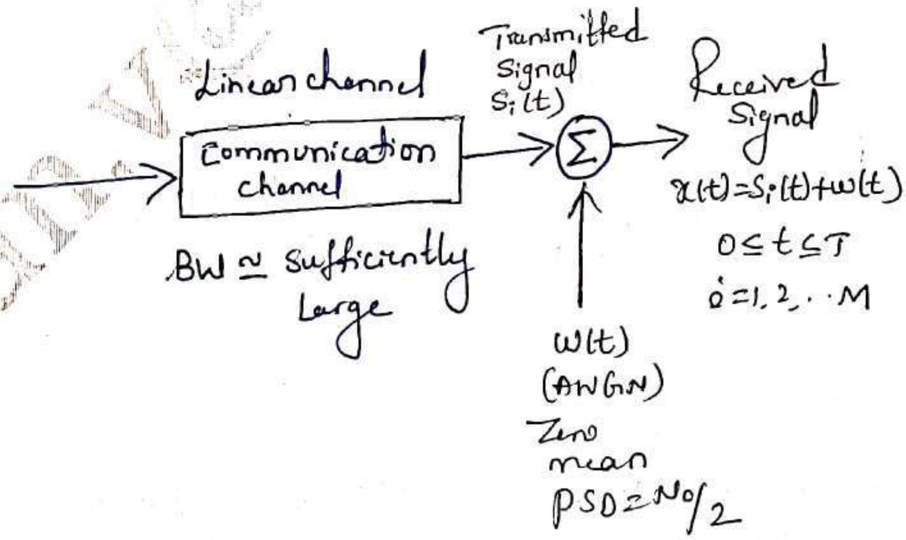
* Modulator convert's $\vec{S_i}$ vector into sequence of finite energy signal $\{S_i(t)\}$, which is fn of time 't'.

* $S_i(t)$ is necessary to be a real valued energy signal.

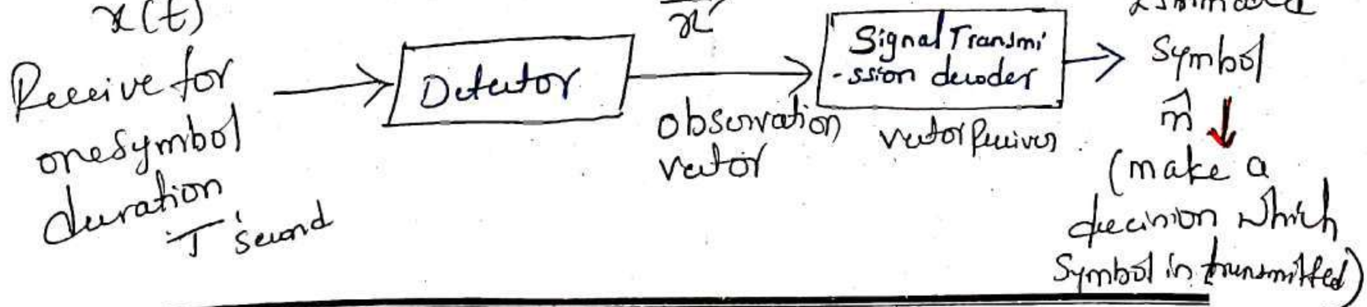
$$\boxed{E_i = \int_0^T S_i^2(t)\, dt} \text{ Joules.} \quad ; i = 1, 2, \ldots M.$$

Step4. Signal is transmitted over a channel.

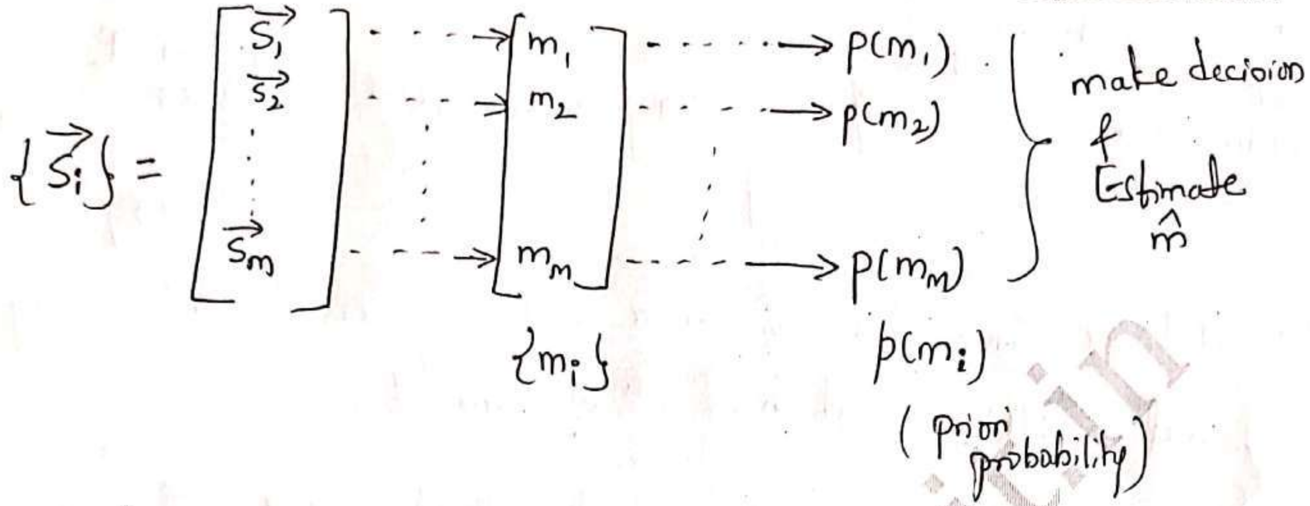
$S_i(t) \cdots \rightarrow E_i$
Such a 'one' symbol (or) signal is transmitted every 'T sec'



Step5 :- (Receiver)



{prior knowledge of signal vector $\{\vec{S_i}\}$}



Due to (noise) AWGN in the channel ⇒ which affect decision making process

↓ gives rise to

Symbol Error ↓ To minimize Symbol Error.

Aim: To minimize the Symbol Error, we use the optimum detector

optimum Receiver ⇒ which Minimize the average probability of Symbol Error.

Types of optimum receiver

i. → coherent detector — Receiver must be in phase Synchronized with transmitter carrier.

ii. → Non-coherent detector — Here in phase synchronization is not required.

orthogonal and orthonormal functions

i. orthogonal functions :-

Consider two basis functions
$\phi_i(t)$ and $\phi_j(t)$

Condition for orthogonality

Constant value.

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} \boxed{K} ; & i=j \\ 0 ; & i\neq j \end{cases}$$

where T - Symbol duration.

Eg:- Consider the Quadrature Carrier's

$$\left.\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T}}\cos 2\pi f_c t \\ \phi_2(t) &= \sqrt{\frac{2}{T}}\sin 2\pi f_c t \end{aligned}\right\}$$

check $\phi_1(t)$ and $\phi_2(t)$ are orthogonal (or) not.

$$\int_0^T \sqrt{\frac{2}{T}}\cos(2\pi f_c t)\cdot\sqrt{\frac{2}{T}}\sin(2\pi f_c t)\,dt$$

$$= \frac{2}{T}\int_0^T \sin(2\pi f_c t)\cdot\cos(2\pi f_c t)\cdot dt$$

Note:- $\sin A\cos B = \frac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$

$$= \frac{2}{T}\int_0^T \frac{1}{2}\left[\sin 0 + \sin(4\pi f_c t)\right]\cdot dt$$

$$= \frac{1}{T}\int_0^T \sin(4\pi f_c t)\cdot dt = \frac{1}{T}\cdot(0) = 0.$$

$\phi_1(t)\cdot\phi_2(t) \Rightarrow$ orthogonal to each other.

orthnormal :-

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = \begin{cases} 1 ; & i = j \\ 0 ; & i \neq j \end{cases}$$

i.e $\displaystyle\overset{i=j}{\int_0^T \phi_i(t)\,\phi_i(t)\,dt} = \int_0^T |\phi_i(t)|^2\,dt = 1 \Rightarrow$ orthonormal Basis fn.

i.e Energy of basis function is unity $\Rightarrow$ orthonormal Basis function.

Eg:- $\phi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_c t$

$$\int_0^T \phi_1^2(t)\,dt = \frac{2}{T}\int_0^T \cos^2(2\pi f_c t)\,dt$$

$$= \frac{2}{T}\int_0^T \left[\frac{1+\cos(4\pi f_c t)}{2}\right]\cdot dt$$

$$= \frac{2}{T}\left[\frac{1}{2}\int_0^T dt + \frac{1}{2}\cancelto{0}{\int_0^T \cos(4\pi f_c t)\,dt}\right]$$

$$= \frac{\cancel{2}}{\cancel{T}}\cdot\frac{1}{\cancel{2}}[\cancel{T}] = 1 \Rightarrow \text{orthonormal.}$$

In otherway,

⇒ $\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$ ⇒ orthonormal basis fn
[i.e unit Energy basis fn].

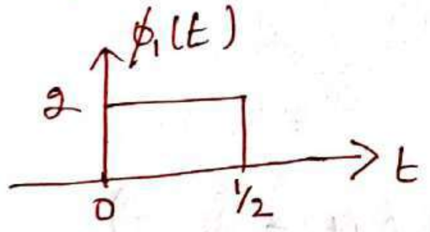
Carrier $c_1(t) = A \cos(2\pi f_c t)$

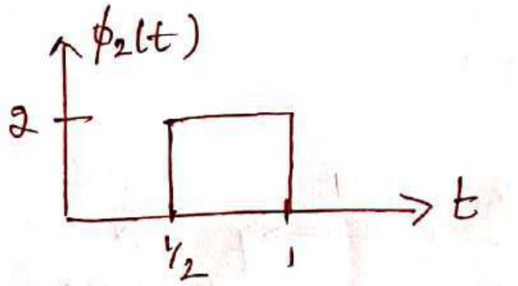
make the Carrier to be unit energy. and one Carrier is orthonormal to other Carrier.

"The process of making the Carrier's into orthonormal is called "Gram-Schmidt orthogonalization procedure" (GSOP)".

Examples:-

check whether the functions $\phi_1(t)$ and $\phi_2(t)$ are orthogonal. also express the given signals $x_1(t), x_2(t)$ and $x_3(t)$ interms of $\phi_1(t)$ and $\phi_2(t)$.





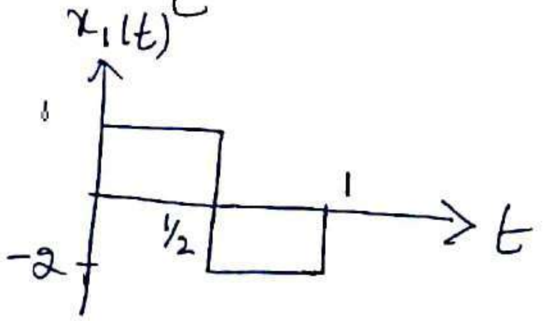
soln:- $\int_0^T \phi_1(t) \cdot \phi_2(t)\, dt = 0$.

$\phi_1(t)$ and $\phi_2(t)$ are not over lapping Signals.

$\therefore \phi_1(t) \cdot \phi_2(t) = 0$

$\Rightarrow \int_0^T \phi_1(t)\phi_2(t)\, dt = 0$. ie No Common Area b/w $\phi_1(t)$ and $\phi_2(t)$

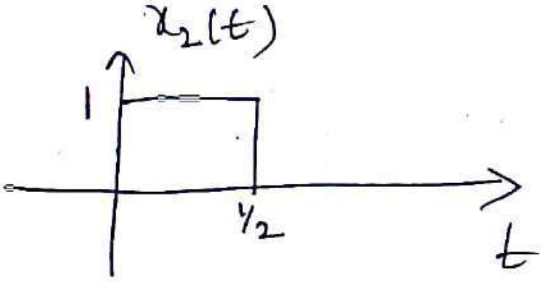
∴ $\phi_1(t)$ and $\phi_2(t)$ are orthogonal.
[i.e Nooverlapping].



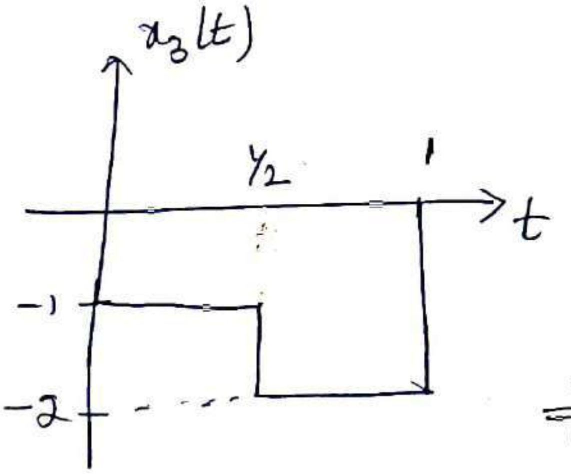
write $x_1(t)$ in terms of basis function's $\phi_1(t)$ and $\phi_2(t)$.

$$x_1(t) = \frac{1}{2}\phi_1(t) + (-1)\phi_2(t)$$

$$\Rightarrow \boxed{x_1(t) = \frac{1}{2}\phi_1(t) - \phi_2(t)}$$



$$\boxed{x_2(t) = \frac{1}{2}\phi_1(t)}$$



$\Rightarrow 0$

$$x_3(t) = -\frac{1}{2}\phi_1(t) - \phi_2(t)$$

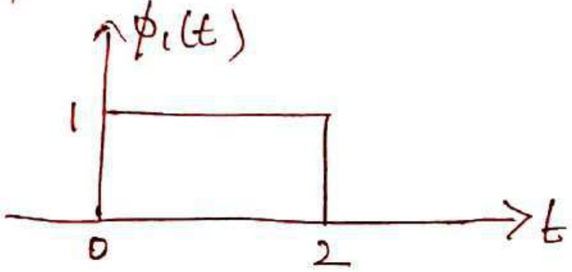
$$\Rightarrow \boxed{x_2(t) = -\frac{1}{2}\phi_1(t) - \phi_2(t)}$$

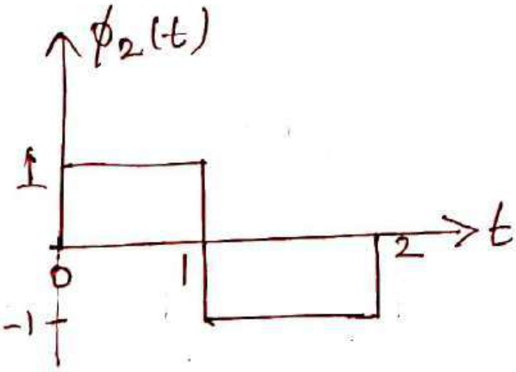
Example 2.

i. check whether the signals $\phi_1(t)$ and $\phi_2(t)$ sketched in fig. are orthogonal.

ii) obtain corresponding orthonormal functions.

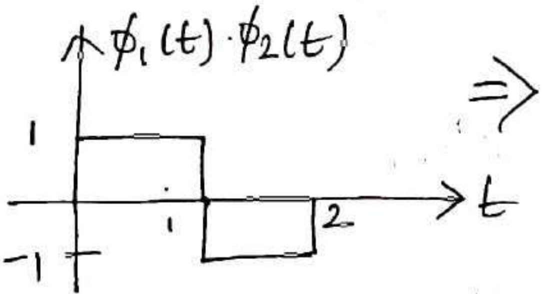
iii. Express the given signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ interms of $\phi_1(t)$ and $\phi_2(t)$.





Soln:-

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = ?$$



$$\Rightarrow \int_0^2 \phi_1(t)\,\phi_2(t)\,dt$$

$$= \int_0^1 (1)\,dt + \int_1^2 (-1)\,dt$$

$$= 1\cdot t\Big|_0^1 - 1\cdot t\Big|_1^2$$

$$= [1-0] - [2-1] = 1 - 1 = 0.$$

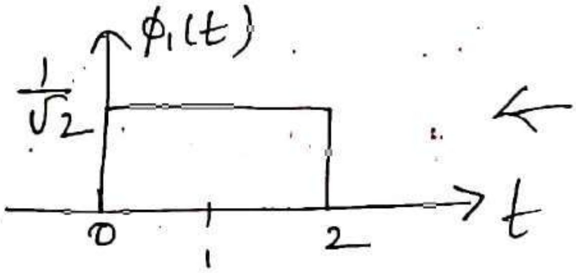
$$\Rightarrow \int_0^T \phi_1(t)\cdot\phi_2(t)\,dt = 0$$ i.e Both the basis functions $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other.

⑤

ii. orthogonal.

$$E_1 = E_{\phi_1(t)} = \int_0^2 |\phi_1(t)|^2 \cdot dt = \int_0^2 (1) \cdot dt = 2 \text{ Joules}.$$

$$\phi_1'(t) = \frac{\phi_1(t)}{\sqrt{E_1}} = \frac{1}{\sqrt{2}} \phi_1(t).$$



← 1st orthonormal basis fn.

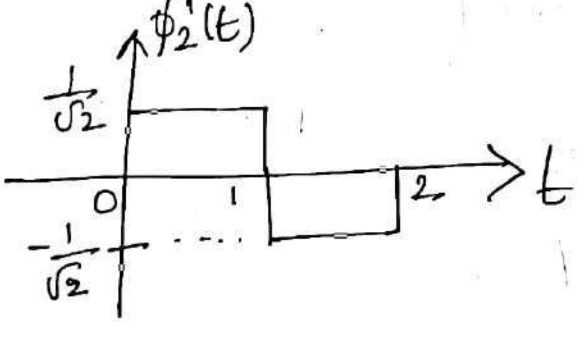
$$E_2 = E_{\phi_2(t)} = \int_0^2 |\phi_2(t)|^2 dt = \int_0^1 (1)^2 dt + \int_1^2 (-1)^2 dt$$

$$= t\Big|_0^1 + t\Big|_1^2 = (1-0) + (2-1)$$

$$= 1 + 1 = 2 \text{ Joules}$$

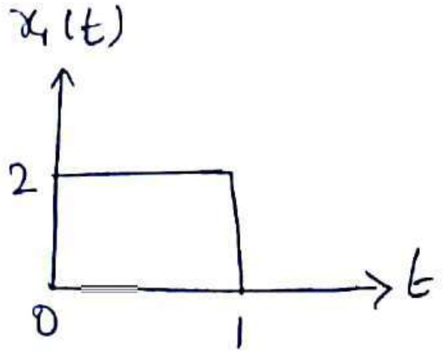
$\phi_2(t)$ is not orthonormal Make it orthonormal

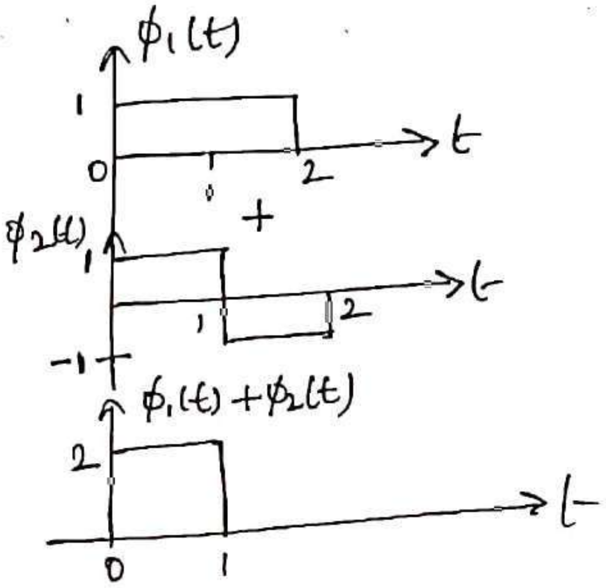
i.e $$\phi_2'(t) = \frac{\phi_2(t)}{\sqrt{E_2}} = \frac{1}{\sqrt{2}} \phi_2(t)$$



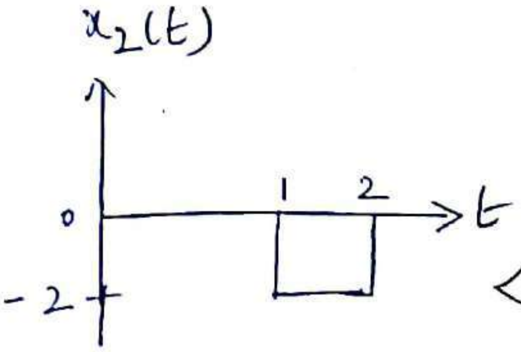
⇐ 2nd orthonormal basis function.

iii. Given.

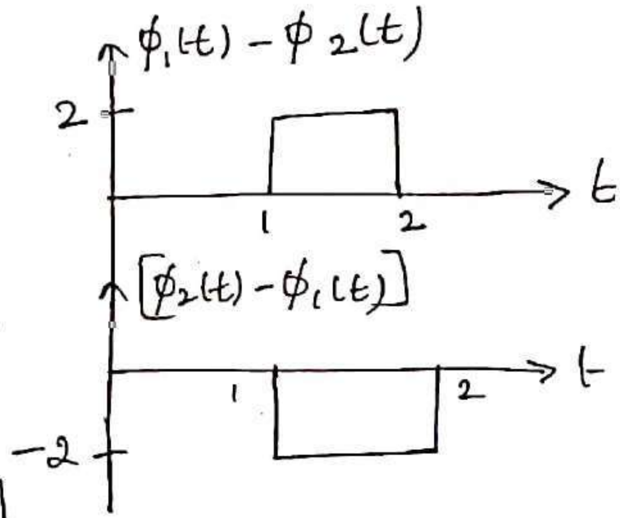




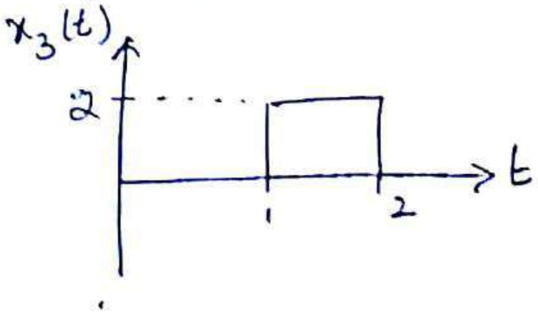
$$\boxed{x_1(t) = \phi_1(t) + \phi_2(t)}$$



$\Longleftrightarrow$



$$\boxed{x_2(t) = \phi_2(t) - \phi_1(t)}$$



$$\boxed{x_3(t) = \phi_1(t) - \phi_2(t)}$$

7

Module-2

2.2 Geometric representation of signals

XP.

1. Explain the geometric representation of set of M energy signals as linear combination of N orthogonal basis functions, illustrate for the case N=2 and M=3, with necessary diagrams and expressions. **(08 Marks) Dec 2018-Jan 2019.**
2. Explain the geometric representation of signals. Show that energy of the signal is equal to the squared length of the vector representing it. **(08 Marks) June-July 2018.**
3. Show that the energy of a signal is equal to squared length of the signal vector. **(08 Marks) Dec2019-Jan 2020**

Soln:- Consider M → possible energy signals (no. of possible transmitted signal).

$$\{x_i(t)\}_{i=1}^{M} \Rightarrow x_1(t), x_2(t), \ldots\ldots x_m(t)$$

These signals are represented interms of 'N' orthonormal basis function.

i.e $\phi_j(t) = \{\phi_1(t), \phi_2(t) \ldots\ldots \phi_N(t)\}$.

and

$N \leq M$.

ideally any signal $x_1(t), x_2(t) \ldots\ldots x_m(t)$ can be represented as Linear Combination of 'N' orthonormal basis function.

Say i.e ith signal

$$x_i(t) = x_{i_1}\phi_1(t) + x_{i_2}\phi_2(t) + \ldots + x_{iN}\phi_N(t)$$

$$\boxed{x_i(t) = \sum_{J=1}^{N} x_{ij}\phi_j(t)}, \quad i = 1, 2, 3 \ldots M$$

→ build a Synthesizer (or) Transmitter block.

P.T.O

In the above equation

$$x_{ij} = \int_0^T x_i(t)\, \phi_j(t)\, dt$$

T - duration of Symbol . $x_i(t)$.

The basis functions $\phi_1(t), \phi_2(t) \cdots \cdots \phi_N(t)$ are orthonormal.

i.e $\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 1 ; & \text{for } i=j \\ 0 ; & \text{for } i \neq j \end{cases}$

when $i=j$ $\Rightarrow$ $\int_0^T \phi_i^2(t) dt = \begin{cases} 1 & ; \text{for } i=j \text{ (unitenergy)} \\ 0 & ; i \neq j \end{cases}$

i.e $\int_0^T \phi_i(t)\, \phi_j(t)\, dt = 0$; for $i \neq j$

This means the basis functions : $\phi_1(t), \phi_2(t) \cdots \phi_N(t)$ are orthogonal to Each other over an interval 0 to T.

Example:- N = No of orthonormal f^n = 2
M = No of message signals = 3.

$$x_i(t) = \sum_{j=1}^{N} x_{ij} \cdot \phi_j(t). \qquad i = 1, 2, 3$$

$$\Rightarrow x_i(t) = \sum_{j=1}^{2} x_{ij}\, \phi_j(t)$$

$$x_i(t) = x_{i1}\, \phi_1(t) + x_{i2}\, \phi_2(t)$$

(8a)

$$x_i(t) = x_{i1}\phi_1(t) + x_{i2}\phi_2(t)$$

$\underline{i=1}$

$$x_1(t) = x_{11}\phi_1(t) + x_{12}\phi_2(t)$$

$\underline{i=2}$

$$x_2(t) = x_{21}\phi_1(t) + x_{22}\phi_2(t).$$

$\underline{i=3}$

$$x_3(t) = x_{31}\phi_1(t) + x_{32}\phi_2(t).$$

obs:- * $x_1(t)$, $x_2(t)$ and $x_3(t)$ are expressed interms of $\phi_1(t)$ and $\phi_2(t)$.

→ The three message vector's x_1, x_2 and x_3 Can be represented on ϕ_2 v/s ϕ_1 plane (or) Signal Space also called Euclidean Space.

→ ϕ_1 and ϕ_2 are orthogonal to Eachother bcz they are $\perp^{r}$ to Eachother.

→ the plot of message vector's x_1, x_2 and x_3 on Signal Space depends on the co-ordinates values i.e x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}.

86

P-TO

$$x_1(t) = x_{11}\phi_1(t) + x_{12}\phi_2(t)$$

$$x_2(t) = x_{21}\phi_1(t) + x_{22}\phi_2(t)$$

$$x_3(t) = x_{31}\phi_1(t) + x_{32}\phi_2(t).$$

assuming the co-ordinates

$$\left.\begin{aligned}(x_{11}, x_{12}) &= (3,1)\\(x_{21}, x_{22}) &= (1,3)\\(x_{31}, x_{32}) &= (2,-2)\end{aligned}\right\}\quad\begin{aligned}x_1 &= 3\phi_1 + \phi_2\\x_2 &= \phi_1 + 3\phi_2\\x_3 &= 2\phi_1 - 2\phi_2.\end{aligned}$$

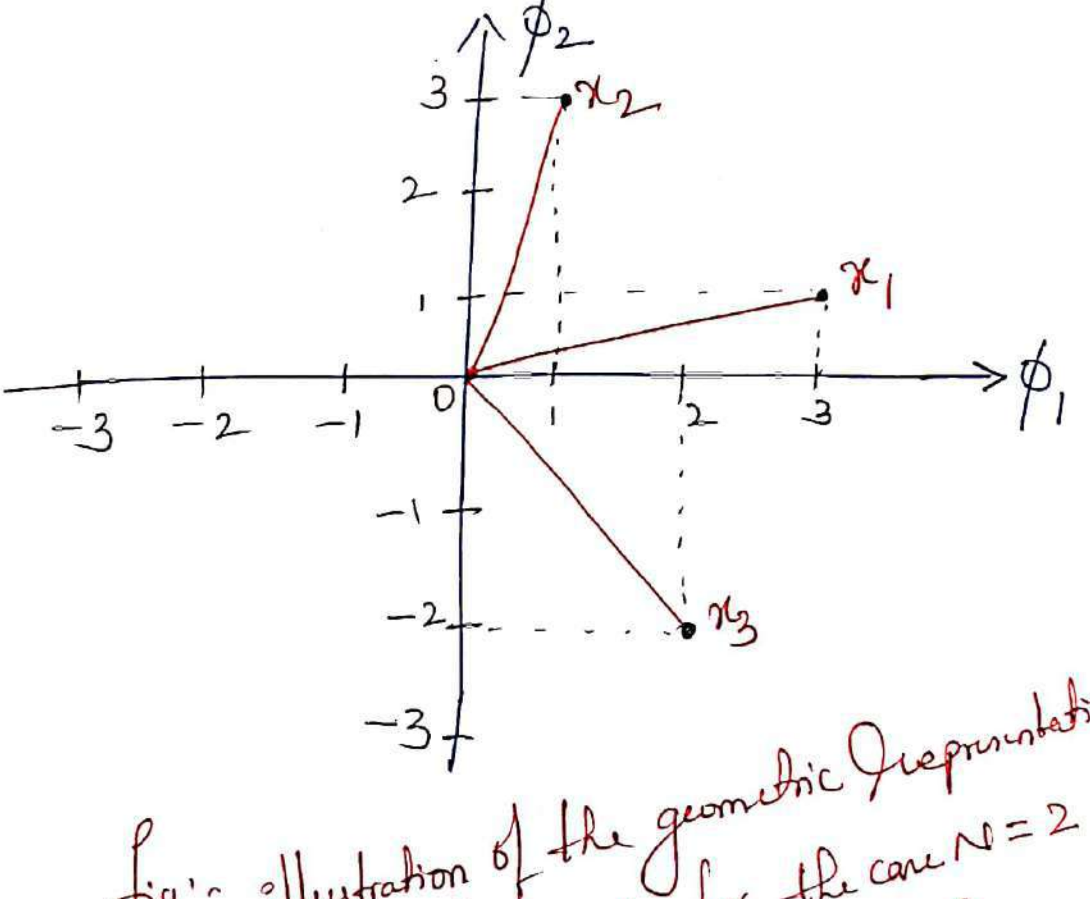
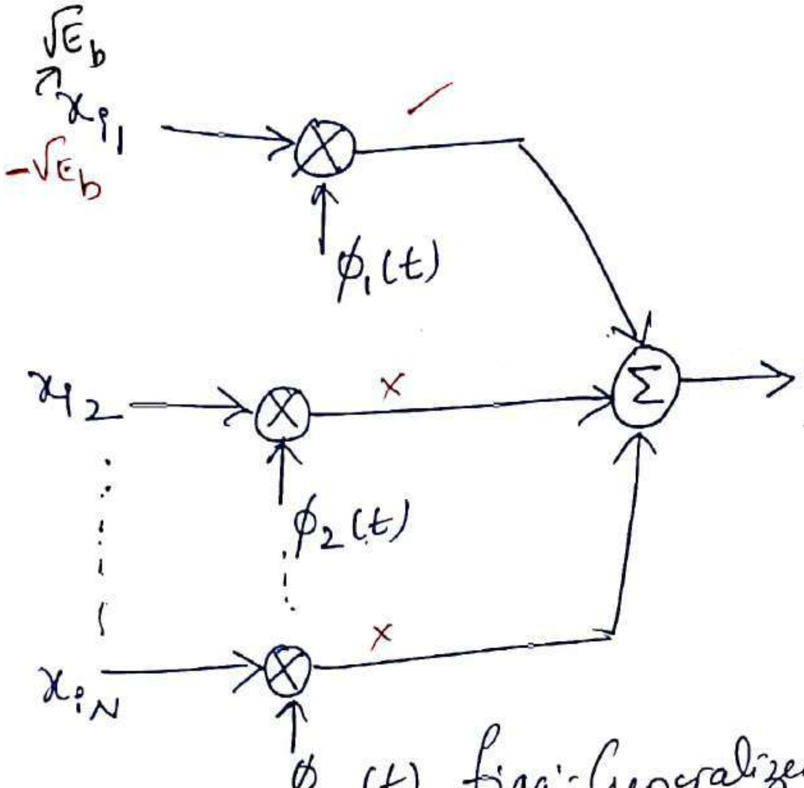


fig:- illustration of the geometric representation of signals for the case N = 2 and M = 3.

ASK } only one
BPSK } basis f^n.



← ASK

$$x_i(t) = \begin{cases} \sqrt{E_b}\,\phi_1(t) \Rightarrow \text{Symbol 1} \\ 0 \Rightarrow \text{Symbol '0'} \end{cases}$$

← BPSK.

$$= \begin{cases} \sqrt{E_b}\,\phi_1(t) \Rightarrow \text{Symbol '1'} \\ -\sqrt{E_b}\,\phi_1(t) \Rightarrow \text{Symbol '0'} \end{cases}$$

fig a:- Generalized Synthesizer (or) Transmitter.

$$x_i(t) = \sum_{j=1}^{N} x_{ij}\,\phi_j(t)$$

at the Receiver we product Modulator (P.M)

step1. $x_i(t)\,\phi_K(t) = \sum_{j=1}^{N} x_{ij}\,\phi_j(t)\,\phi_K(t)$

step2. o/p of P.M is passed through integrator.

$$\int_0^T x_i(t)\,\phi_K(t)\,dt = \sum_{j=1}^{N} x_{ij} \int_0^T \phi_j(t)\,\phi_K(t)\,dt$$

$$= \begin{cases} 1 \; ; \text{ only } j=K \\ 0 \; ; \text{ o/w.} \end{cases}$$

orthonormal basis fn's.

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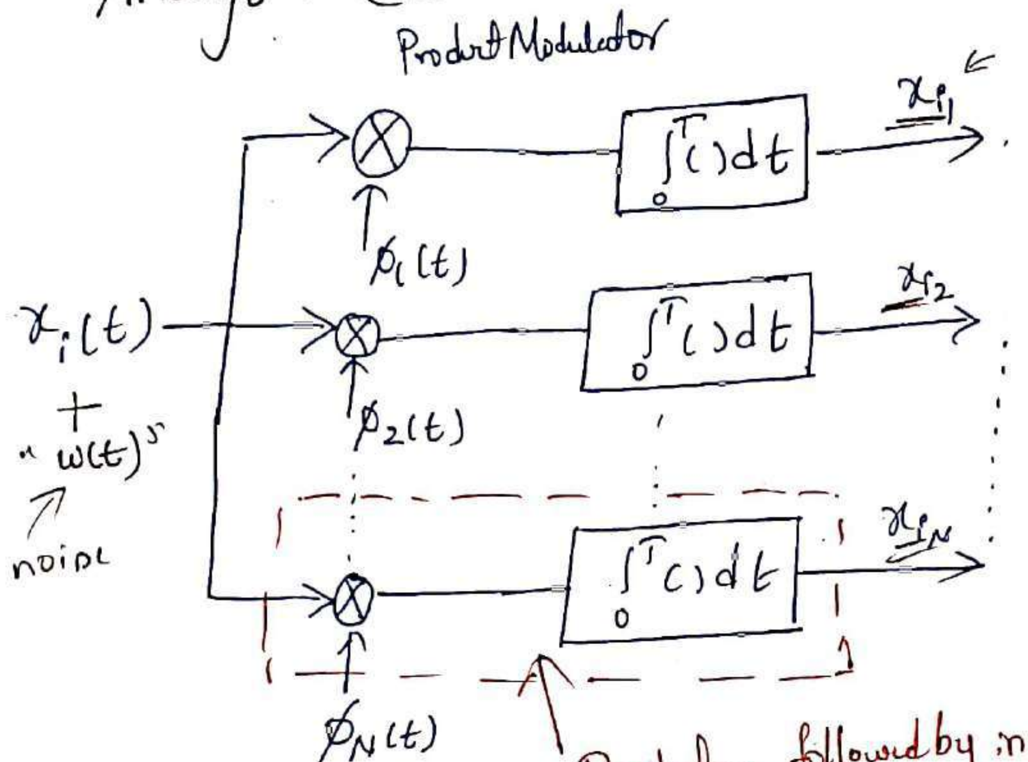
$$\int_0^T x_i(t)\,\phi_k(t)\,dt = x_{ik}$$

$$\boxed{x_{ij} = \int_0^T x_i(t)\,\phi_j(t)\cdot dt}\ ,\ j = 1, 2, 3 \dots N$$

Analyzer (or) Receiver.



Product modulator followed by integrator is called "coherent Receiver" (or) Correlation Receiver.

Any Signal can be represented as Linear combination of 'N' orthonormal Signal:

$$\overline{x_i} = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{iN} \end{bmatrix} \Rightarrow$$ Signal Space "N dimensional" signal space.

Eg:- for ASK.

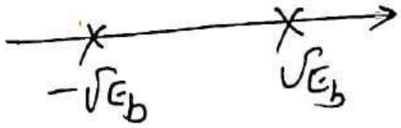
$$\overline{x_i} = \begin{bmatrix} x_{i_1}(\sqrt{E_b}, 0) \\ x_{i_2}-0 \\ x_{i_3}\to 0 \\ \vdots \\ x_{i_N}\to 0 \end{bmatrix}$$

Signalspace is one dimensional

BPSK

$$\overline{x_i} = \begin{bmatrix} x_{i_1} \to (\sqrt{E_b}, -\sqrt{E_b}) \\ x_{i_2}=0 \\ x_{i_3}=0 \\ \vdots \\ x_{i_N}=0 \end{bmatrix}$$

onedimentional.



Relation between signal energy and its vectors -

the Energy of $x_i(t)$ is

$$E_i = \int_0^T x_i^2(t)\,dt$$

$$= \int_0^T \left[\sum_{j=1}^{N} x_{ij}\,\phi_j(t)\right]\left[\sum_{k=1}^{N} x_{ik}\,\phi_k(t)\right] dt$$

$$= \sum_{j=1}^{N}\sum_{k=1}^{N} x_{ij}\,x_{ik} \underbrace{\int_0^T \phi_j(t)\,\phi_k(t)\,dt} \quad \begin{cases} 1 : j=k \\ 0 : j\neq k \end{cases}$$

$$E_i = \sum_{j=1}^{N} x_{ij}^2 = x_{i_1}^2 + x_{i_2}^2 + \dots + x_{i_N}^2$$

$$= \|\overline{x_i}\|^2$$

obs:- Energy of the signal (E_i) = Squared Length of N-dimensional signal vector.

2.2.1 Gram-Schmidt Orthogonalization procedure (GSOP)

Explain the Gram-Schmidt othogonalization procedure. (08 Marks) June-July 2018.

* Any signal canbe represented interms of orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$.

* Gram-schmidt orthogonalization procedure is the tool to obtain the orthonormal basis functions $\phi_i(t)$.

Consider a set of m energy Signals $x_1(t), x_2(t) \ldots x_m(t)$

the first basis function is

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_1}}$$

$\phi_1(t)$ - orthonormal basis function i.e unit energy fn. $[E_{\phi_1(t)} = 1J]$

$$E_1 = \int_0^T x_1^2(t)\,dt$$

Eg:- for BPSK
No. of symbols m=2
Symbol
$1 \rightarrow x_1(t)$
$0 \rightarrow x_2(t)$

Ingeneral for M-ary PSK
$\phi_1(t)$
$\phi_2(t)$
$\vdots$
$\phi_m(t)$
No. of basis functions canbe found by (GSOP)

assume that all the signals $x_1(t), x_2(t) \ldots x_m(t)$ are real-valued signals.

where E_1 is the energy of the signal $x_1(t)$.

To get unit energy basis function, we are normalizing the signal $x_1(t)$ by $\sqrt{E_1}$.

P.T.O

$$x_1(t) = \sqrt{E_1}\,\phi_1(t) \longleftarrow ⓐ$$

W.k.t

$$\underset{\substack{\uparrow \\ \text{ith Signal representation}}}{x_i(t)} = \sum_{j=1}^{N} x_{ij}\,\phi_j(t) \quad - i=1,2,\ldots M$$

When N=1.

$$x_i(t) = \sum_{j=1} x_{ij}\,\phi_j(t), \quad i=1,2,\ldots M$$

i=1

$$x_1(t) = x_{11}\,\phi_1(t) \longleftarrow ⓑ$$

Comparing eqⁿ ⓐ and eqⁿ ⓑ

$$x_{11} = \sqrt{E_1}.$$

for defining other basis functions, a new intermediate function is introduced ①

$$\boxed{g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\,\phi_j(t)}$$

$i = 1, 2, 3 \cdots M$

where,

$$\boxed{x_{ij} = \int_0^T x_i(t)\,\phi_j(t)\,dt} \quad ②$$

$j = 1, 2, \cdots i-1$

$i \rightarrow x_1(t)$
$x_2(t)$
$\vdots$
$x_M(t)$

$j \rightarrow$ basis function
$j = 1 \quad \phi_1(t)$
$\phi_2(t)$
$\phi_N(t)$
$N \leq M.$

from a given $g_i(t)$, a new set of basis functions are defined as:

$$\boxed{\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}}, \quad i = 2, 3 \cdots N.$$

$$\text{Next basis fn} = \frac{\text{intermediate fn}}{\text{Sqrt of Energy of intermediate fn}}$$

2nd basis function.

$i = 2$ [from eqⁿ ①]

$$g_2(t) = x_2(t) - \sum_{j=1}^{1} x_{2j}\,\phi_j(t)$$

$$g_2(t) = x_2(t) - x_{21}\,\phi_1(t)$$

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$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$$

$$\int_0^T g_2^2(t)\,dt = \int_0^T [x_2(t) - x_{21}\phi_1(t)]^2\,dt$$

$$= \int_0^T \left[x_2^2(t) + x_{21}^2\,\phi_1^2(t) - 2x_2(t)x_{21}\,\phi_1(t)\right]dt$$

$$= \int_0^T x_2^2(t)\,dt + x_{21}^2 \overset{1}{\int_0^T \phi_1^2(t)\,dt} - 2x_{21}\overset{x_{21}}{\int_0^T x_2(t)\phi_1(t)\,dt}$$

$$\int_0^T g_2^2(t)\,dt = E_2 + x_{21}^2 - 2x_{21}\cdot(x_{21})$$

$$= E_2 + x_{21}^2 - 2x_{21}^2 = E_2 - x_{21}^2$$

$$\therefore \boxed{\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} = \frac{x_2(t) - x_{21}\phi_1(t)}{\sqrt{E_2 - x_{21}^2}}}$$

$\phi_1(t)$ and $\phi_2(t)$ are orthogonal

$$\int_0^T \phi_1(t)\phi_2(t)\,dt = \int_0^T \left[\frac{x_2(t) - x_{21}\phi_1(t)}{\sqrt{E_2 - x_{21}^2}}\right]\phi_1(t)\,dt$$

$$= \frac{1}{\sqrt{E_2 - x_{21}^2}}\left[\int_0^T x_2(t)\,\phi_1(t)\,dt - \int_0^T x_{21}\phi_1(t)\cdot\phi_1(t)\,dt\right]$$

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$$= \frac{1}{\sqrt{E_2 - x_{21}^2}} \left[\int_0^T \underbrace{x_2(t)\,\phi_1(t)\,dt}_{x_{21}} - x_{21} \int_0^T \underbrace{\phi_1^2(t)\,dt}_{1 \text{ (unit energy)}} \right]$$

$$= \frac{1}{\sqrt{E_2 - x_{21}^2}} \left[x_{21} - x_{21} \right] = 0.$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$$

$$\int_0^T \phi_2^2(t)\,dt = \frac{\cancel{\int_0^T g_2^2(t)\,dt}}{\cancel{\int_0^T g_2^2(t)\,dt}} = 1 \Rightarrow \text{orthonormal.}$$

i.e basis function $\phi_2(t)$ is orthonormal.

Note:-

Generalized equations for orthonormal basis functions are:-

orthonormal basis function $\phi_i(t) = \frac{g_i(t)}{\sqrt{E_{g_i(t)}}}$; $i = 1, 2, .. N$ ← ①

where $g_i(t)$ – intermediate fn.

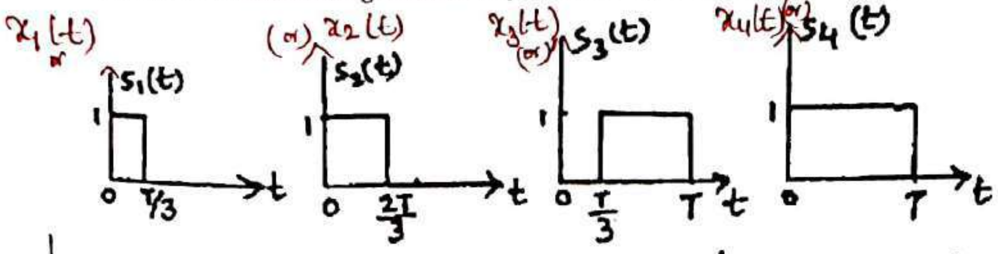
$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\,\phi_j(t) \leftarrow ②$$

where the coefficient

$$x_{ij} = \int_0^T x_i(t)\,\phi_j(t)\,dt \leftarrow ③$$

$j = 1, 2, 3 .. (i-1)$

For the signals $s_1(t), s_2(t), s_3(t)$ and $s_4(t)$ shown in Fig, find a set of orthonormal basis functions using Gram-Schmidt orthogonalization procedure. **(08 Marks) June-July 2019.**



Soln:- Given M=4, N = ? [i.e find no. of basis function's required to represent the given four signals].

and ii. what are the orthonormal basis function's $\phi_1(t)$, $\phi_2(t)$, $\phi_3(t)$ and $\phi_4(t)$ ..?

w.k.t

$$N \leq M$$

no. of basis fun → N; M ← no. of message signals.

and $\boxed{N_{max} = M = 4}$

Step 1. first basis function

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{E_1}}$$

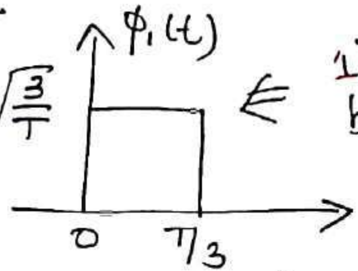
$$E_1 = \int_0^{T/3} x_1^2(t)\,dt = \int_0^{T/3} (1)^2 dt = 1 \cdot t\Big|_0^{T/3} = (T/3 - 0)$$

$\boxed{E_1 = T/3}$ Joules

and $\boxed{x_1(t) = \sqrt{E_1}\,\phi_1(t) = \sqrt{T/3}\,\phi_1(t)}$

$$\phi_1(t) = \frac{x_1(t)}{\sqrt{T/3}}$$

$$\boxed{\phi_1(t) = \sqrt{\frac{3}{T}}\, x_1(t)} \quad ; \; 0 \le t \le T/3$$



$\Leftarrow$ 1st orthonormal basis fn $\left[\text{ie} \int_0^{T/3} (\phi_1^2(t))\,dt = 1\right]$.

$E_{\phi_1(t)} = 1$ Joule.

Step 2

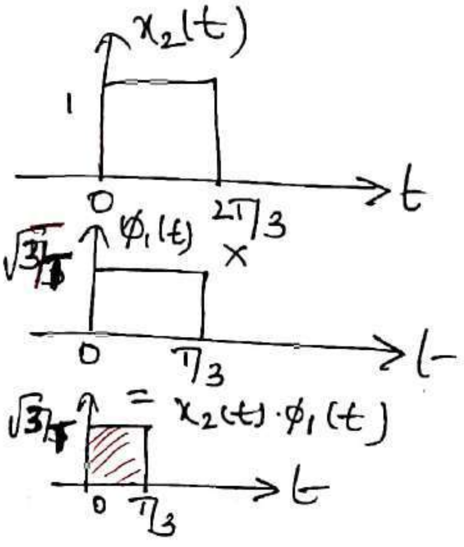
To find other basis fn, we need to go for intermediate signal w.r.t

$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\, \phi_j(t)$$

$i = 2$.

$$g_2(t) = x_2(t) - \sum_{j=1}^{1} x_{2j}\, \phi_j(t)$$

$$g_2(t) = x_2(t) - x_{21}\, \phi_1(t)$$



$$x_{21} = \int_0^T x_2(t)\, \phi_1(t)\, dt$$

$$= \int_0^{T/3} (1)\left(\sqrt{3/T}\right) dt$$

$$= \sqrt{\frac{3}{T}} \cdot t \Big|_0^{T/3} = \sqrt{\frac{3}{T}} \cdot T/3$$

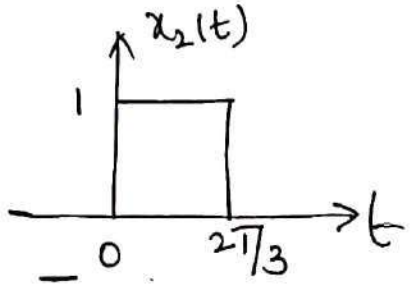
$$= \sqrt{\frac{3}{T}} \cdot \left(\sqrt{\frac{T}{3}}\right)^2 = \sqrt{\frac{3}{T}} \cdot \sqrt{\frac{T}{3}} \cdot \sqrt{T/3}$$

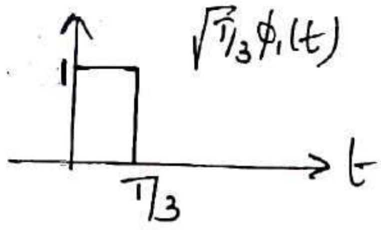
$$= \sqrt{\frac{3}{T} \cdot T/3} \cdot \sqrt{T/3} = \sqrt{T/3}$$

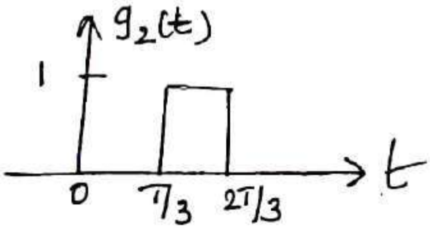
$$\boxed{x_{21} = \sqrt{T/3}}$$

plot $g_2(t)$.

$$g_2(t) = x_2(t) - \sqrt{\frac{T}{3}}\, \phi_1(t)$$







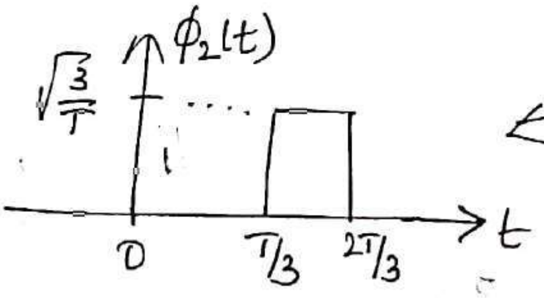
$$E_{g_2(t)} = \int_0^T g_2^2(t)\,dt$$

$$= \int_{T/3}^{2T/3} (1)^2\,dt$$

$$= t\Big|_{T/3}^{2T/3} = \left[\frac{2T}{3} - T/3\right]$$

$$\boxed{E_{g_2(t)} = T/3} \text{ Joules.}$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2(t)}}} = \frac{g_2(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\, g_2(t)$$



2nd orthonormal basis fn. $E_{\phi_2(t)} = 1$.

i.e $\int_0^T \phi_2^2(t)\,dt = 1$

Note:- No. overlap b/w $\phi_1(t)$ & $\phi_2(t)$.

$\Rightarrow$ ∴ both are orthogonal also.

i.e $\int_0^T \phi_1(t) \cdot \phi_2(t) \cdot dt = 0$.

$$\phi_1(t) = \begin{cases} \sqrt{3/T} ; & 0 \le t \le T/3 \\ 0 : & ow \end{cases}$$

$$\phi_2(t) = \begin{cases} \sqrt{3/T} ; & T/3 \le t \le \frac{2T}{3} \\ 0 : & ow \end{cases}$$

$$\therefore \phi_2(t) = \begin{cases} \sqrt{3/T} ; & T/3 \le t \le 2T/3 \\ 0 ; & ow. \end{cases}$$

Step 3 :- 3rd basis function.

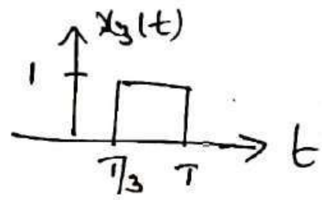
$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\,\phi_j(t)$$

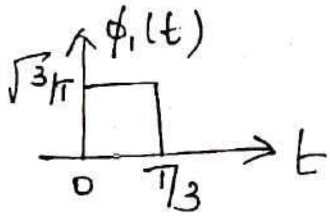
$i = 3$.

$$g_3(t) = x_3(t) - \sum_{j=1}^{2} x_{3j}\,\phi_j(t)$$

$$g_3(t) = x_3(t) - x_{31}\phi_1(t) - x_{32}\phi_2(t)$$

$$x_{31} = \int_0^T x_3(t)\cdot\phi_1(t)\,dt.$$

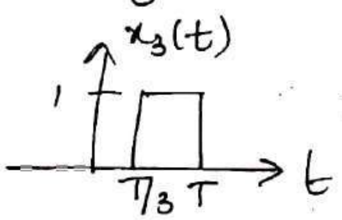


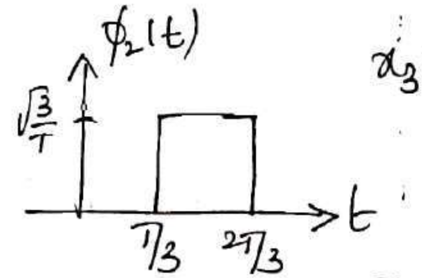


$\Rightarrow x_3(t)\cdot\phi_1(t) = 0$
(bcz no overlap)
(or) No common Area.

$$\boxed{x_{31} = 0}$$

$$x_{32} = \int_0^T x_3(t)\,\phi_2(t)\,dt$$





$$x_3(t)\cdot\phi_2(t) = \begin{cases} \sqrt{3/T} ; & T/3 \le t \le \frac{2T}{3} \\ 0 ; & ow \end{cases}$$

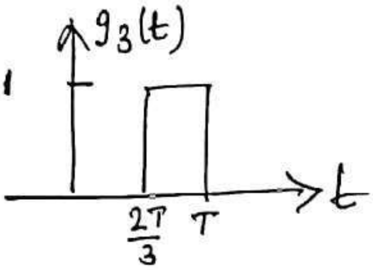
$$x_{32} = \int_{T/3}^{2T/3} (1)(\sqrt{\tfrac{3}{T}})\cdot dt = \sqrt{\tfrac{3}{T}}\cdot t\Big|_{T/3}^{2T/3} = \sqrt{\tfrac{3}{T}}\left[\tfrac{2T}{3} - T/3\right]$$

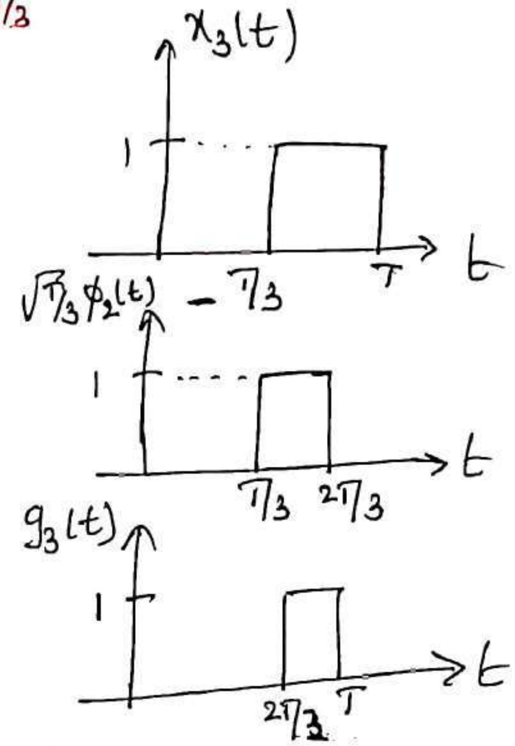
$$= \sqrt{\tfrac{3}{T}}\cdot T/3 = \sqrt{T/3}$$

$$\boxed{x_{32} = \sqrt{T/3}}$$

$$\therefore\ g_3(t) = x_3(t) - \cancelto{0}{x_{31}}\,\phi_1(t) - \underset{\downarrow\ \sqrt{T/3}}{\cancel{x_{32}}}\,\phi_2(t)$$

$$\boxed{g_3(t) = x_3(t) - \sqrt{T/3}\,\phi_2(t)}$$





$$E_{g_3(t)} = \int_0^T g_3^2(t)\cdot dt$$

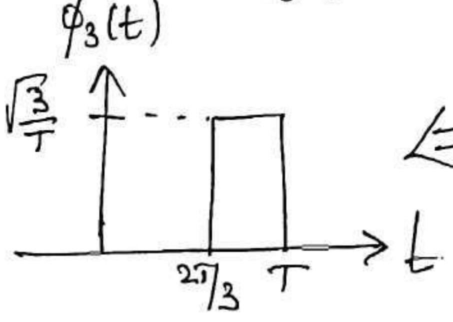
$$= \int_{2T/3}^{T} (1)^2\,dt = t\Big|_{2T/3}^{T}$$

$$= \left[T - 2T/3\right]$$

$$= \frac{3T-2T}{3} = T/3$$

$$\boxed{E_{g_3(t)} = T/3}\ \text{Joules}$$

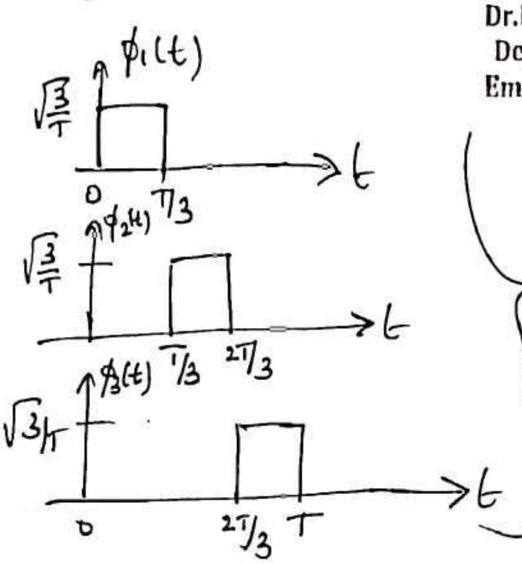
$$\phi_3(t) = \frac{g_3(t)}{\sqrt{E_{g_3(t)}}} = \frac{g_3(t)}{\sqrt{T/3}} = \sqrt{\frac{3}{T}}\,g_3(t)$$



$\Longleftarrow$ 3rd basis function (orthonormal).

$$\phi_3(t) = \begin{cases} \sqrt{3/T}\ ; & \frac{2T}{3} \le t \le T \\ 0\ ; & \text{ow} \end{cases}$$

Note:-



No overlap b/w basis f/s they are orthonormal (ie unitenergy) basis f/n and also orthogonal basis functi-ons (ie orthogonal to eachother).

Step 4:-

4th basis function (may or may not be exist. bcz $N \leq M$] . for $M=4$.

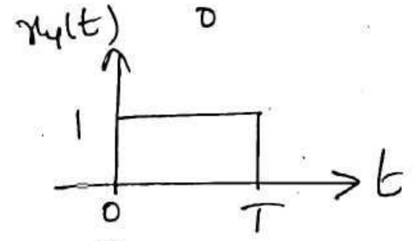
$$g_i(t) = x_i(t) - \sum_{j=1}^{i-1} x_{ij}\,\phi_j(t)$$

$i=4$

$$g_4(t) = x_4(t) - \sum_{j=1}^{4-1} x_{4j}\,\phi_j(t) = x_4(t) - \sum_{j=1}^{3} x_{4j}\,\phi_j(t)$$

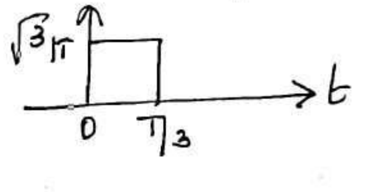
$$g_4(t) = x_4(t) - x_{41}\phi_1(t) - x_{42}\phi_2(t) - x_{43}\phi_3(t).$$

$$x_{41} = \int_0^T x_4(t)\cdot\phi_1(t)\,dt = \int_0^{T/3} (1)(\sqrt{3/T})\,dt = \sqrt{\frac{3}{T}}\cdot t\Big|_0^{T/3}$$

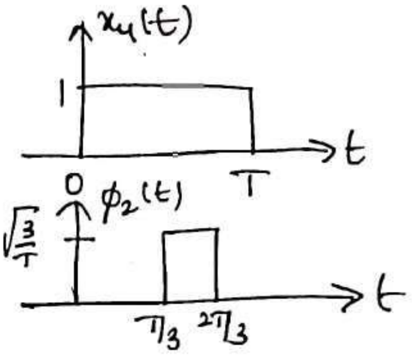


$$= \sqrt{\frac{3}{T}}\left[T/3 - 0\right] = \sqrt{T/3}$$

$$\boxed{x_{41} = \sqrt{T/3}}$$



$$x_{42} = \int_0^T x_4(t)\,\phi_2(t)\,dt$$



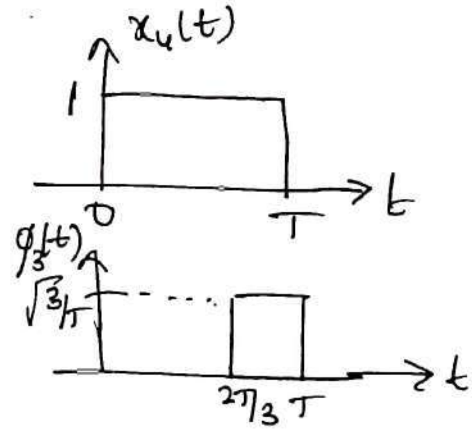
$$x_{42} = \int_{T/3}^{2T/3} (1)\cdot\left(\sqrt{\tfrac{3}{T}}\right)\cdot dt$$

$$= \sqrt{\tfrac{3}{T}}\; t\Big|_{T/3}^{2T/3} = \sqrt{\tfrac{3}{T}}\left[\tfrac{2T}{3} - T/3\right]$$

$$= \sqrt{\tfrac{3}{T}}\cdot(T/3) = \sqrt{T/3}$$

$$\boxed{x_{42} = \sqrt{\tfrac{T}{3}}}$$

$$x_{43} = \int_0^T x_4(t)\,\phi_3(t)\,dt$$



$$= \int_{2T/3}^{T} (1)\left(\sqrt{\tfrac{3}{T}}\right)\cdot dt$$

$$= \sqrt{\tfrac{3}{T}}\cdot t\Big|_{2T/3}^{T}$$

$$x_{43} = \sqrt{\tfrac{3}{T}}\left[T - 2T/3\right] = \sqrt{\tfrac{3}{T}}\left[T/3\right] = \sqrt{T/3}$$

$$\boxed{x_{43} = \sqrt{T/3}}$$

$$g_4(t) = x_4(t) - x_{41}\,\phi_1(t) - x_{42}\,\phi_2(t) - x_{43}\,\phi_3(t)$$

$$g_4(t) = x_4(t) - \sqrt{\tfrac{T}{3}}\left[\underset{\substack{\downarrow\\ 0\le t\le T/3}}{\phi_1(t)} + \underset{\substack{\downarrow\\ T/3\le t\le 2T/3}}{\phi_2(t)} + \underset{\substack{\downarrow\\ 2T/3\le t\le T}}{\phi_3(t)}\right]$$

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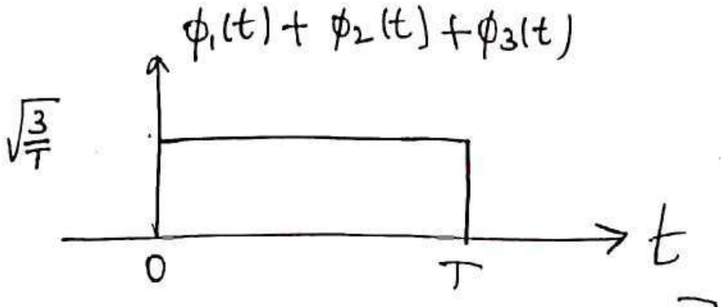
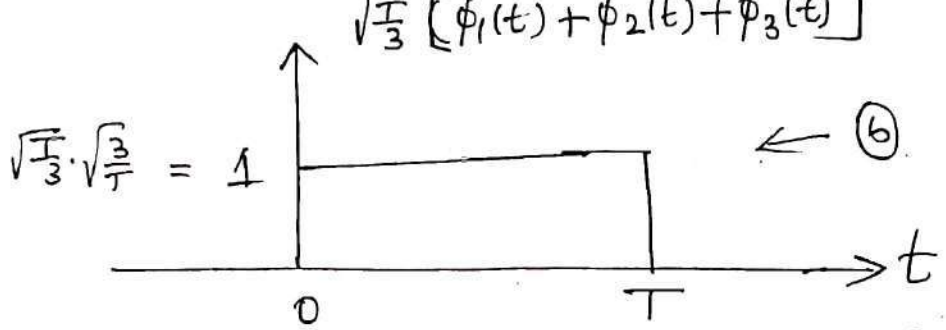
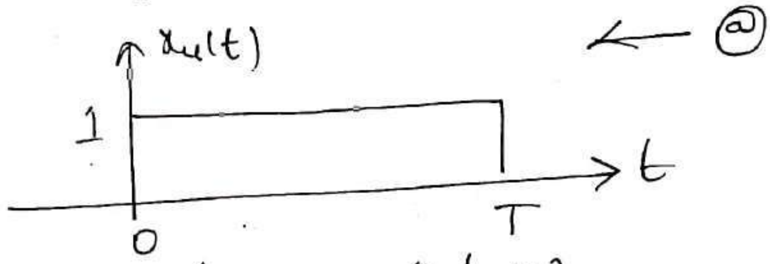






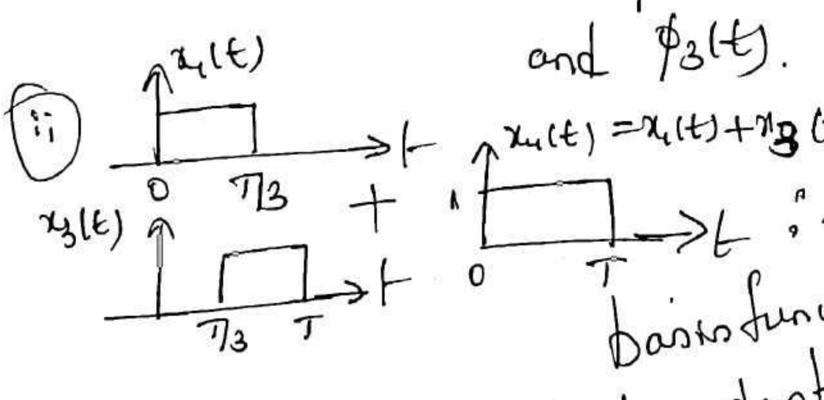
graph ⓐ − graph (b)

$$x_4(t) - \sqrt{T/3}\left[\phi_1(t)+\phi_2(t)+\phi_3(t)\right] = 0.$$

$$\boxed{\phi_4(t) = 0}$$

Obs:- (i) No. of orthonormal basis functions is equal to 3. ie $\phi_1(t)$, $\phi_2(t)$ and $\phi_3(t)$.

(ii)



∴ the no. of orthonormal basis functions is equal to no. of independent message signals.

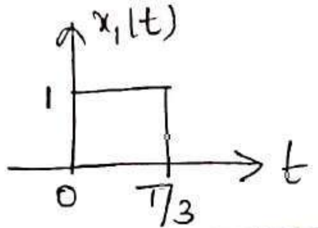
N = 3 = no of independent message signals ie $x_1(t)$, $x_2(t)$, & $x_3(t)$.

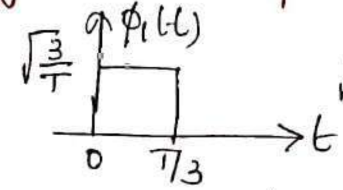
iii.

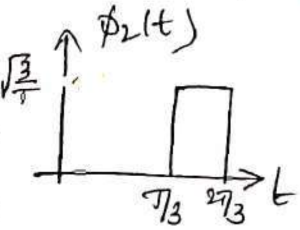
i.e only three signals $x_1(t)$, $x_2(t)$ and $x_3(t)$ are Linearly independent. and the Last signal i.e $x_4(t)$ is Linearly dependent [bcz $x_4(t) = x_1(t) + x_3(t)$].

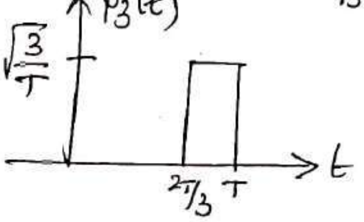
$$\therefore \boxed{\text{No. of basis fn } (N) = \text{No of independent msg Signals} = 03}$$

How to represent given four signals interms of basis functions?

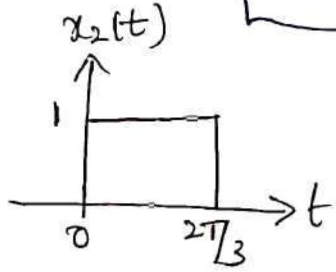






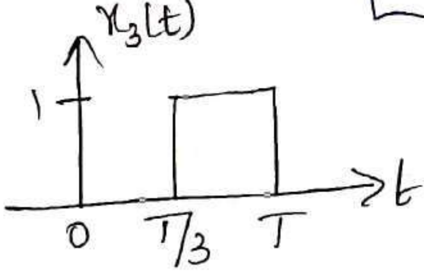


$$\Rightarrow \boxed{x_1(t) = \sqrt{\frac{T}{3}}\phi_1(t)}$$

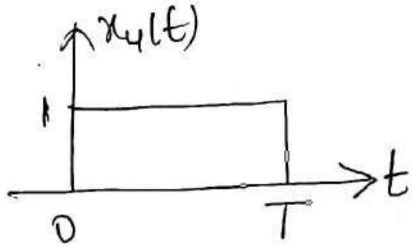


$$x_2(t) = \sqrt{\frac{T}{3}}[\phi_1(t) + \phi_2(t)]$$

$$\boxed{x_2(t) = \sqrt{T/3}\,\phi_1(t) + \sqrt{T/3}\,\phi_2(t)}$$



$$\boxed{x_3(t) = \sqrt{T/3}\,\phi_2(t) + \sqrt{T/3}\,\phi_3(t)}$$

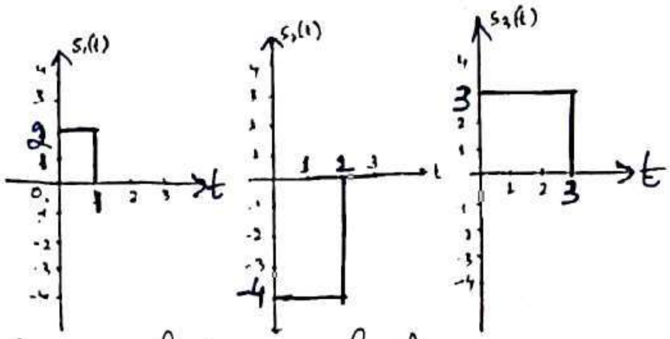


$$x_4(t) = x_1(t) + x_3(t)$$

$$= \sqrt{\frac{T}{3}}\phi_1(t) + \sqrt{\frac{T}{3}}\phi_2(t) + \sqrt{T/3}\,\phi_3(t)$$

$$\boxed{x_4(t) = \sqrt{\frac{T}{3}}[\phi_1(t) + \phi_2(t) + \phi_3(t)]}$$

Using the Gram-Schmidt orthogonalization procedure, find a set of orthonormal basis functions to represent the three $s_1(t), s_2(t)$ and $s_3(t)$ shown in Fig. Also express each of these signals in terms of the set of basis functions. **(08 Marks) Dec 2018-Jan 2019.**



i. $N = 2$

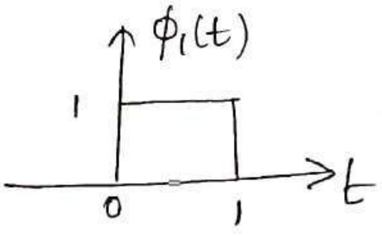
ii. Expren $S_1(t), S_2(t), S_3(t)$ interm of basis functions.

Soln:- Step1 :- first basis function

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$E_1 = \int_0^T S_1^2(t)\,dt = \int_0^1 (2)^2 dt = 4[1-0] = 4 \text{ Joules}$$

$$\phi_1(t) = \frac{S_1(t)}{\sqrt{4}} = \frac{1}{2} S_1(t)$$



$$\phi_1(t) = \begin{cases} 1 ; & 0 \le t \le 1 \\ 0 : & ow \end{cases}$$

Step2 :- 2nd basis function.

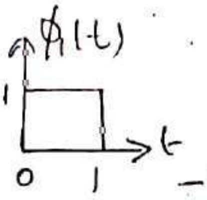
intermediate signal

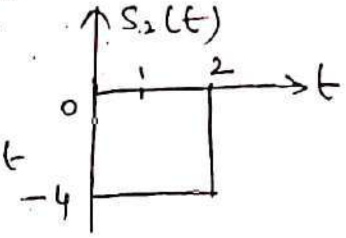
$$g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\,\phi_j(t)$$

$i=2$ $$g_2(t) = S_2(t) - \sum_{j=1} S_{2j}\,\phi_j(t)$$

$g_2(t) = s_2(t) - s_{21}\phi_1(t)$

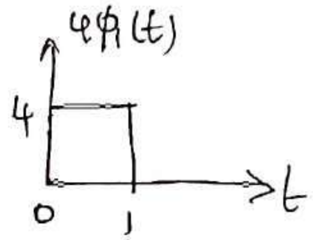
$s_{21} = \int_0^T s_2(t)\phi_1(t)\,dt$;





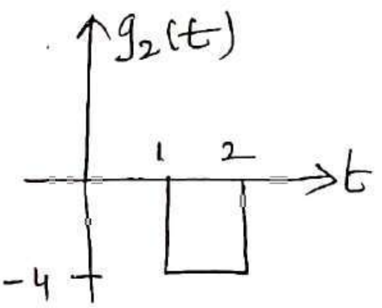
$S_{21} = \int_0^1 (-4)(1)\cdot dt = -4\, t|_0^1 = -4(1-0) = -4$

$\boxed{S_{21} = -4}$



$g_2(t) = s_2(t) - (-4)\phi_1(t)$

$g_2(t) = s_2(t) + 4\phi_1(t)$.

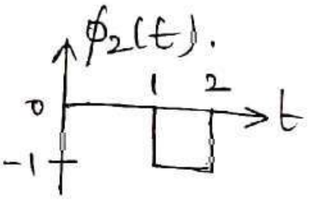


$E_{g_2(t)} = \int_1^2 g_2^2(t)\,dt = \int_1^2 (-4)^2 dt$

$= 16[2-1] = 16$ Joules.

2nd basis fn

$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2(t)}}} = \frac{g_2(t)}{\sqrt{16}} = \frac{1}{4} g_2(t)$



$\phi_2(t) = \begin{cases} -1 ; & 1 \le t \le 2 \\ 0 ; & \text{ow.} \end{cases}$

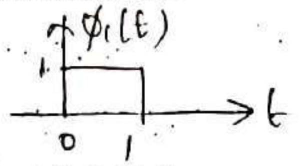
Note:- No overlap b/w $\phi_1(t)$ and $\phi_2(t)$ ∴ they are ortho-normal basis fn as well as orthogonal.

$i=3$

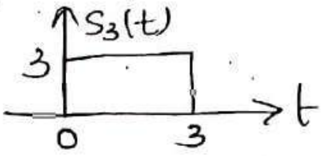
$g_3(t) = s_3(t) - \sum_{j=1}^{2} s_{3j}\phi_j(t)$

$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$

$$S_{3_1} = \int_0^T s_3(t)\,\phi_1(t)\,dt$$

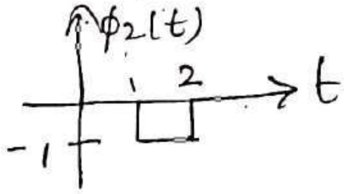


$$= \int_0^1 (3)(1)\,dt = 3[1-0] = 3.$$



$$\boxed{S_{3_1} = 3}$$

$$S_{3_2} = \int_0^T s_3(t)\cdot\phi_2(t)\,dt$$



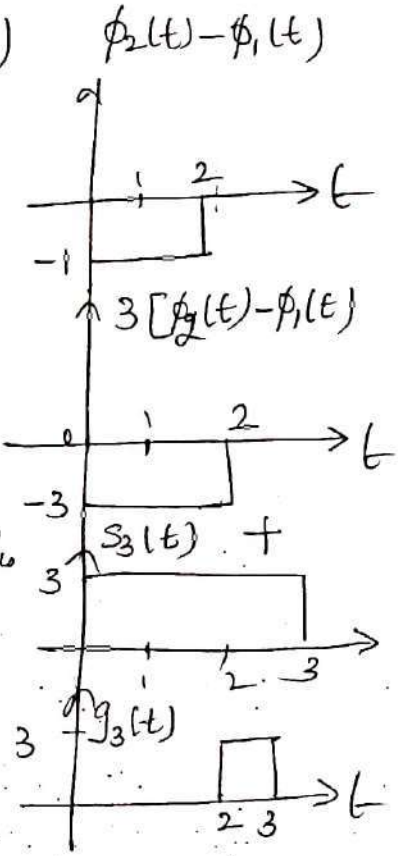
$$= \int_1^2 (3)(-1)\,dt = -3\,t\Big|_1^2 = -3(2-1) = -3.$$

$$\boxed{S_{3_2} = -3}$$

$$\therefore g_3(t) = S_3(t) - 3\phi_1(t) - (-3)\phi_2(t)$$

$$= S_3(t) - 3\phi_1(t) + 3\phi_2(t)$$

$$= S_3(t) + 3[\phi_2(t) - \phi_1(t)].$$



$$E_{g_3(t)} = \int_2^3 g_3^2(t)\,dt = \int_2^3 (3)^2\,dt = 9\cdot t\Big|_2^3$$

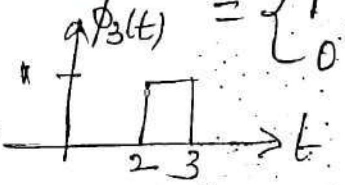
$$= 9[3-2] = 9 \text{ Joule}$$

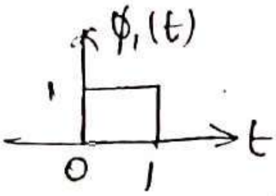
3rd basis fn

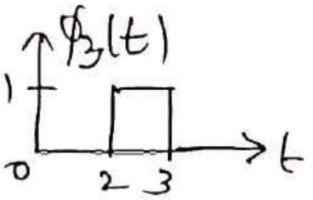
$$\phi_3(t) = \frac{g_3(t)}{\sqrt{E_{g_3(t)}}} = \frac{g_3(t)}{\sqrt{9}}$$

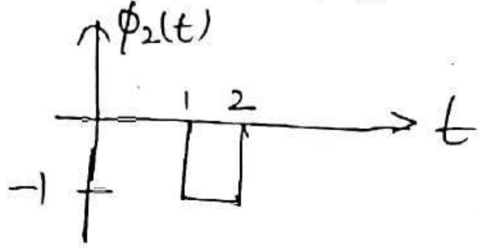
$$\boxed{\phi_3(t) = \frac{1}{3}\,g_3(t)}$$

$$= \begin{cases} 1; & 2 \le t \le 3 \\ 0; & ow \end{cases}$$







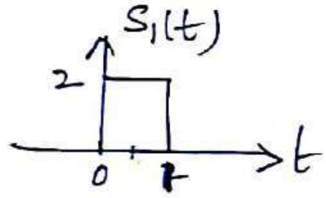


No-overlapping
∴ all the three basis fun are orthogonal to each other. and they have unit energy ∴ they are orthonormal.

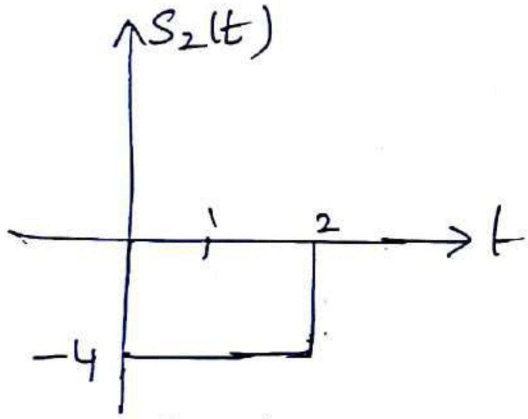
$M = 3, \quad N = 3.$

No. of basis f^{n} (N) = No. of independent signal^{s} = 3 [i.e. $S_1(t)$, $S_2(t)$ & $S_3(t)$ all are independent to each other]. [Cannot be expressed as a Linear Comb^{n}]

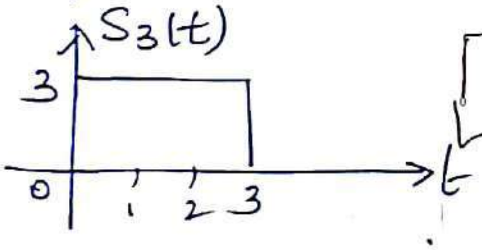
$S_1(t)$, $S_2(t)$ and $S_3(t)$ in terms of Basis fun?



$$\boxed{S_1(t) = 2\phi_1(t)}$$



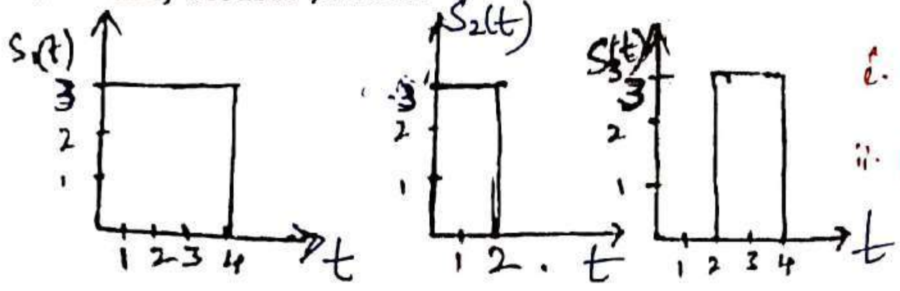
$$\boxed{S_2(t) = -4\phi_1(t) + 4\phi_2(t)}$$



$$\boxed{S_3(t) = 3\phi_1(t) - 3\phi_2(t) + 3\phi_3(t)}$$

no. of basis fun required $N = 3$

Three signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ shown in Fig. Apply Gram Schmidt procedure to obtain an orthonormal basis for the signals. Express signals $s_1(t)$, $s_2(t)$ and $s_3(t)$ in terms of orthonormal basis functions. **(08 Marks) Dec 2018-Jan 2019.**



i. N = ? (No of basis fu's.
ii. Expres s_1, s_2, s_3 interms of basis fu'n.

Soln:-

no. of orthonormal function's (N) = no. of independent signals = 2.

[i.e $s_2(t)$ & $s_3(t)$ are independent]

and ⇒ ∴ $s_1(t)$ is dependent
bcz $s_1(t) = s_2(t) + s_3(t)$
[i.e linear Combination].

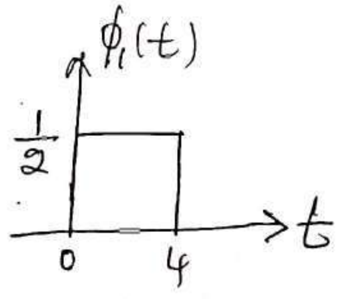
Step1. first basis fu

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

$$E_1 = \int_0^T s_1^2(t)\,dt = \int_0^4 (3)^2 dt = 9 \cdot t\Big|_0^4 = 9[4-0]$$

$$\boxed{E_1 = 36 \text{ Joules}}$$

$$\therefore \phi_1(t) = \frac{s_1(t)}{\sqrt{36}} = \frac{1}{6}\,s_1(t)$$



$$\phi_1(t) = \begin{cases} \frac{1}{2} \; ; & 0 \le t \le 4 \\ 0 \; ; & \text{ow} \end{cases}$$

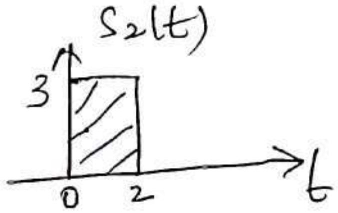
Step2. To find 2nd orthonormal fn.

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t)$$ ← 2nd intermediate Signal.

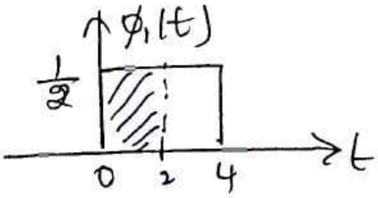
i=2

$$g_2(t) = s_2(t) - \sum_{j=1}^{1} s_{2j}\,\phi_j(t)$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$



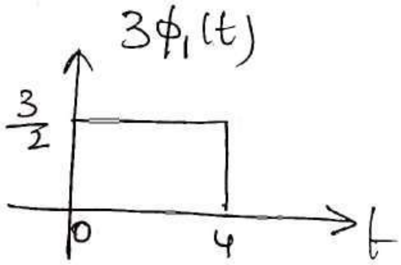
$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$$



$$= \int_0^2 (3)\left(\frac{1}{2}\right)dt = \frac{3}{2}\,t\Big|_0^2$$

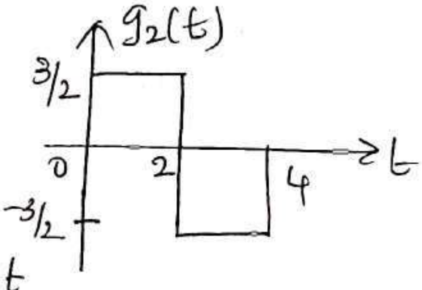
$$= \frac{3}{2}\left[2-0\right] = 3$$

$$\boxed{s_{21} = 3}$$



$$g_2(t) = s_2(t) - 3\,\phi_1(t)$$

$$= \begin{cases} \cancel{(3-3/2)}^{-3/2} \; ; & 0 \le t \le 2 \\ (0-3/2) \; ; & 2 \le t \le 4 \end{cases}$$



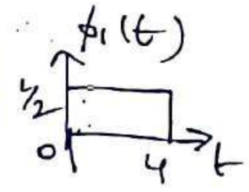
$$E_{g_2(t)} = \int_0^T g_2^2(t)\,dt = \int_0^2 (3/2)^2 dt + \int_2^4 (-3/2)^2 dt$$

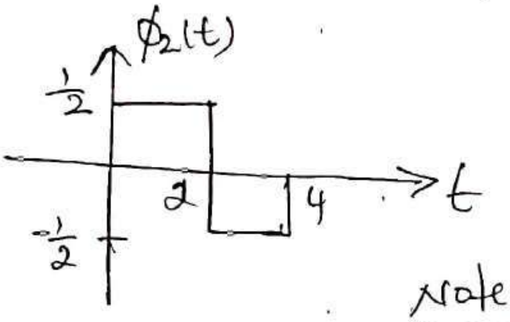
$$= \frac{9}{4}\cdot t\Big|_0^2 + 9/4\; t\Big|_2^4 = \frac{9}{4}\left[(2-0)+(4-2)\right]$$

$$E_{g_2(t)} = \frac{9}{4}\left[2+2\right] = 9/4 \times 4 = 9$$ Joules.

2nd basis function.

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2(t)}}} = \frac{g_2(t)}{\sqrt{9}} = \frac{1}{3}\, g_2(t)$$





$$\phi_2(t) = \begin{cases} 1/2 \, ; & 0 \le t \le 2 \\ -1/2 \, ; & 2 \le t \le 4. \end{cases}$$

Note:- $\int_0^4 \phi_1(t) \cdot \phi_2(t)\, dt = 0$.

$$= \int_0^2 (1/2)(\tfrac{1}{2})\, dt + \int_2^4 (-\tfrac{1}{2})(-\tfrac{1}{2})\, dt$$

$$= \frac{1}{4}\, t\Big|_0^2 - \frac{1}{4}\, t\Big|_2^4$$

$$= \frac{1}{4}[2-0] - \frac{1}{4}[4-2]$$

$$= \frac{1}{4}[2-2] = \underline{0}.$$

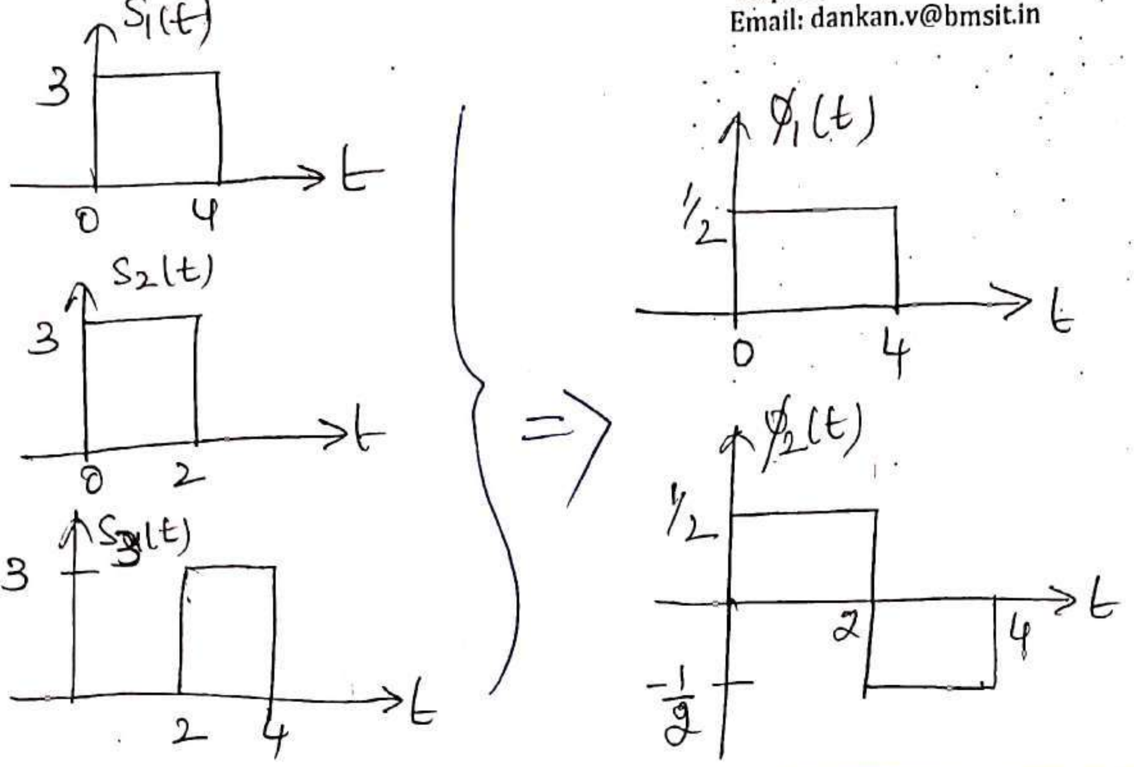
i.e $\phi_1(t)$ and $\phi_2(t)$ are orthogonal to each other.

3rd basis fn

$\phi_3(t) = 0$.

bcz only two basis fn's are exist since no. of independent message signals are two.

Express $S_1(t)$, $S_2(t)$ and $S_3(t)$ in terms of basis functions.



$$\boxed{S_1(t) = 6\phi_1(t)}$$

$$S_2(t) = 3[\phi_1(t) + \phi_2(t)]$$

$$\boxed{S_2(t) = 3\phi_1(t) + 3\phi_2(t)}$$

To draw Signal Constellation diagram.

$S_1(t) = 6\phi_1(t)$

$S_2(t) = 3\phi_1(t) + \phi_2(t)$

$S_3(t) = 3\phi_1(t) - 3\phi_2(t)$

w.k.t $S_3(t) = S_1(t) - S_2(t)$

$$= 6\phi_1(t) - 3\phi_1(t) - 3\phi_2(t)$$

$$\boxed{S_3(t) = 3\phi_1(t) - 3\phi_2(t)}$$

there are two basis fn's (ϕ_1, ϕ_2) ∴ it is two dimensional.

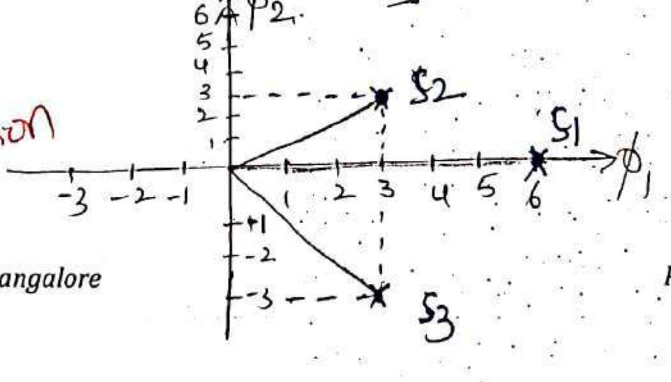


fig:- 2D-Signal Constellation diagram.

Representation of Finite energy Signals as Vectors:-

2.3 Conversion of the continuous AWGN channel into a vector channel

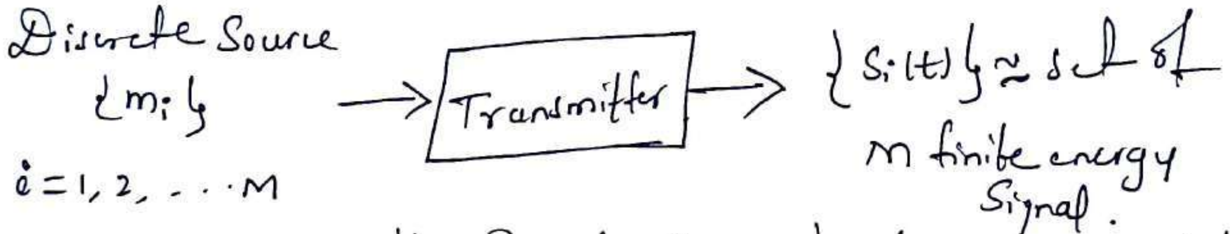
Derive the expression for mean and variance of the correlator outputs. Also show that the correlator outputs are statistically independent. **(08 Marks) June-July 2018.**

Show that for a noisy input, the mean value of the j^{th} correlator output X_j depends only on S_{ij} and all the correlators outputs X_j, j=1,2,.......N, have a variance equal to the PSD $^{N_0}/_2$ of the additive noise process w(t). **(08 Marks) June-July 2019.**

a. Find Mean of X_l.
b. Variance of X_l
c. S.T o/p is Mutually uncorrelated (or) (independent)

Soln:- * Representation of Finite energy Signals as vectors is the basic step in digital Communication System and this process of representation is called "Gram-schmidt orthogona-lization" procedure [GSOP].

* w.k.t at transmitter End



Transmitter converts Discrete Source {m_i} message symbols into set of 'M' finite energy signal having 'M' dimension.

* Using GSOP.
$S_i(t)$ is represented by Linear combination of N-ortho-normal basis function. with dimension N. which is [$N \leq M$].

@

N-orthonormal basis functions represented by

$\{\phi_1(t), \phi_2(t), \ldots \phi_N(t)\}$ $(N \leq M)$.

'N' no. of Linearly independent function's are in the set of M Signals.

Note:-

i.e N-orthonormal basis function $\simeq$ Each of duration T Sec and having unit Energy called Basis function.

According to GSOP:-

$$\int_0^T \phi_l(t)\, \phi_m(t)\, dt = \begin{cases} 1 : \text{if } l = m \\ 0 : \text{if } l \neq m. \end{cases}$$

which is used to represent $S_i(t)$.

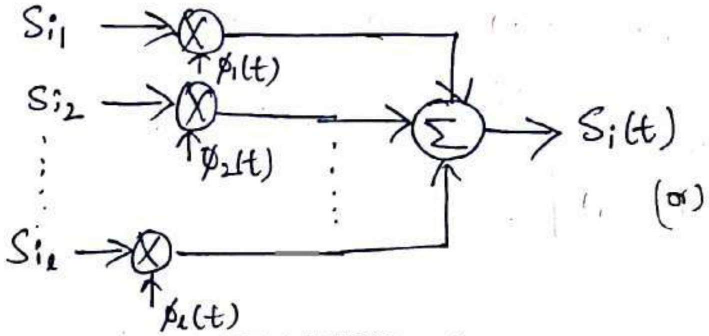
Consider set of 'M' finite energy signal

$$\{S_i(t)\} = \underbrace{S_{i1}\phi_1(t) + S_{i2}\phi_2(t)}_{\uparrow} + \cdots + S_{il}\,\phi_l(t)$$

$i = 1, 2, 3 \ldots M$

$S_i(t)$ is a Linear combination of N-orthonormal basis function $\{\phi_l(t)\}$ where $l = 1, 2, 3 \ldots N$.

where $S_{i1}, S_{i2}, \ldots S_{il}$ are Signal coefficient's.

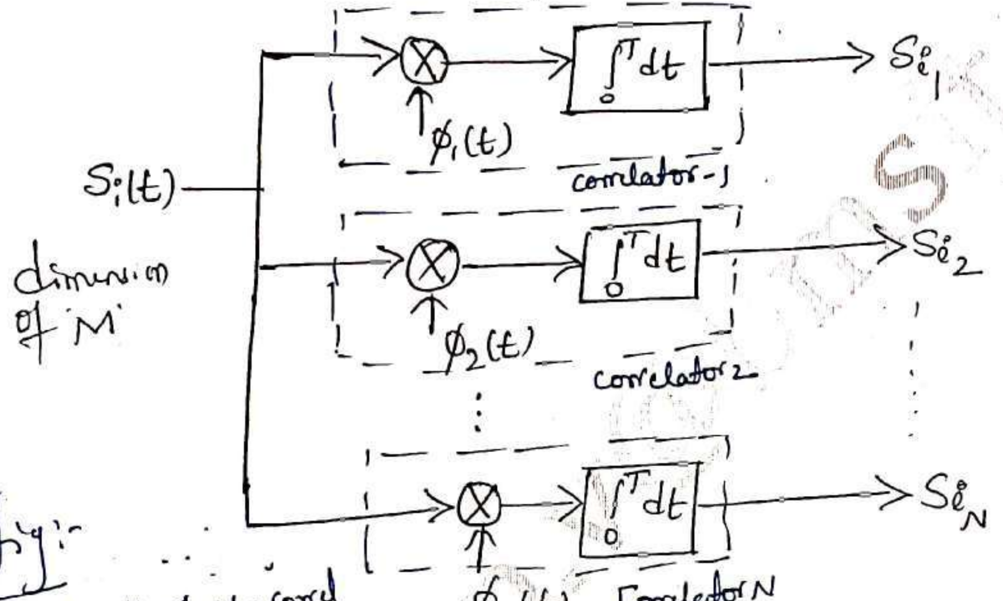


$$\text{(or)} \quad S_i(t) = \sum_{l=1}^{M} S_{il}\, \phi_l(t)$$

* There 'M' Finite energy Signals are transmitted over a channel having (duration Every 'T' sec) duration Every 'T' Seconds.

At Receiver side :-

* at Receiver End, there is a BANK OF N - CORRELATORS



Signal vector

$$\vec{S_i} = \begin{bmatrix} S_{i_1} \\ S_{i_2} \\ \vdots \\ S_{i_N} \end{bmatrix}$$

In N-dimensional Euclidean space (Signal space)

fig:- BANK of N-correlator

Each correlator output $\boxed{S_{i_l} = \int_0^T S_i(t)\, \phi_l(t)\, dt} \leftarrow$ ⓐ

$i = 1, 2, 3 \ldots M$

$l = 1, 2, 3 \ldots N$

and $[N \leq M]$

Where.. $\boxed{S_i(t) = \sum_{l=1}^{N} S_{il}\, \phi_l(t)} \leftarrow$ ⓑ

eqⁿ ⓑ in eqⁿ ⓐ

$$\int_0^T \left(\sum_{l=1}^{N} s_{il}\, \phi_l(t) \right) \phi_l(t)\, dt$$

$$= \sum_{l=1}^{N} s_{il} \int_0^T \cancelto{1}{\phi_l(t)\phi_l(t)}\, dt.$$

S_{il}, $l = 1, 2, 3 \ldots N$

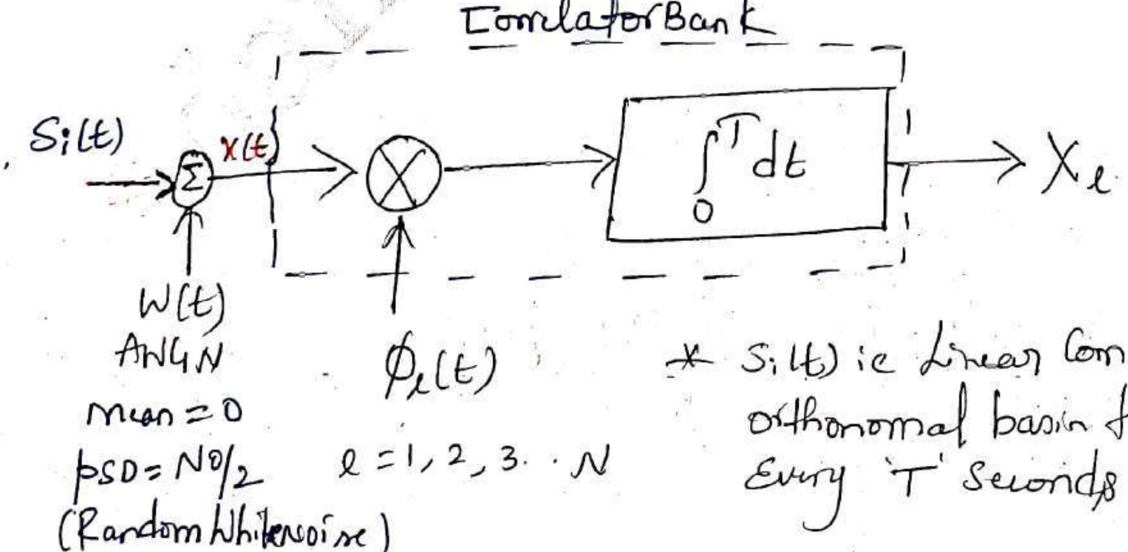
$$= \sum_{l=1}^{N} S_{il}$$

* Representation of signals in N-dimensional signal space is called Euclidean space.

xx. Response of Correlator Bank in presence of Noisy input

Consider a Bank of N correlator with input $x(t)$.

Correlator Bank



* $S_i(t)$ ie Linear Combⁿ of N-orthonormal basis fⁿ transmits Every 'T' seconds.

* The correlator input x(t) is given by

$$\boxed{x(t) = S_i(t) + W(t)}$$

$0 \le t \le T$, $i = 1, 2, \cdots M$.

* The correlator output denoted by X_l and given by

$$X_l = \int_0^T x(t)\,\phi_l(t)\cdot dt \qquad l = , 1, 2, 3 \cdots N$$

$$= \int_0^T \left[S_i(t) + w(t) \right] \cdot \phi_l(t)\, dt$$

$$X_l = \underbrace{\int_0^T S_i(t)\,\phi_l(t)\,dt}_{S_{il}} + \int_0^T \underline{W(t)\,\phi_l(t)\,dt}.$$

1st term Deterministic quantity (Transmitted signal $S_i(t)$ is known) denoted by (S_{il})

2nd term is Random variable due to the AWGN. denoted by (W_l)

Correlator input:-

* $$\underbrace{x(t)}_{\text{Gaussian Random process (RP)}} = \underbrace{S_i(t)}_{\text{Deterministic Signal}} + \underbrace{W(t)}_{\text{Gaussian Noise}}$$

* Correlator output :-

$$\underset{\substack{\uparrow \\ \text{Gaussian} \\ \text{Random} \\ \text{Variable.}}}{X_l} = S_{il} + W_l$$

* The output of the correlator is Characterised by Mean and Variance.

a. **Mean** of $x_l = S_{il} + W_l$

$$E[x_l] = E[S_{il} + W_l]$$

$$= E[S_{il}] + E[W_l]$$

$$\uparrow \text{Deterministic Signal}$$

$$E[x_l] = S_{il} + 0$$

$$\boxed{E[x_l] = S_{il}}$$

Note:-
$E[\text{Constant}] = \text{Constant}$
$V[\text{Constant}] = 0.$

$E[W_l]$ = Gaussian noise with zero mean.
$= 0$

b. **Variance** :- $\sigma_{x_l}^2 = Var[x_l] = Var[S_{il} + W_l]$

$$= Var[S_{il}] + Var[W_l]$$

$$\uparrow \text{Deterministic} \qquad \uparrow PSD = \frac{N_0}{2} (AWGN)$$

$$= 0 + \frac{N_0}{2}$$

$$\boxed{Var[x_l] = N_0/2}$$

Conclusion :- The output correlator having Mean value of $E[x_l] = S_{il}$ which in deterministic Signal and Variance $Var[x_l] = \sigma_{x_l}^2 = N_0/2$ that in the PSD of AWGN.

Note:- i. Correlator output is given by

$$\boxed{X_l = S_{il} + W_l}$$

Mean value $\boxed{E[X_l] = S_{il}}$: which is deterministic signal.

$$\boxed{Var[x_l] = \sigma_{x_l}^2 = N_0/2}$$; which is psd of AWGN.

ii. Correlator output X_l ; $l = 1, 2, 3 \ldots N$ have mean value of S_{il} and equal variance $= N_0/2$ i.e P.S.D of AWGN.

iii. Thus the output of correlator (X_l) under Noisy input depends on the mean value S_{il} and variance equal to the PSD $N_0/2$ of the additive noise process $w(t)$.

c. To show that X_l are Mutually uncorrelated (or) correlator o/p X_l are statistically independent.

$$Cov[X_l X_k] = \frac{N_0}{2} \int_0^T \phi_l(t)\, \phi_k(t)\, dt$$

$$= \frac{N_0}{2} (0)$$

bcz $\phi_l(t)$ & $\phi_k(t)$ are orthogonal to each other.

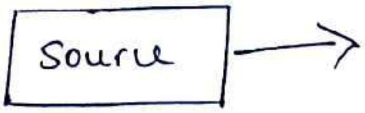
$$\boxed{Cov(X_l, X_k) = 0}$$

∴ Correlator o/p X_l are statistically independent.

Estimation of Known Signal in AWGN channel :-

(or) Optimum Receiver using coherent detection.

Step 1.



M messages / Symbols = $\{m_1, m_2, m_3 \cdots m_M\}$

also Each message Symbols are converted into

$m_1 \longrightarrow S_1(t)$

$m_2 \longrightarrow S_2(t)$

$\vdots \qquad \vdots \qquad \vdots$

$m_M \longrightarrow S_M(t)$

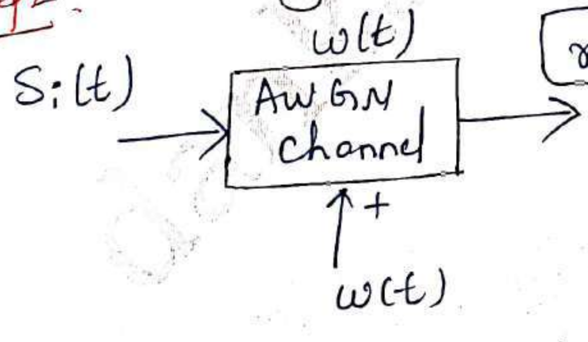
$S_i(t)$

Each message is equiprobable i.e probability $= \frac{1}{M}$.

Corresponding Transmitted Signals $S_i(t)$ Each of duration T sec.

→ During Transmission through channel, Noise will be get added.

Step 2.



$$\boxed{x(t) = S_i(t) + w(t)}$$

$0 \leq t \leq T$.

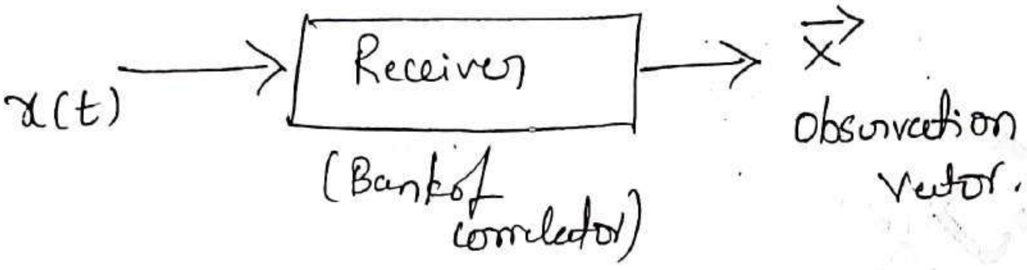
$$x(t) = S_i(t) + w(t) \quad ; \; 0 \leq t \leq T$$

$x(t)$ → Sample function of received Rp $x(t)$

$w(t)$ → Sample fn of AWGN $w(t)$. Mean = 0 Variance PSD $= \frac{N_0}{2}$.

I.

Step 3. Received Signal $x(t)$ is passed through a Bank of correlators.



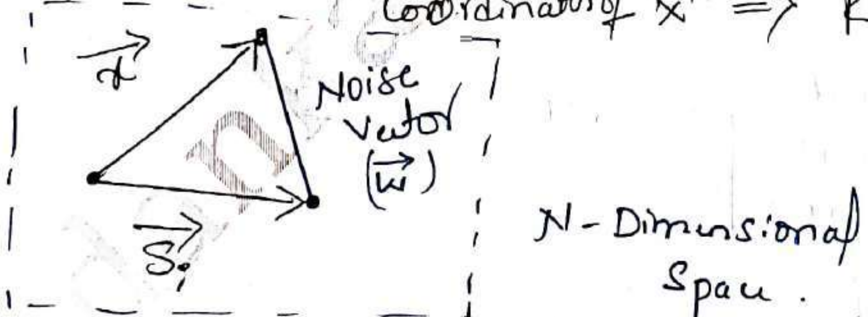
$\vec{X}$ - represents a sample value of random vector

$$\boxed{\vec{X} = \vec{S_i} + \vec{W}}$$

Observation Space:-

Co-ordinates of $\vec{S_i}$ => transmitted signal point

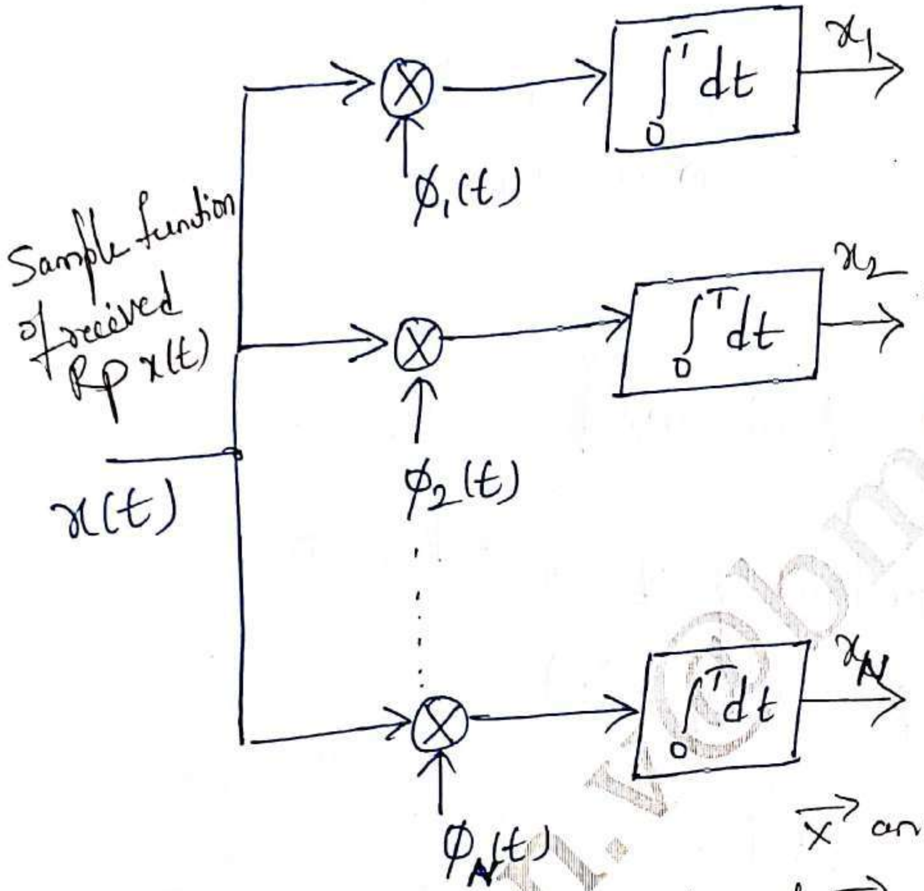
Coordinates of $\vec{x}$ => Received signal point.



Difference b/w $\vec{x}$ and $\vec{S_i}$ represents $\vec{w}$.

At the Receiver End :-

BANK - OF - N - Corrdators.



observation vector

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_N \end{bmatrix}$$

$$\boxed{\vec{x} = \vec{s_i} + \vec{w}}$$

$\vec{x}$ and $\vec{w}$ are Sample value of $\vec{x}$ & $\vec{w}$ called Random vectors.

→ using $\vec{x}$ $S_i(t)$ canbe determined.

$$S_i(t) \xrightarrow[\text{RULE}]{\text{DECISION}} \hat{m} \text{ (Estimate value)}$$

ie transmitted Signal

Decision Rule :-

i. In order to calculate estimate value $\hat{m}$ corresponding to the transmitted signal $s_i(t)$, the Average probability of Symbol Error should be Minimum. [minimized].

let us Consider

* Suppose estimate value $\hat{m}$ is equal to transmitted Symbol m_i

$$\boxed{\hat{m} = m_i}$$

then the average probability of correct decision is given by the conditional probability (P_c)

Conditional Probability (P_c) - Represents the probability of estimation of transmitted signal m_i then observation vector $\vec{x}$ is already received.

mathematically

$$P_c = P(m_i \text{ sent} \mid \vec{x} \text{ received})$$

using the concept of Conditional probability, Average Probability of Symbol Error is obtained and is denoted by

$$\boxed{P_e = 1 - P_c}$$

$$\boxed{P_e = 1 - P(m_i \text{ sent} \,|\, \vec{x} \text{ received})}$$

According to the decision rule Avg. probability of Symbol Error ~~is~~ must be minimized.

$$\boxed{P_e \simeq \text{minimized.}}$$

$$\Big\downarrow \text{ Can be done only when (Required) possible.}$$

$$\boxed{P_c \simeq \text{Maximized.}}$$

This results to the optimum Decision Rule. which is mathematically written as

$$\boxed{P(m_k \text{ sent} \,|\, \vec{x} \text{ received}) \simeq \text{Maximum.}}$$

called as optimum decision rule.

So, In order to Estimate received symbol $\hat{m}$ is equal to m_i [i.e $\hat{m} = m_i$]

$$\boxed{P(m_i \text{ sent} \,|\, \vec{x} \text{ received}) \geq P(m_k \text{ sent} \,|\, \vec{x} \text{ received})}$$

transmitted Symbol

for all $k = 1, 2, \ldots M$
$k \neq i$

V

$$\boxed{P(m_i \text{ sent} \mid \vec{x} \text{ received}) \geq P(m_k \text{ sent} \mid \vec{x} \text{ received})}$$

for all $k = 1, 2, 3 \cdots M$

$k \neq i$

This decision rule can also be represented interms of Likelihood function

$$\boxed{f_{\vec{X}}(\vec{x}/m_i) = \prod_{l=1}^{N} f_{X_l}(x_l/m_i)}$$

$i = 1, 2, 3 \cdots M$

$l = 1, 2, \cdots N$

Conditional pdf and received vector $\vec{x}$
(Given m_i was transmitted)

V1

2.4 Optimum receivers using coherent detection

2.4.1 ML Decoding

Obtain the Maximum Likelihood decision rule for the signal detection problem. **(10 Marks) June-July 2018.**

soln:- Let the observation vector be x. The decision is made as $\hat{m} = m_i$.

The average probability of symbol error is this decision is

$$P_e(m_i, x) = P(m_i \text{ not sent} \mid x)$$

$$= 1 - P(m_i \text{ sent} \mid x) \leftarrow ①$$

where $P_e(m_i, x)$ indicates average probability of symbol Error where 'x' is the observation vector and message m_i is selected.

- To minimize the Error probability $P_e(m_i, x)$ given by above equation the optimum decision rule can be stated as,

$$\text{Set } \hat{m} = m_i \text{ if } P(m_i \text{ sent}/x) \geq P(m_k \text{ sent}/x)$$

$$\text{for all } k \neq i. \leftarrow ②$$

- This decision rule can be represented graphically. Let R-denote the N-dimensional space of all possible

(2)-1

vectors 'x'. and this region be partitioned into 'M' decision regions. The decision rule can then be written as,

Vector x lies in region R_i if $[\ln f_X(x|m_k)]$ is maximum for $k=i$. ← ③

Where $f_X(x|m_k)$ is the Likelihood function which results when symbol m_k is transmitted. This rule called Maximum Likelihood and the corresponding detector which uses this rule is called "Maximum Likelihood detector".

* The decision rule of eqⁿ ③ can be re-written alternatively as, Vector x Lies in region R_i if $||x-s_k||$ is Minimum for $k=i$.

Here $||x-s_k||$ is the distance b/w the received signal point and the message point.

The Maximum Likelihood decision rule chooses the message point closest to the received signal point.

2-2

Maximum Likelihood decision Rule.

Likelihood function :- $f_{\vec{X}}(\vec{x}/m_i) = \prod_{l=1}^{N} f_{X_l}(x_l/m_i)$

Decision rule to estimate :- $f_{\vec{X}}(\vec{x}/m_i) \approx$ Maximum for $k = l$.

$\hat{m} = m_i$

$$\Big\Downarrow$$ Using Log function (monotonically increasing function)

$$\boxed{\ln f_{\vec{X}}(\vec{x}/m_i) \text{ is Maximum for } k = l}$$

i.e Maximum Likelihood Decision = Maximum Likelihood Receiver.

$$f_{\vec{X}}(\vec{x}/m_i) = (\pi N_0)^{-N/2} \left\{ e^{-\frac{1}{N_0}\sum_{l=1}^{N}(x_l - S_{kl})^2} \right\}$$

$k = 1, 2, 3 \cdots M$

taking Natural Log.

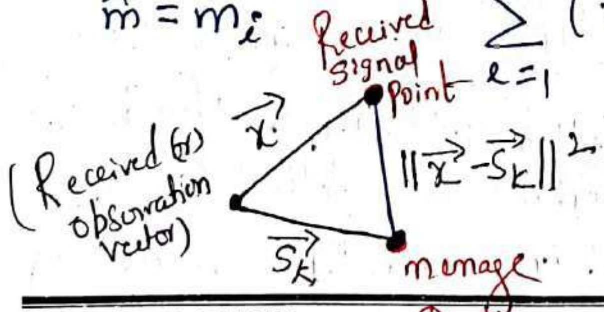
$$\underset{\text{Max}}{\underline{\ln f_{\vec{X}}(\vec{x}/m_k)}} = \underset{\text{Constant}}{\underline{-N/2 \ln(\pi N_0)}} - \frac{1}{N_0} \underline{\sum_{l=1}^{N} (x_l - S_{kl})^2}.$$

To make Max

$\sum_{l=1}^{N}(x_l - S_{kl})^2$ to be Minimum (as -ve term).

Decision Rule $\qquad$ Minimum

$$\hat{m} = m_i \qquad \underline{\quad} \sum_{l=1}^{N}(x_l - S_{kl})^2 \quad \text{for } k = l$$



$$\sum_{l=1}^{N}(x_l - S_{kl})^2 = \|\vec{x} - \vec{S}_{kl}\|^2$$

(Signal vector) Point

N-Dimension of Euclidean Space called observation Space.

α-3

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Expanding Summation

neglecting term.

$$\sum_{l=1}^{N}(x_l - s_{kl})^2 = \sum_{l=1}^{N} x_l^2 + \sum_{l=1}^{N} s_{kl}^2 - 2\sum_{l=1}^{N} x_l s_{kl}$$

⇒ Energy of received signal, which is independent of index k

(E_k) Energy of signal

inner product of received signal $\vec{x}$ and signal vector $\vec{S_k}$

$$\boxed{\sum_{l=1}^{N}(x_l - s_{kl})^2 = \|\vec{x} - \vec{s_k}\|^2}$$

Vector $\vec{S_k}$ which in Square of the distance from vector $\vec{x}$ having Minimum value ≈ Min(RHS)

$$\|\vec{x} - \vec{s_k}\| = \sum_{l=1}^{N} s_{kl}^2 - 2\sum_{l=1}^{N} x_l s_{kl}$$

$$= \sum_{l=1}^{N} x_l s_{kl} - \frac{1}{2}E_k \qquad \Big| \quad \sum_{l=1}^{N} s_{kl}^2 = E_k$$

inner product

→ Condition for Maximum Likelihood Receiver.

$$= x^T s_k - \frac{1}{2}E_k$$

$$k = 1, 2, \cdots M$$

An Example of Maximum Likelihood decision for $N=2$, and $M=4$.

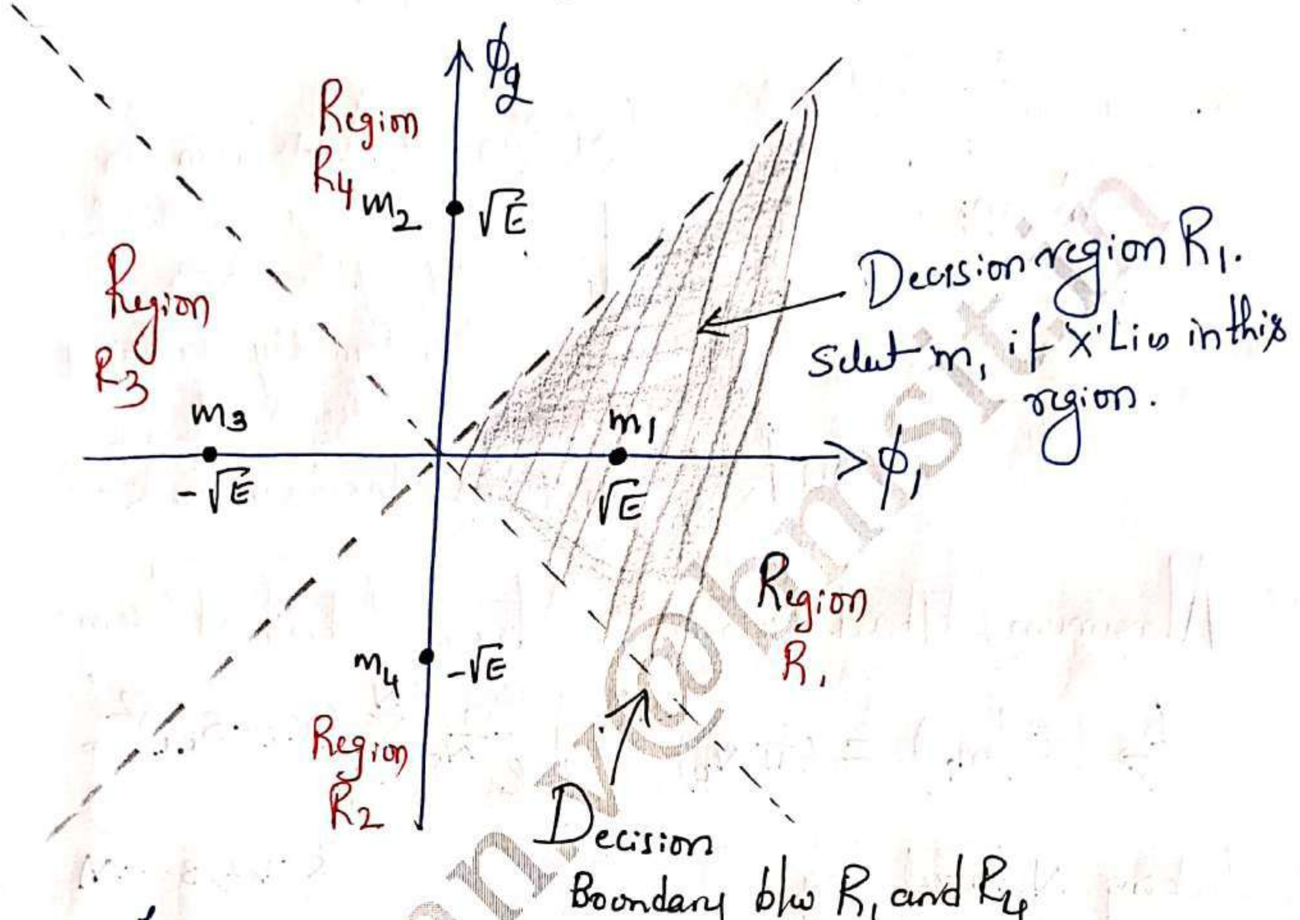


fig:- Decision Regions for $N=2$.

- Fig. shows the decision regions and boundaries for $N=2$ and $M=4$ Symbols.
- The region $\phi_1 - \phi_2$ i.e 2dimensional. There are four regions R_1, R_2, R_3 and R_4 Corresponding to four message points m_1, m_2, m_3 and m_4. The decision boundaries are shown shown by the dotted lines.
- if the observation vector falls in region R_1 then message m_1 is Selected.

2.5

2.4.2 Correlation receiver (or) Maximum Likelihood Receiver (or) Optimum Receiver.

Explain the correlation receiver with neat diagrams and explanation of detector and the maximum-likelihood decoder blocks. **(08 Marks) Dec 2018-Jan 2019.**

→ Refer Section 2.4.1

Obtain the decision rule for maximum likelihood decoding and explain the correlation receiver. **(08 Marks) June-July 2019/ (08 Marks) Dec2019-Jan 2020**

Explain the correlation receiver using product integrator and matched filter. **(08 Marks) Dec2019-Jan 2020**

Soln :- * For an AWGN, when the transmitted signals $S_1(t), S_2(t), \ldots S_m(t)$ are equally likely is called a Correlation receiver.

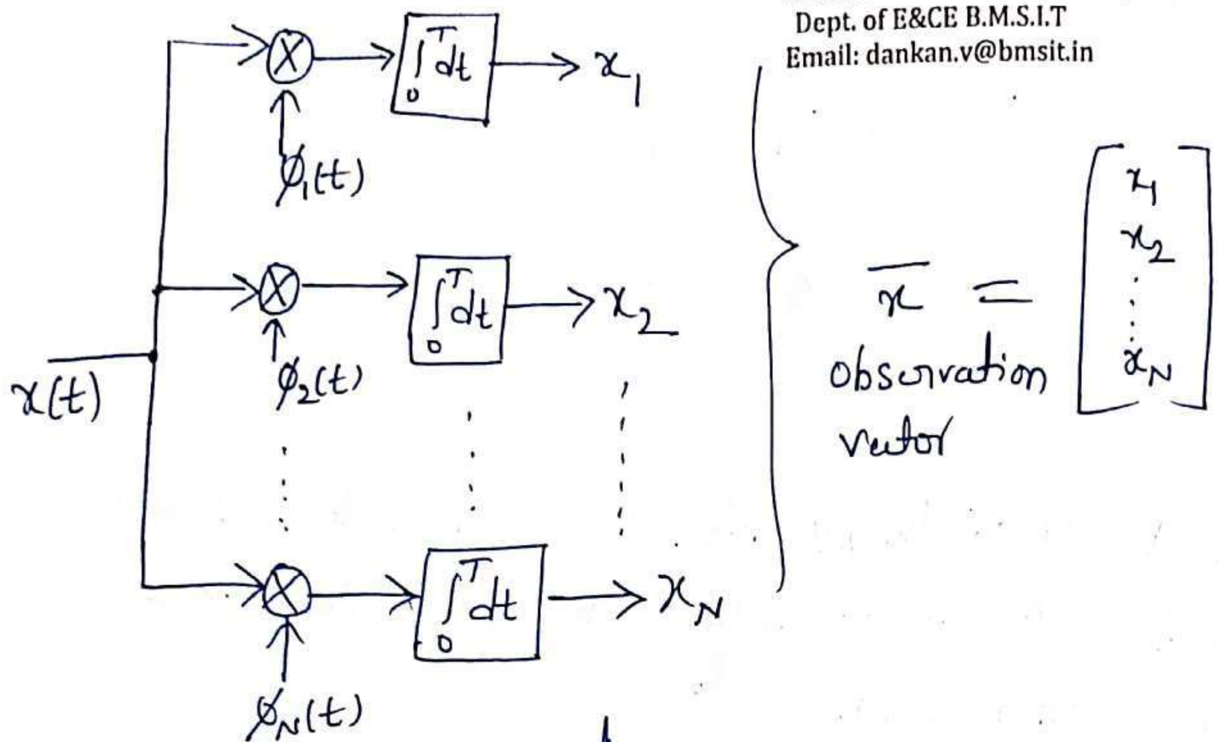
* it Consists of Consists of two Subsystems. which are
  i. Detector
  ii. Maximum-likelihood decoder.

i. Detector :-

* Detector which consists of M correlators Supplied with a set of orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ that are generated Locally.

* This bank of correlators operates on the received signal $x(t)$, $0 \leq t \leq T$, to produce the observation vector $\vec{x}$.

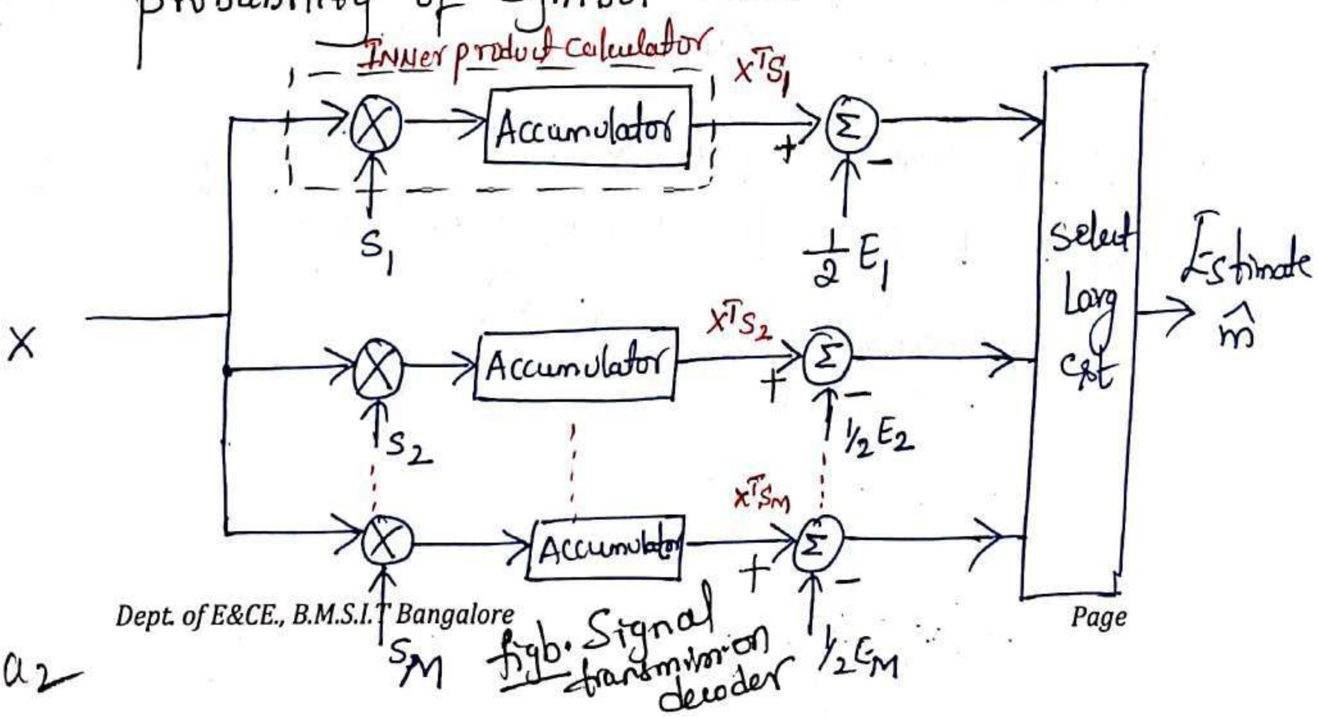
a1



figa. Detector (or) Demodulator.

ii. Maximum-likelihood decoder:-

* Which operates on the observation vector x to produce an estimate $\hat{m}$ of the transmitted Symbol m_i, $i=1,2,---M$, in Such a way that the average probability of Symbol Error is minimized.



figb. Signal transmission decoder

a2

* The observation Vector x is Multiplied by M signal vectors $S_1, S_2 \dots S_M$ and the resulting products are Successively Summed in accumulators to for the corresponding set of inner products $\{x^T S_k,\ k=1,2,3 \cdot\cdot M\}$.

* Finally, the Largest in the resulting set of numbers is selected and a Corresponding decision is Made on the transmitted Signal.

* The optimum receiver is commonly referred to as a correlation receiver.

a3

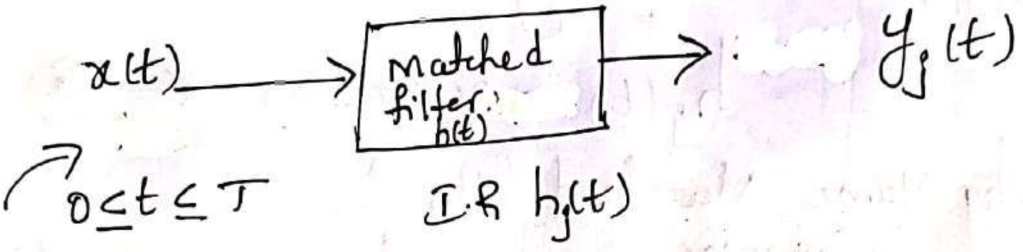
2.4.3 Matched filter receiver

Explain the matched filter receiver. Obtain the expression for the impulse response of the matched filter. **(08 Marks) Dec 2018-Jan 2019**

Explain with neat diagram and necessary equations the matched filter receiver. **(07 Marks) June-July 2019.**

Soln:- The Matched filter is used for detection of signals in baseband and passband transmission.

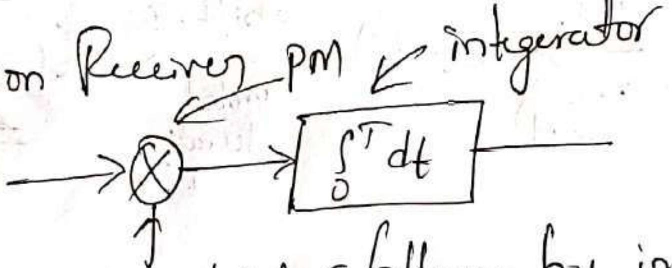
*



finite energy Signal.

* Basically matched filter is alternative to correlation receiver.

In a Correlation Receiver

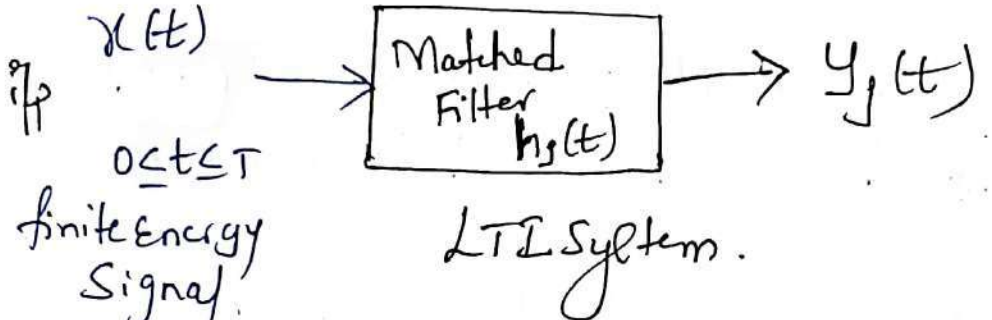


[ie Product modulator follows by integrator]

* order implementation of integrator is difficult, so instead of integrator if there is an filter, order implementation of filter is easy.

* Whatever the o/p correlator receiver gives the same o/p Will give the Matched filter.

- ∴ the matched filter Receiver and correlation Receiver both are same.
- Matched filter is the optimal filter. for AWGN.



$$h_j(t) = x(T-t) \quad \leftarrow ⓐ$$

Why Name Matched filter ?

the Impulse response of the matched filter is matched with the input (or) incoming signal . matched filters are used at Receiver side.

$$x(t) \xrightarrow[\text{(Time reversal)}]{T.R} x(-t) \xrightarrow[\text{Symbol duration 'T'}]{\text{shift by}} x(-t+T)$$

$$= \underline{\underline{x(T-t)}}$$

C_{IR} of matched filter.

- o/p of the matched filter is given by

$$\boxed{y_j(t) = x(t) * h_j(t)}$$

b2

* The output of matched filter

$$y_j(t) = x(t) * h_j(t) = \int_{-\infty}^{\infty} x(\tau) h_j(t-\tau)\, d\tau \longleftarrow ①$$

*1 assume that, the impulse response of the filter is

$$h_j(t) = \phi_j(T-t) \longleftarrow ②$$

where T is the symbol duration

and $\phi_j(T-t)$ is the time reversed and time delayed version of $\phi_j(t)$

$$h_j(t-\tau) = \phi_j[T-(t-\tau)]$$

$$\boxed{h_j(t-\tau) = \phi_j(T-t+\tau)} \longleftarrow ③$$

eqⁿ③ in eqⁿ①

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)\, \phi_j(T-t+\tau)\, d\tau \longleftarrow ④$$

* The Limits of integration are changed b'cz $\phi_j(t)$ is Non Zero only in the interval (0,T)

Sampling this o/p at time t = T, we get

$$y_j(T) = \int_0^T x(\tau)\, \phi_j(\cancel{T}-\cancel{T}+\tau)\, d\tau$$

b3

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$$y_j(T) = \int_0^T x(\tau)\phi_j(\tau)\,d\tau \leftarrow ⑤$$

$\hookrightarrow$ o/p equal to ④ same of correlator.

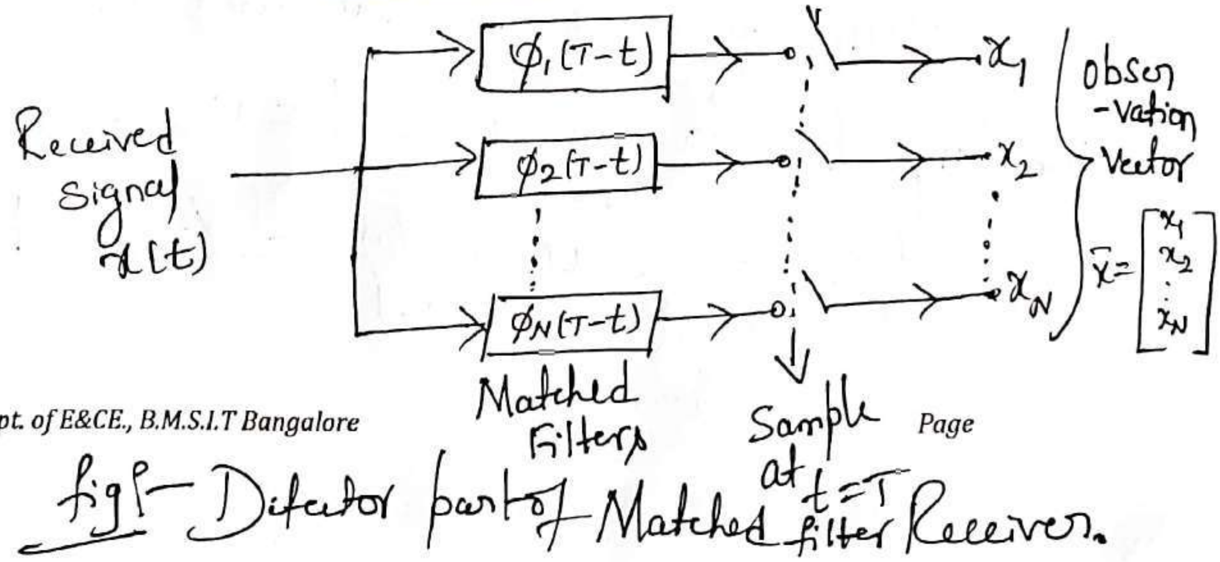
* A Filter having an impulse response as Suggested in eqⁿ ② is known as Matchedfilter.

* The I.R $\boxed{h_j(t) = \phi_j(T-t)}$ for $0 \le t \le T$
$j = 1, 2, 3 .. M$

"Given a pulse Signal $\phi(t)$ occupying the interval $0 \le t \le T$, a Linear time-invariant filter is said to be Matched to the signal $\phi(t)$ if its impulse response $h(t)$ satisfies the condition

$$\boxed{h(t) = \phi(T-t)} \quad \text{for } 0 \le t \le T.$$

* A time variant filter defined in this way is called matched filter. Correspondingly, an optimum receiver using matched filters in place of correlators is called a "matched filter receiver".



by fig:- Detector part of Matched filter Receiver.

Requirements of detection Receiver

i. signal to noise ratio of the receiver must be Large.

ii. Error probability should be Minimum.

Note:-

* matched filter's satisfies all the above requirements.

* it is called matched filter since its impulse response is matched to the shape of input signal.

Properties of Matched filter

$$\text{IR: } h(t) = x(T-t) \quad ; \; 0 \le t \le T$$

$$y(t) = x(t) * h(t) = x(t) * x(T-t)$$

$$\updownarrow \text{F.T}$$

$$\boxed{Y(f) = \underbrace{|X(f)|^2}_{\text{ESD}} e^{-j2\pi fT}} \leftarrow ①$$

property 1. The spectrum of the o/p of the matched filter with input signal being the signal to which it is matched is proportional to the energy spectral density (ESD) of the i/p signal except for a time delay factor.

i.e $Y_0(f) \propto |~~X~~(f)|^2$

$\quad\uparrow$ ESD of i/p.

Property 2 :-

The output signal of a matched filter is proportional to a shifted version of the autocorrelation function of the input signal to which the filter is matched.

$$x(t) \longrightarrow R_x(\tau)$$

F.T pairs

$$R_x(\tau) = \int_{-\infty}^{\infty} |X(f)|^2 \cdot e^{j2\pi f\tau}\, df \leftarrow ②$$

$$|X(f)|^2 = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, df \leftarrow ③$$

taking I.F.T eqn ① [from ② - ③]

$$\boxed{y_0(t) = R_x(t-T)}$$

Note:- output of the matched filter at the sampling instant is the energy of the input signal.

i.e $R_x(0) = \int_{-\infty}^{\infty} |X(f)|^2 \cdot df \Rightarrow$ Energy of the signal E

$$\boxed{y_0(T) = R_x(0) = E}$$

66

Property 3

$$(SNR)_{0,max} = \frac{2E}{N_0} = \frac{E}{(N_0/2)} = \frac{\text{Energy of the signal}}{\text{PSD of white noise}}.$$

Output SNR of a matched filter depends only on the ratio of the Signal energy to the power Spectral density of the white noise at the filter input.

Module-3

DIGITAL MODULATION TECHNIQUES

- Topic1 **PSK:** Phase shift Keying techniques using coherent detection: generation, detection and error probabilities of BPSK and QPSK, M–ary PSK, M–ary QAM (Relevant topics in Text 1 of 7.6, 7.7).
- Topic2 **FSK:** Frequency shift keying techniques using Coherent detection: BFSK generation, Detection and error probability (Relevant topics in Text 1 of 7.8).
- Topic3 **Non coherent orthogonal modulation techniques:** BFSK, DPSK Symbol representation, Block diagrams treatment of Transmitter and Receiver, Probability of error (without derivation of probability of error equation) (Text 1: 7.11, 7.12, 7.13).

Text Book:

Simon Haykin, —Digital Communication System, John Wiley & sons, First Edition, 2014, ISBN 978-0-471-64735-5.

PSK

1. Explain the signal space representation for binary phase shift keying modulation. Also derive the expression for the probability of error for the binary phase shift keying. **(10 Marks) June-July 2018.**

2. With a neat block diagram, explain the generation and coherent detection of QPSK signals. **(06 Marks) June-July 2018.**

3. Derive the expression for error probability of binary PSK using coherent detection. **(06 Marks) Dec 2018-Jan 2019./ (06 Marks) June-July 2019.**

4. Binary data are transmitted over a microwave link at the rate of 10^6 bits/sec and the power spectral density of the noise at the receiver input is 10^{-10} watts/Hz. Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for the following cases. Binary PSK using coherent detection DPSK
   Note : Take erfc(2.63) = 2×10^{-4}, Q(3.7) = 10^{-4}. **(06 Marks) Dec 2018-Jan 2019.**

5. Define bandwidth efficiency. Tabulate and comment on the bandwidth efficiency of M-ary PSK signals for different values of M. **(06 Marks) Dec 2018-Jan 2019.**

6. What is the advantage of M-ary QAM over M-ary PSK system? Obtain the constellation of QAM for M = 4 and draw signal space diagram. **(04 Marks) Dec 2018-Jan 2019.**

7. Sketch the QPSK waveform for the sequence 01101000. **(06 Marks) June-July 2019.**

8. Obtain the constellation of QAM for M=16 and Draw the signal Space Diagram. **(04 Marks) June-July 2019.**

9. With necessary diagrams, explain the generation and reception of BPSK signal. **(10 Marks) Dec 2019-Jan 2020.**

FSK

1. Derive an expression for probability of error of binary frequency shift keying technique. Also draw the block diagrams of BFSK transmitter & coherent receiver. **(10 Marks) June-July 2018./ (8 Marks) Dec 2019-Jan 2020.**
2. Explain the generation and coherent detection of BFSK system. **(06 Marks) June-July 2019.**
3. An FSK system transmits binary data at the rate of $2x10^6$ bps. During the source of transmission, AWGN of zero mean and two sided PSD 10^{-20} Watts/Hz is added to the signal. The amplitude of the received sinusoidal wave for digit 1 or 0 is 1µv. Determine the average probability of symbol error assuming non-coherent detection. **(04 Marks) June-July 2019.**

Non coherent orthogonal modulation techniques

1. With a neat diagram, explain the non coherent detection of binary frequency shift keying technique. **(04Marks) June-July 2018.**
2. For the binary sequence given by 10010011, illustrate the operation DPSK. **(02 Marks) June-July 2018.**
3. With neat diagram and expressions, explain binary FSK generation and noncoherent detection scheme. **(06 Marks) Dec 2018-Jan 2019.**
4. Explain the generation and optimum detection of differential phase – shift keying with neat block diagram. **(06 Marks) Dec 2018-Jan 2019./ (8 Marks) Dec 2019-Jan 2020.**

5. The binary sequence **1100100010** is applied to the DPSK transmitter.
   i. Sketch the resulting wave form at the transmitter output.
   ii. Applying this waveform to the DPSK receiver, show that in the absence of noise, the original binary sequence is reconstructed at the receiver output.
   (06 Marks) June-July 2019.
6. Given the binary data **10010011**, draw the BPSK and DPSK waveform. **(06 Marks) Dec 2019-Jan 2020.**

Digital Modulation Techniques

Data transmission
- Baseband data transmission (wired)
- Passband data transmission (Freespace)

i. Baseband data transmission :-

* The digital data is transmitted over the channel directly without any modulation.
* Suitable for short range communication.
* Channel is Fiber optical cable.
* Whenever the data is having a significant frequency then it is called as a Baseband data.

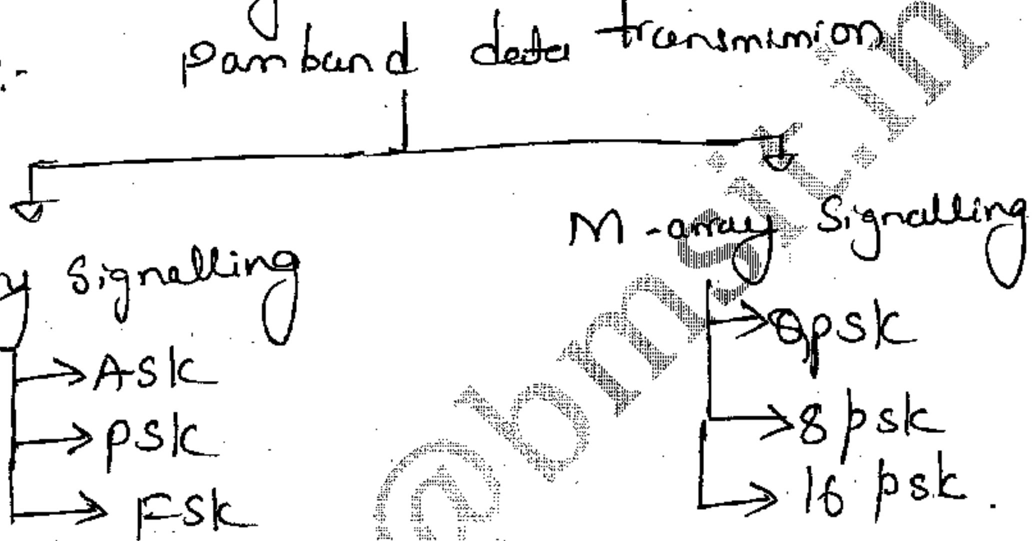
Eg:- Voice (0-4kHz), TV (0-6MHz)

* A Signal may be sent in its baseband ~~format~~ when a dedicated wired channel is available otherwise it must be Converted to passband.
* Types

Baseband data transmission
- pulse Analog Commn
  - PAM
  - PWM
  - PPM
- pulse digital Modln
  - PCM (pulse code modln)
  - DPCM (Differential pulse code modln)
  - DM (Delta modln)
  - ADM (adaptive delta modln)

ii. Passband data transmission

- * The digital data Modulates high frequency sinusoidal Carrier.
- * These techniques are suitable for transmission over Long distance.
- * Types:-



- * Channel is free space.

Advantages of passband data transmission:-

i. Simpler and cheaper because of advances made in IC Technology.

ii. In Digital communication the speech, video, and other data may be merged and Transmitted over the common channel using Multiplexing.

iii. Data encryption (privacy) i.e only permitted receivers may be allowed to detect the transmitted data.

iv. Low noise interference because the data transmission is digital.

v. Error detection and correction is possible. (channel coding is used).

Disadvantages

i. Due to Analog to Digital Conversion, the data rate becomes high ∴ more bandwidth is required.

ii. Needs Synchronization.

Benefits of passband data transmission over Base band data transmission

i. Long distance transmission.

ii. Multiplexing techniques can be used for bandwidth conservation.

iii. problems such as ISI and Crosstalks are absent.

iv. pass band transmission can take place over wireless channels also.

v. Minimum Circuit Complexity.

vi. Large number of modulation techniques are available.

Note:- Inter Symbol Interference (ISI) is a form of a signal in which one symbol interferes with subsequent symbols.

Definition of Modulation :-

Modulation is a process by which some characteristics of a carrier wave is varied in accordance with a modulating (message) signal.

Types :- i. Amplitude shift keying (ASK)
ii. Frequency shift keying (FSK)
iii. Phase shift keying (PSK).

note : $A_c \cos(\omega_c t \pm \phi) = \overset{ASK}{A_c} \cos(2\pi \overset{FSK}{f_c} t \pm \overset{psk}{\phi})$

xx. The choise of Modulation technique is based on the following parameters

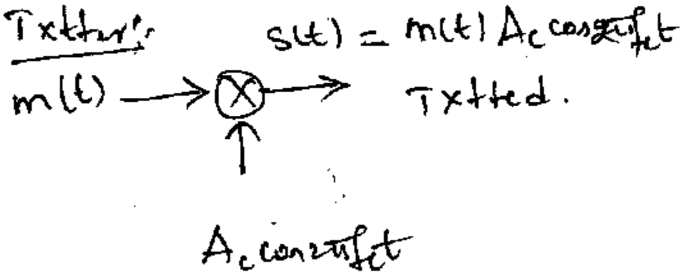
i. Maximum data rate
ii. Minimum probability of symbol error.
iii. Minimum transmitted power.
iv. Minimum channel Bandwidth.
v. Maximum resistance to interfering signals
vi. Minimum circuit complexity.

Demodulation of passband Signals

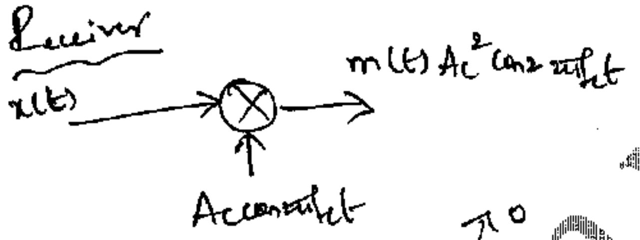
Types

Coherent detection
(Synchronous detection)

Txtter:



Receiver



$$= \underline{\underline{\frac{m(t) A_c^2}{2}}} + \overset{\to 0}{\cancel{\frac{m(t) A_c^2}{2} \cos 2\pi 2 f_c t}}$$

← Extract m(t).

* In this method, the Local Carrier generated at the receiver is in phase with the Carrier at the transmitter Hence is also Called Synchronous detector.
* In this method, Error probability decreases but the System becomes complex.

Non-coherent detection
(Envelope detection)

* In this method the received Carrier need not be inphase with transmitter Carrier. Hence it is also called Envelope detection.

* Non coherent detection is Simple but it has higher probability of Error.

Digital Modulation

* it is a Special kind of Modulation where the message Signal (modulating Signal) is of digital in nature (Binary) and the Carrier wave is in Analog nature.

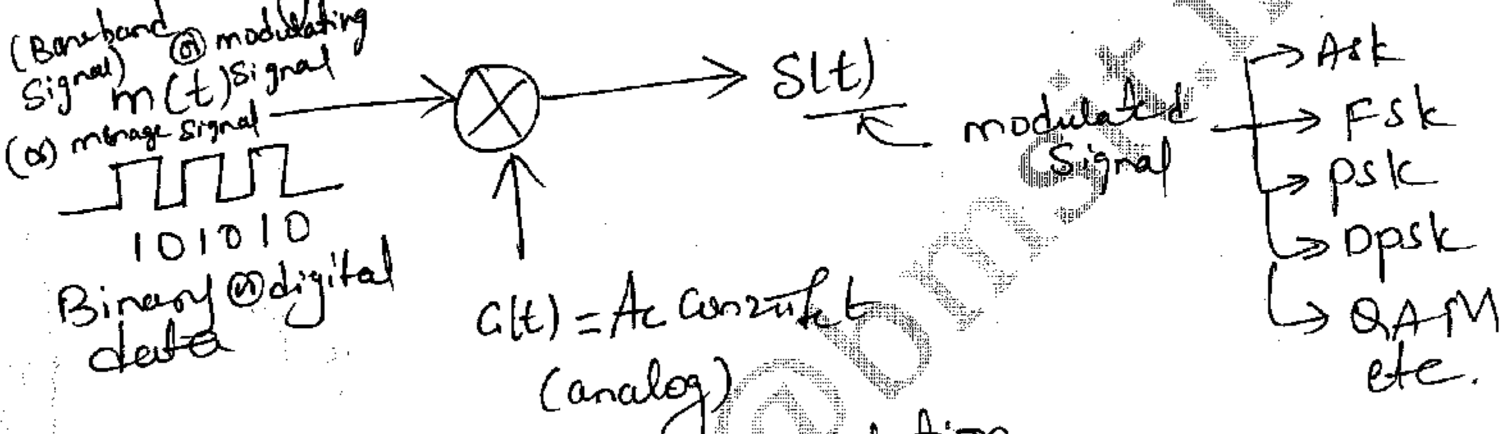


fig:- Digital modulation.

* In digital Modulation Switching | keying of the amplitude, frequency (or) phase of the carrier wave is done.

* The ASK, psk and FSK are analogous to AM, pm and FM respectively.

* The difference is that in digital modulation techniques (ASK, psk and FSK) the modulating Signal is of digital in nature while in AM, pm and FM Modulating Signal is Analog.

	message Signal/ Baseband/modulating Signal $m(t)$	Modulated Signal $S(t)$	Carrier signal
Digital Comm^{n}	digital (or) Binary data	"Analog" $m(t) A_c \cos 2\pi f_c t$	"Analog"
Analog Comm^{n}	Analog	"Analog" $m(t)(A_c \cos 2\pi f_c t)$	"Analog".

i. Amplitude shift keying :-

* In ASK, the amplitude of the carrier wave is changed (switched) according to the digital input signal (modulating signal).
* ∴ ASK is analogous to Amplitude Mod^{n} (AM).

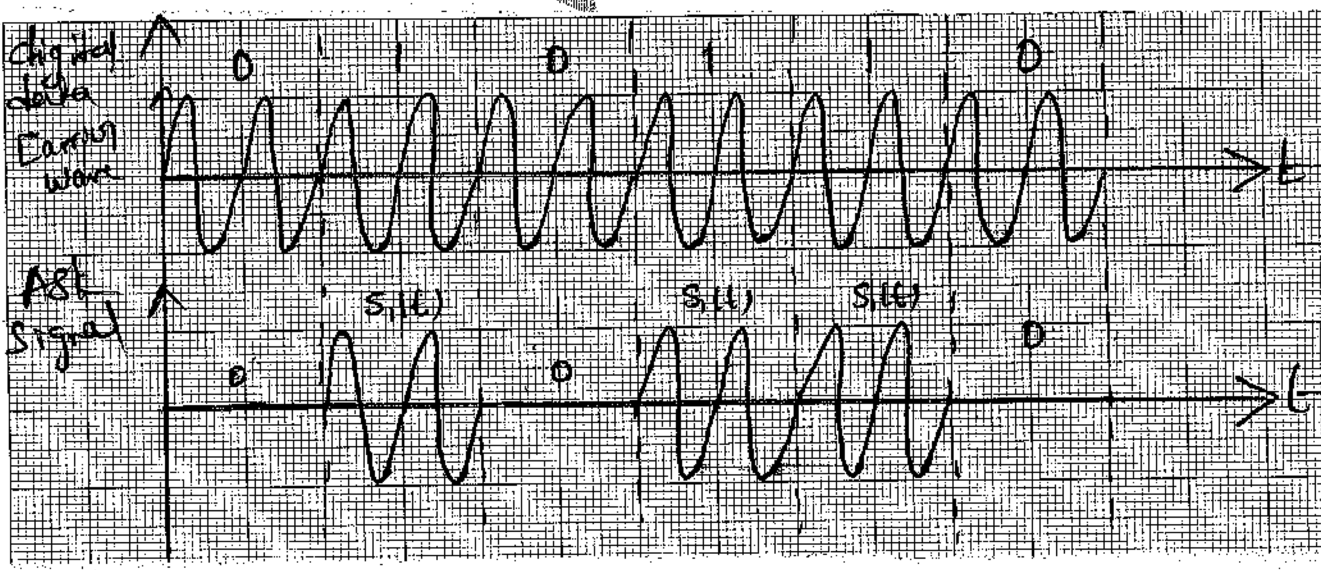
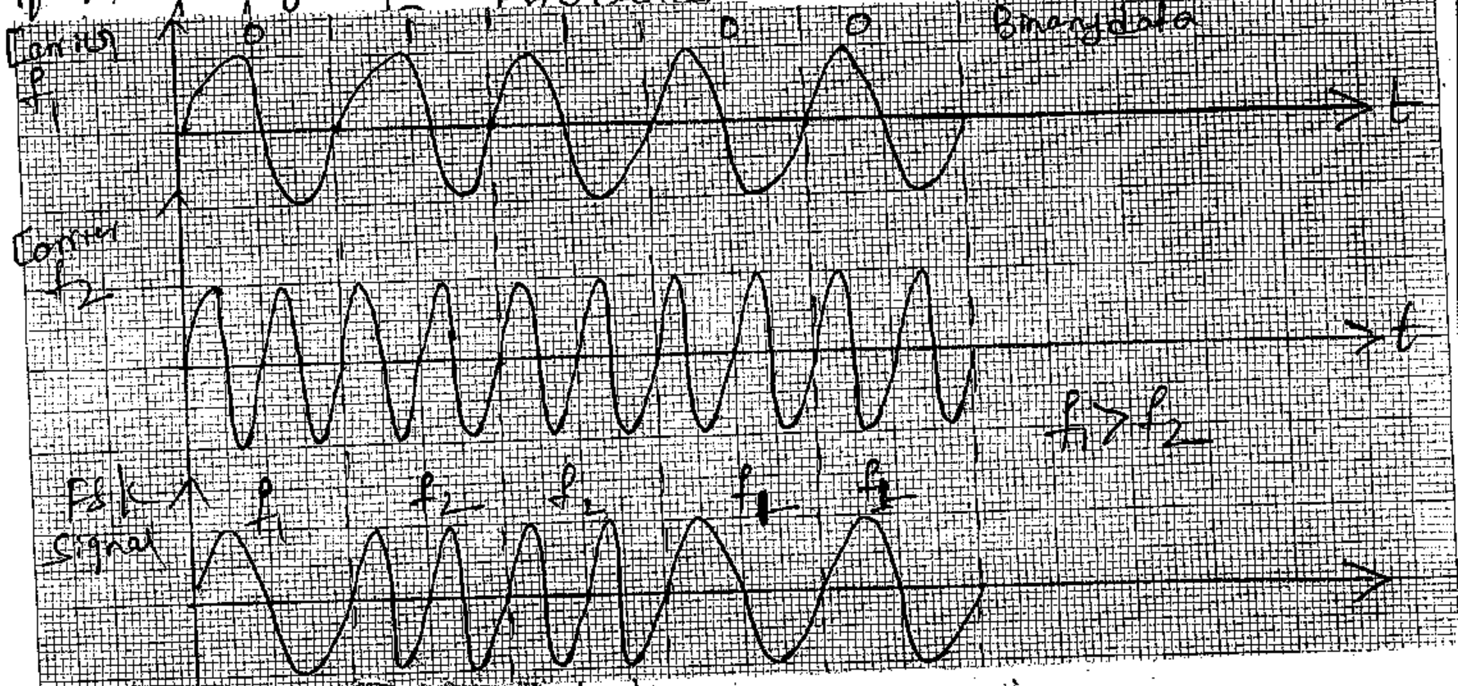


fig :- ASK Signal.

i. Frequency shift keying (FSK)

* if the frequency of Sinusoidal carrier wave is varied (Switched) depending on the digital input signal, then it is known as the Frequency shift keying (FSK).

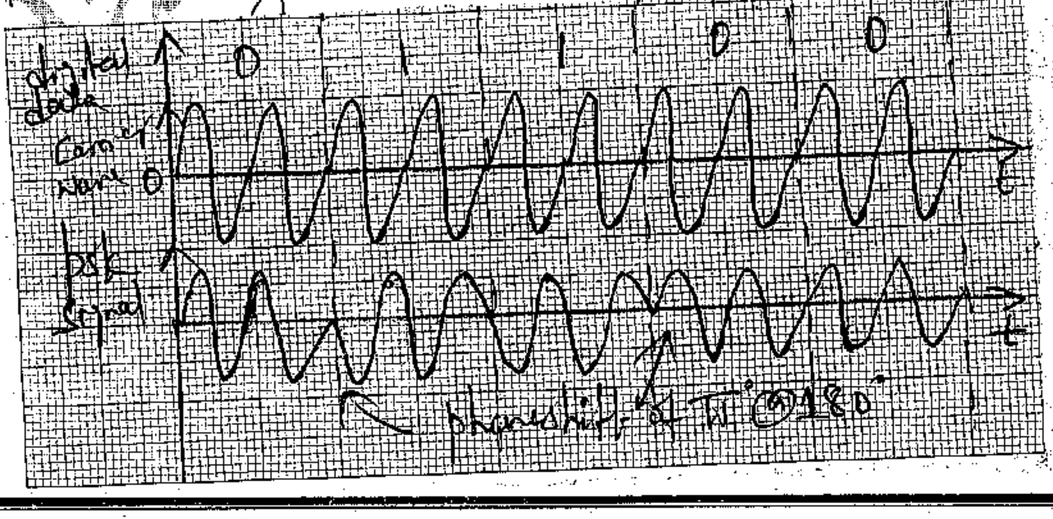
* it is analogous to Frequency Modln (FM).



ii. Phase Shift keying (PSK)

* In phase shift keying, phase of the carrier wave (analog) is switched as per the input digital signal.

* PSK is analogous to phase Modulation (PM).



Topic 1 :- Mathematical Analysis of BPSK

* The phase of the carrier wave is modulated by the binary symbols '1' and '0' in Binary phase shift keying (BPSK).

* if Carrier wave $c(t) = A_c \cos 2\pi f_c t$

avg. Power $P_{avg} = \frac{A^2}{2}$ watts

$A^2 = 2P$ $A = \sqrt{2P}$

and $P_{avg} = \frac{E_b}{T_b}$ J/sec

∴ we know that in BPSK phase of the carrier is changed by π (180°) when the symbol changes from 0 to 1 (or) 1 to 0.

for Symbol 1 modulated Signal

$\cos(\theta + \pi) = -\cos\theta$

$$S_1(t) = \sqrt{2P} \cos 2\pi f_c t \longleftarrow ①$$

then for Symbol 0

$$S_2(t) = \sqrt{2P} \cos(2\pi f_c t + \pi)$$

$$S_2(t) = -\sqrt{2P} \cos 2\pi f_c t \longleftarrow ②$$

Generation of Bpsk (10 M)

Dec/Jan 2020

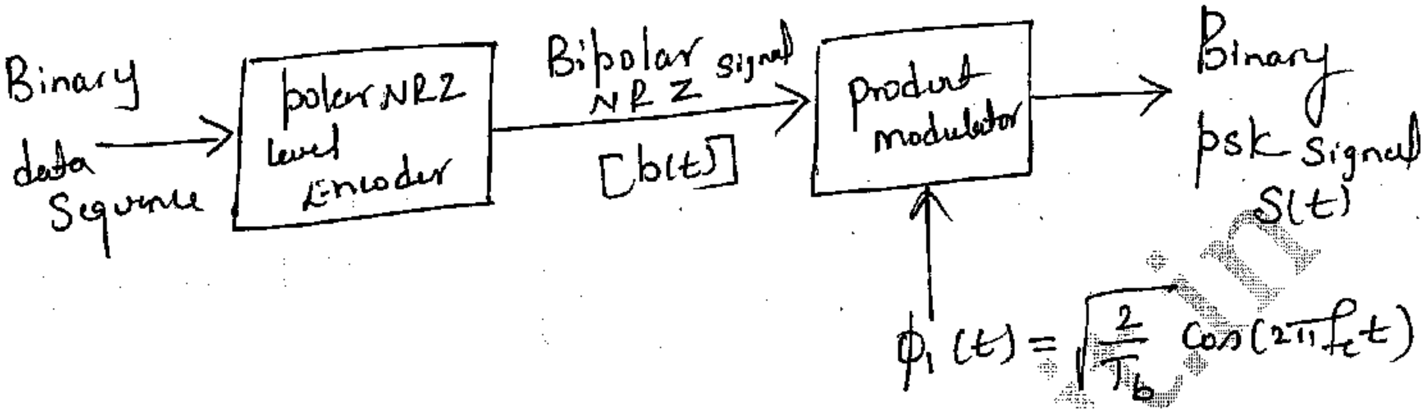


fig:- Block diagram of generation of Bpsk.

* The generator Consists of two components

i. polar NRZ level Encoder

ii. product Modulator.

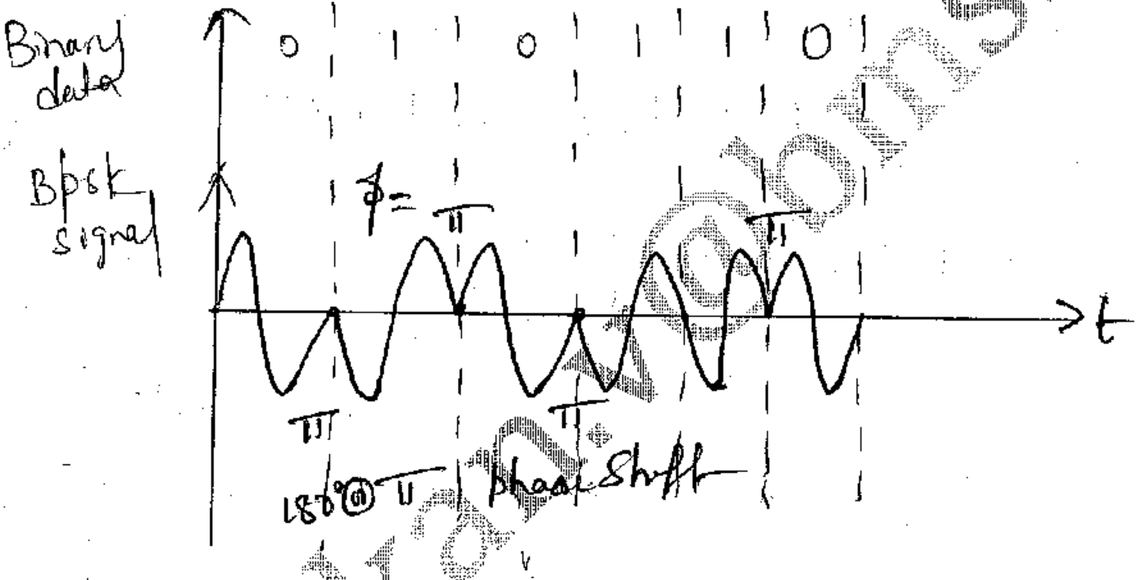
* polar NRZ level Encoder, which represents Symbols 1 and '0' of the incoming binary sequence by amplitude levels $+\sqrt{E_b}$ and $-\sqrt{E_b}$ respectively.

* product modulator, which multiplies the output of the polar NRZ Encoder by the basis function $\phi_1(t)$.

By combining eq^n ① and eq^n ②
Bpsk Signal as

$$\boxed{S(t) = b(t)\sqrt{2P}\cos 2\pi f_c t}$$

where $b(t) = \begin{cases} +1 & : \text{when transmitting symbol is '1'} \\ -1 & : \text{When transmitting Symbol is '0'.} \end{cases}$



** BPSK Receiver / coherent detection of BPSK

Dec/Jan 2020

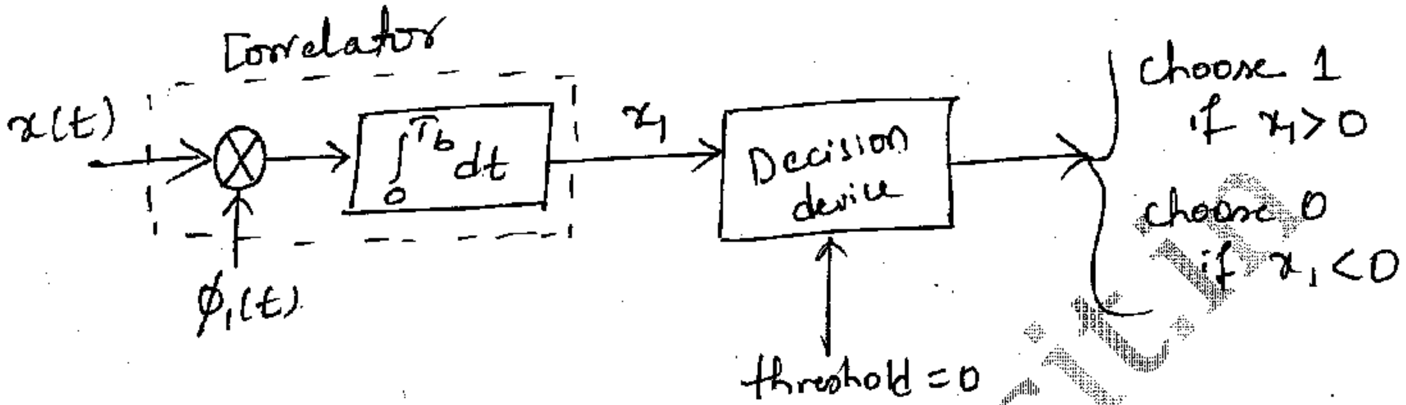


fig:- coherent Binary PSK receiver.

* The Receiver is synchronized with the transmitter as shown in the block diagram.

* the two Basic Components on the BPSK receiver are

i. Correlator

ii. Decision device.

* Correlator, which correlates the received Signal $x(t)$ with the basis function $\phi_1(t)$ on a bit by bit basis.

* Decision device, which compares the correlator output against a Zero threshold assuming that binary symbols '1' and '0' are equiprobable.

* if the threshold is Exceeded a decision is made in favour of Symbol '1', if not the decision is made in favour of Symbol '0'.

6

J/J 2018

6A. Signal Space Diagram of Binary PSK

* In a Binary psk system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0 respectively.

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad 0 \le t \le T_b$$

$$s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b$$

$= A \cos 2\pi f_c t$
$= P = A^2/2$
$A = \sqrt{2P}$
& $P_{avg} = E_b/T_b$
$\therefore A = \sqrt{\frac{2E_b}{T_b}}$
$= \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$

where T_b is the bit duration and E_b is the transmitted signal Energy per bit.

* The basis function of unit Energy

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \quad 0 \le t \le T_b$$

$$\therefore \quad s_1(t) = \sqrt{E_b}\, \phi_1(t) \ ; 0 \le t \le T_b$$

$$s_2(t) = -\sqrt{E_b}\, \phi_1(t) \quad 0 \le t \le T_b$$

* A binary psk system is ∴ characterized by having a signal space that is one dimensional with a signal constellation consisting of two message points ($M=2$).

* The representation of Co-ordinates of two message points are

$$s_{11} = \int_0^{T_b} s_1(t)\, \phi_1(t)\, dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\, \phi_1(t)\, dt = -\sqrt{E_b}$$

* The message point corresponding to $S_1(t)$ is Located at $S_{11} = +\sqrt{E_b}$ and the message point corresponding to $S_2(t)$ is Located at $S_{21} = -\sqrt{E_b}$.

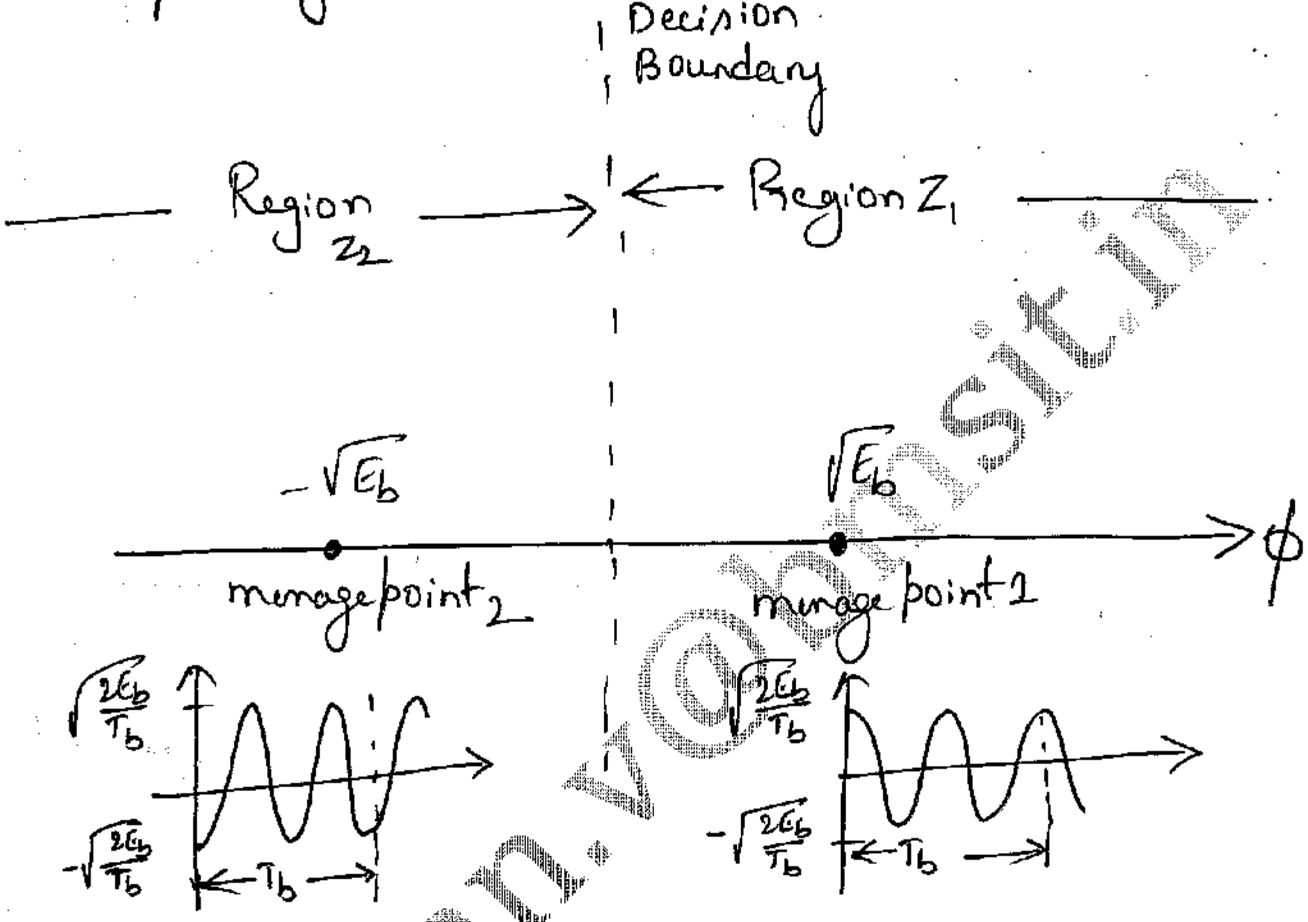


fig:- Signal space diagram.

Band width of Bpsk Signal :-

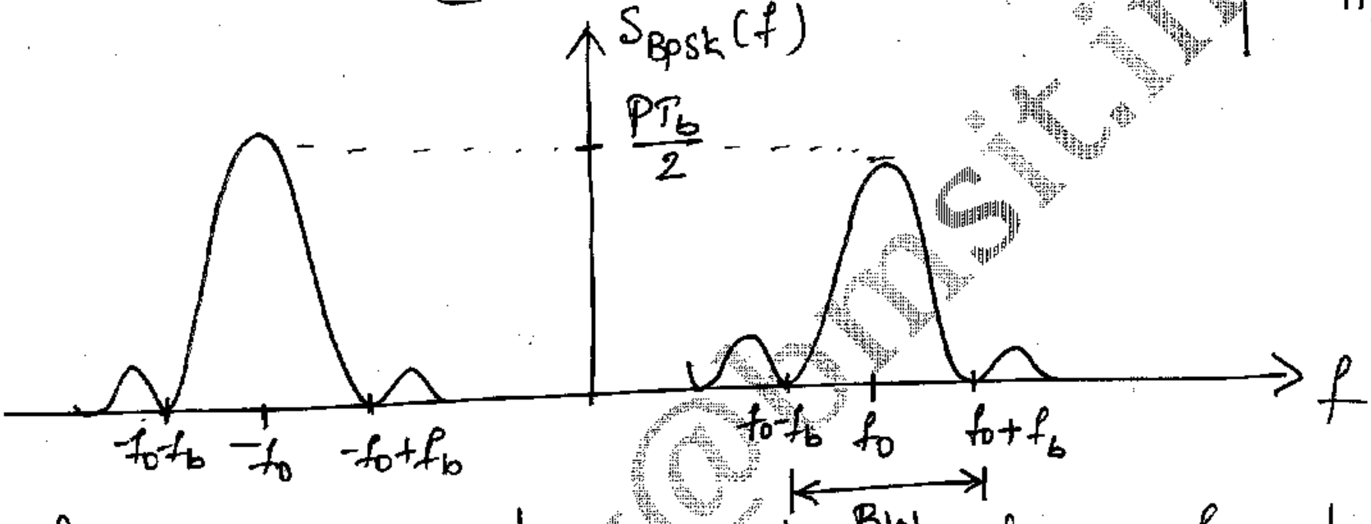
* The Spectrum of the Bpsk Signal is Centered around the Carrier frequency f_0.

$f_b = \frac{1}{T_b}$, then for BPSK the maximum frequency in the baseband Signal will be f_b.

* the PSD (power Spectral density)

$$S_{Bpsk}(f) = \frac{PT_b}{2}\left\{\left[\frac{\sin\pi(f-f_0)T_b}{\pi(f-f_0)T_b}\right]^2+\left[\frac{\sin\pi(f+f_0)T_b}{\pi(f+f_0)T_b}\right]^2\right\}$$

$$= \frac{PT_b}{2}\left\{\text{sinc}^2(f-f_0)T_b + \text{sinc}^2(f+f_0)T_b\right\} \quad \Bigg| \; \text{sinc}\lambda = \frac{\sin\pi\lambda}{\pi\lambda}$$



* The mainLobe is Centered around Carrier frequency f_0 and Extends from f_0-f_b to f_0+f_b.

$$\therefore (BW)_{Bpsk} = f_U - f_L$$

$$= f_0+f_0-(f_0-f_b) = 2f_b$$

$$\boxed{(BW)_{Bpsk} = 2f_b} \text{ Hz.}$$

* Thus the minimum bandwidth of Bpsk Signal is equal to twice the highest frequency Contained in baseband Signal.

Q1) problem: Det the minimum BW for a BPSK modulator with a Carrier frequency of 40MHz and an input bit rate of 50kbps.

soln: $f_b = 500\text{kbps} = 500\text{kHz}$.

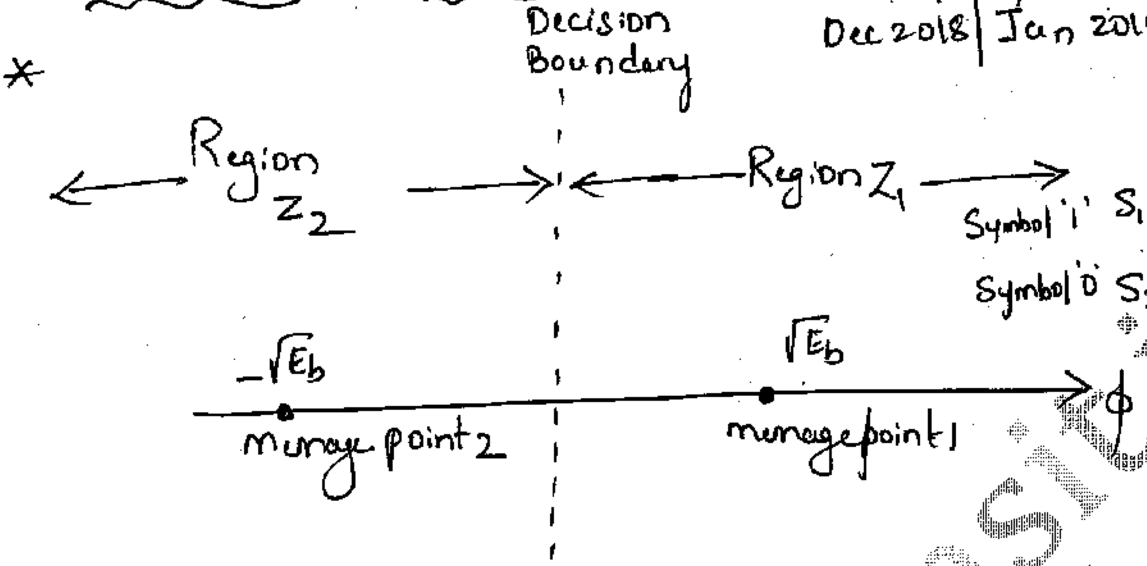
$$BW = 2f_b = 2\times 500k = 1000kHz$$

$$\therefore \boxed{(BW)_{Bpsk} = 1MHz}$$

Probability of Error of Bpsk

June/July 2018
June/July 2019
Dec 2018/Jan 2019.

*



Symbol '1' $S_1(t) = \sqrt{E_b}\,\phi_1(t)$

Symbol '0' $S_2(t) = -\sqrt{E_b}\,\phi_1(t)$

fig a. Signal Space diagram of Bpsk

*

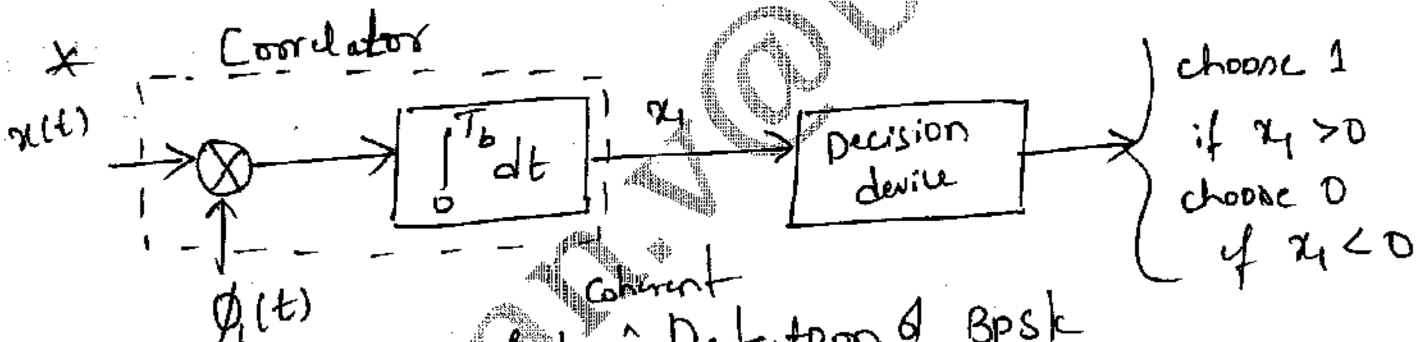


fig b: Coherent Detection of Bpsk

* Let $x(t)$ be the received Signal

$$x(t) = S(t) + w(t) \longleftarrow ①$$

where $w(t)$ is the additive white guassian noise (AWGN) with Zero mean $(\mu = 0)$ and variance of $N_0/2$.

$$* \quad x(t) = \begin{cases} S_1(t) = \sqrt{E_b}\,\phi_1(t) + w(t) & \text{for Symbol '1'} \\ S_2(t) = -\sqrt{E_b}\,\phi_1(t) + w(t) & \text{for Symbol '0'.} \end{cases}$$

* Let us assume that the Symbol '0' is transmitted. the output of the correlator is

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} [s_2(t) + w(t)]\,\phi_1(t)\,dt$$

$$x_1 = \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \underbrace{\int_0^{T_b} w(t)\,\phi_1(t)\,dt}$$

$$x_1 = \int_0^{T_b} -\sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt + W_1$$

$$x_1 = -\sqrt{E_b}\int_0^{T_b} \underbrace{\phi_1^2(t)\,dt}_{1 \text{ (b/c unit Energy basis fn)}} + W_1$$

Note:
1. E[constant] = constant
2. Variance[constant] = 0.

$$\therefore \boxed{x_1 = -\sqrt{E_b} + W_1}$$

* Mean of the random Variable x_1 is

$$E[x_1] = E[-\sqrt{E_b} + W_1]$$

$$= E[-\sqrt{E_b}] + \cancel{E[W_1]} \rightarrow 0 \text{ [given } \mu = 0 \text{ of AWGN]}$$

$$\boxed{E[x_1] = -\sqrt{E_b}}$$

* Variance of x_1 is

$$Var[x_1] = Var[-\sqrt{E_b} + w_1]$$

$$= \underset{\downarrow}{Var[-\sqrt{E_b}]} + \underset{\swarrow}{Var[w_1]}$$

$$= 0 + \frac{N_0}{2}$$

$Var[w_1] = N_0/2$ [given variance of AWGN]

$$\therefore \boxed{Var\{x_1\} = \frac{N_0}{2}}$$

* Conditional pdf (probability density function) when Symbol '0' is transmitted is given as

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \longleftarrow ②$$

mean $E[x_1] = \mu = -\sqrt{E_b}$ and $Var[x_1] = \sigma^2 = N_0/2$

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{\cancel{2}\pi \cdot N_0/\cancel{2}}} e^{-\frac{(x+\sqrt{E_b})^2}{\cancel{2} \cdot N_0/\cancel{2}}}$$

$$f_{x_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x+\sqrt{E_b})^2}{N_0}}$$

$(\sqrt{N_0})^2 = [N_0]$

$$\boxed{f_{x_1}(x_1/0) = \frac{1}{\sqrt{\pi N_0}} e^{-\left(\frac{x+\sqrt{E_b}}{\sqrt{N_0}}\right)^2}} \longleftarrow ③$$

* Let $P_e(0)$ denotes the conditional probability of deciding in favour of symbol '1' when '0' is transmitted.

Region Z_1 :- (Symbol '1') $\quad 0 \leq x_1 \leq \infty$

$$\therefore \quad P_e(0) = \int_0^{\infty} f_{x_1}(x_1|0)\, dx_1$$

$$P_e(0) = \frac{1}{\sqrt{\pi N_0}} \int_{x_1=0}^{\infty} e^{-\left(\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}\right)^2} dx_1$$

put $Z = \frac{x_1+\sqrt{E_b}}{\sqrt{N_0}}$

$dz = \frac{dx_1}{\sqrt{N_0}} + 0$

$$\boxed{dx_1 = \sqrt{N_0}\, dz}$$

$\underset{\sim}{L.L}$ when $x_1 = 0$

$\Rightarrow Z = \sqrt{E_b/N_0}$

$\underset{\sim}{UL}$ when $x_1 = \infty$

$Z = \infty$.

$$\therefore P_e(0) = \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}}\, e^{-Z^2} [\sqrt{N_0}]\, dZ$$

$$= \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi}\,\cancel{\sqrt{N_0}}}\, e^{-Z^2}\, \cancel{\sqrt{N_0}}\, dZ$$

$$= \frac{1}{\sqrt{\pi}} \int_{z=\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz = \frac{1}{\sqrt{\pi}} \int_{z}^{\infty} e^{-z^2} dz$$

Note:- Def^n of Complementary Error function

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2} du$$

$$\boxed{\frac{1}{2}\,\text{erfc}(u) = \frac{1}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2} du}$$

using above eq^u

re-written as

$$P_e(0) = \frac{1}{2}\,\text{erfc}(z)$$

where $z = \sqrt{E_b/N_0}$

$$\therefore \boxed{P_e(0) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

* Lly we can calculate probability of Error for 2nd kind i.e transmitted is '1' and received is '0'.

$$\text{i.e} \quad \boxed{P_e(1) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

let probability of transmitting Symbol '0' and Symbol '1' is respectively $p(0)=p(1)=1/2$.
(Equiprobable).

* the average probability of Error

$$P_e = p(0)\, p_e(0) + p(1)\, p_e(1)$$

$$P_e = \frac{1}{2}\left\{\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right\} + \frac{1}{2}\left\{\frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)\right\}$$

$$\boxed{P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

Note:- relation b/w erfc(x) and Q(x) is

$$Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

∴ probability of error interms of Q function is

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{2E_b}{N_0}}\Big/\sqrt{2}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

So..

$$\boxed{P_{e_{BPSK}} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

Note:-
i. $\frac{N_0}{2}$ _ Noise density (W/Hz)
ii. E_b - energy per bit (Joules)
iii. $P_{avg} = \frac{E_b}{T_b}$: watt's
iv. $R_b = \frac{1}{T_b}$ bps

problem :-

Binary data are transmitted over a microwave link at the rate of 10^6 bits/sec and the power Spectral density of the noise at the receiver input is 10^{-10} W/Hz. find the average carrier power required to maintain an average probability of error $P_e \le 10^{-4}$ for the Binary psk system.

Note:- $\text{erfc}(2.63) = 2\times10^{-4}$, $Q(3.7) = 10^{-4}$

Dec 2018 | Jan 2019.

Soln:-

$$P_{e,\,Bpsk} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \leftarrow ① \le 10^{-4}$$

given $Q(3.7) = 10^{-4} \leftarrow ②$

Comparing eq^n ① and ②

$$\sqrt{\frac{2E_b}{N_0}} = 3.7$$

$$\frac{2E_b}{N_0} = (3.7)^2$$

and noise density given

$$\frac{N_0}{2} = 10^{-10}\ \text{W/Hz}$$

$$R_b = 10^6\ \text{bps}$$

$$T_b = \frac{1}{R_b} = 10^{-6}\ \text{sec}$$

$$\Rightarrow \frac{E_b}{(N_0/2)} = 3.7^2$$

$$E_b = (3.7)^2 \times \left(\frac{N_0}{2}\right) = 3.7^2 \times 10^{-10}$$

$$E_b = 13.69\times10^{-10} \Rightarrow \boxed{E_b = 1.369\ \eta\ \text{Joules}}$$

$$P_{avg} = \frac{E_b}{T_b} = \frac{1.369\times10^{-9}}{10^{-6}} = \underline{1.369\times10^{-3}\ \text{Watts}}$$

$$\boxed{P_{avg} = 1.369\ \text{mW}}$$

Problem 2 · Binary data are transmitted at a rate of 10^6 bps over a microwave link. assuming channel noise is AWGN with zero mean and power spectral density at the receiver input ($N_0/2$) is 10^{-10} w/Hz. find the average Carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent binary psk.

Note:- $\text{erfc}(2\cdot 8) = 2\times 10^{-4}$.

Soln:-

$$R_b = 1\times 10^6 \text{ bps}$$

$$T_b = \frac{1}{R_b} = 10^{-6} \text{ sec}$$

$$\frac{N_0}{2} = 10^{-10} \text{ w/Hz} \qquad P_e \leq 10^{-4}$$

$$P_{e,psk} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$10^{-4} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$\left.\begin{aligned} 2\times 10^{-4} &= \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \leftarrow ① \\ 2\times 10^{-4} &= \text{erfc}(2\cdot 8) \leftarrow ② \end{aligned}\right\} \text{comparing}$$

$$\sqrt{\frac{E_b}{N_0}} = 2\cdot 8$$

$$E_b = (2\cdot 8)^2 \cdot N_0 = 7\cdot 84\times 2\times 10^{-10}$$

$$\boxed{E_b = 15\cdot 68\times 10^{-10}} \text{ Joules}$$

$$P_{avg} = \frac{E_b}{T_b} = \frac{15\cdot 68\times 10^{-10}}{10^{-6}} = \underline{\underline{1\cdot 568\times 10^{-3}\,W}}$$

$$\boxed{P_{avg} = 1\cdot 568 \text{ m Watts}}$$

Topic 2

Frequency Shift Keying (FSK)

Q. Explain the generation and coherent detection of BFSK system. J|J 2019.

Generation of BFSK

* The binary FSK wave can be represented as

$$S(t) = \begin{cases} S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t \; : 0 \le t \le T_b & \text{for Symbol '1'} \\ S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t \; ; 0 \le t \le T_b & \text{for Symbol '0'} \end{cases}$$

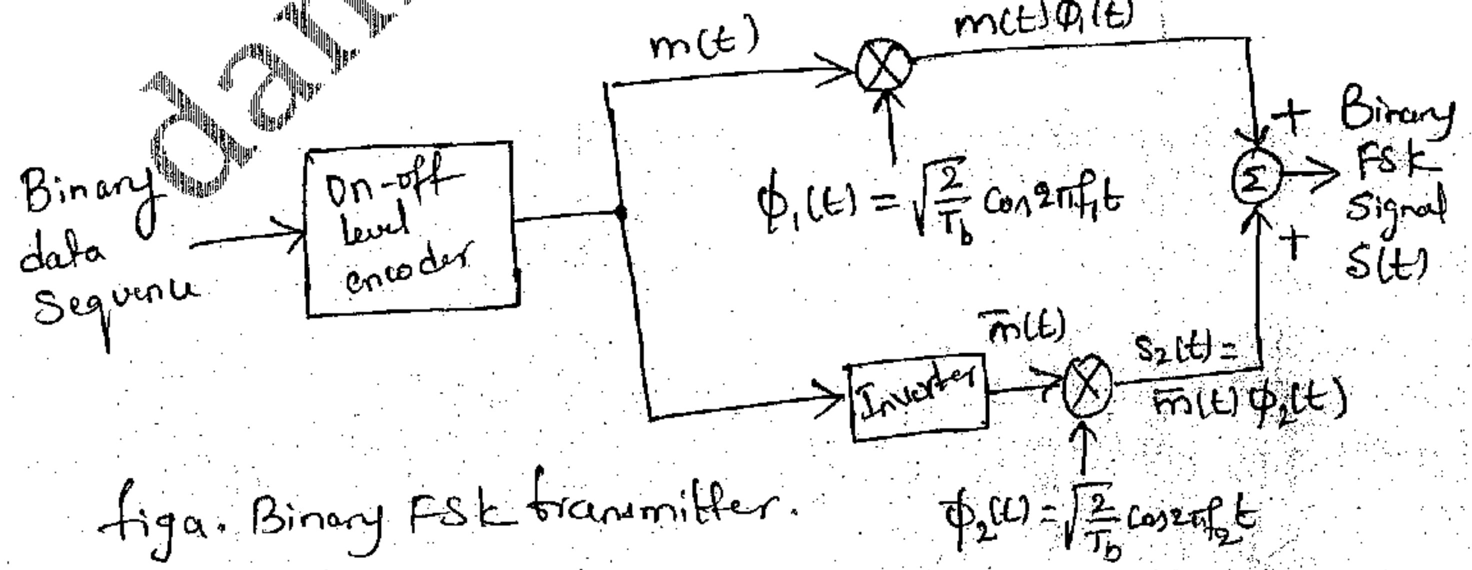
* Let $\phi_1(t)$ and $\phi_2(t)$ are the basis function defined as

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$$

$$\therefore \; S(t) = \begin{cases} S_1(t) = \sqrt{E_b}\, \phi_1(t) & \text{for Symbol '1'} \\ S_2(t) = \sqrt{E_b}\, \phi_2(t) & \text{for Symbol '0'} \end{cases}$$

BFSK transmitter:-



figa. Binary FSK transmitter.

- Input binary data is Unipolar NRZ format applied to the multiplier-1 along with Carrier Signal $\phi_1(t)$.

  i.e $S_1(t) = m(t) \cdot \phi_1(t)$..

- The input m(t) is applied to multiplier-2 through Inverter along with another Carrier Signal $\phi_2(t)$.

  i.e $S_2(t) = \overline{m}(t) \phi_2(t)$

- when Symbol 1 is transmitted, O/p appears from multiplier-1 i.e $S_1(t)$ and no output from multiplier-2

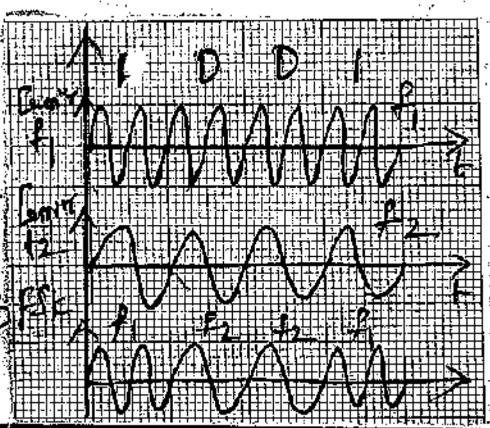
  i.e $S_2(t) = 0$.

  thus frequency f_1 is transmitted for Symbol 1.

- when Symbol 0 is transmitted, O/p appears from multiplier-2 i.e $S_2(t)$ and no output from the multiplier-1 i.e $S_1(t) = 0$.

  Thus frequency f_2 is transmitted for Symbol 0.

- Frequency $f_1 > f_2$ is choosen and the phase of the FSK remains Continuous ∴ FSK is also known as Continuous phase FSK.



$f_1 > f_2$

fig. BFSK Signal

xi. Coherent detection | Receivers of Binary FSK

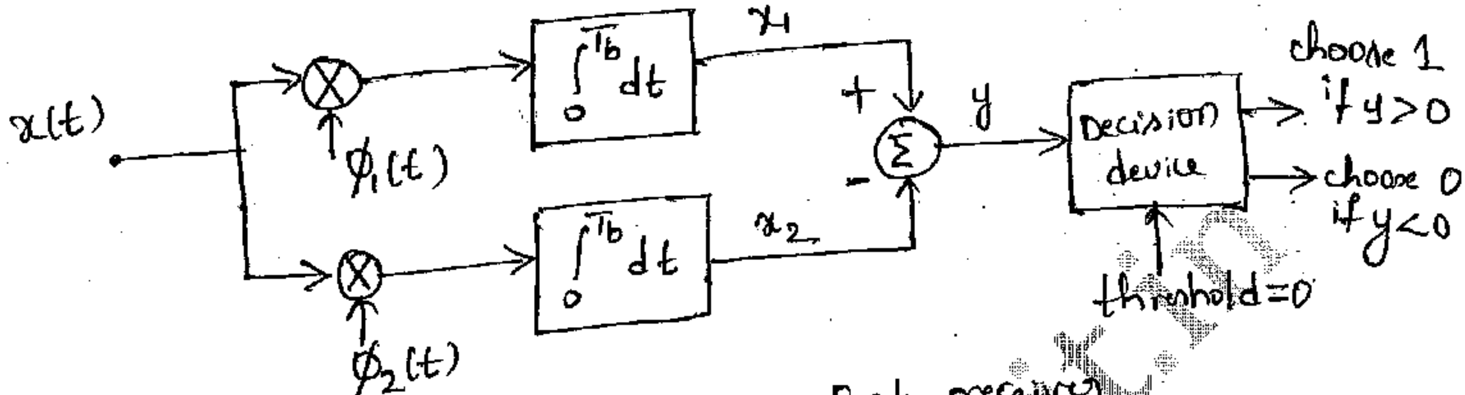


fig b. Coherent binary FSK receiver

- * The detector consists of two correlators that are individually tuned to two different frequencies.
- * A correlator consists of a multiplier followed by an Integerator (LPF).
- * The output of top and bottom correlator be denoted by x_1 and x_2 respectively. the x_1 and x_2 are given to subtractor. the o/p of subtractor $y = x_1 - x_2$.
- * if $(x_1 - x_2) > 0$ i.e $y > 0$, decision is taken in favour of symbol '1'.
- * if $(x_1 - x_2) < 0$ i.e $y < 0$, decision is taken in favour of Symbol '0'.
- * if $(x_1 - x_2) = 0$ ie $y = 0$, the decision is arbitrary.

Probability of Error BFSK

J/J 2018
Dec 2019 | Jan 2020.

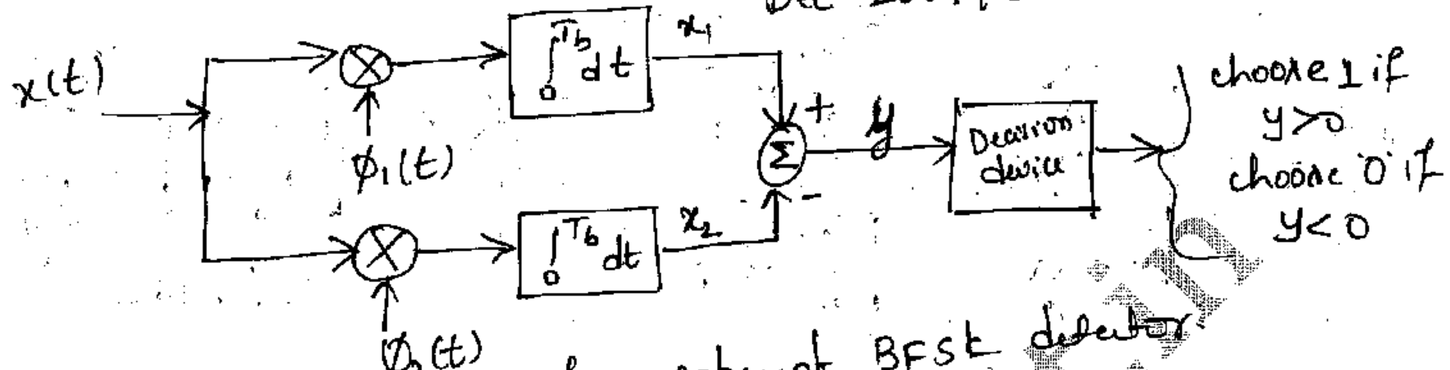


fig:- coherent BFSK detector

* let $x(t)$ be the received ~~Bp~~BFSK signal given by

$$x(t) = s(t) + w(t)$$

$$x = \begin{cases} S_1(t) + w(t) & \text{for symbol '1'} \\ S_2(t) + w(t) & \text{for symbol '0'} \end{cases}$$

where $w(t)$ is AWGN noise having mean $(\mu) = 0$ and variance $\sigma = N_0/2$.

* Let Consider the transmission of symbol '0' then the received signal is

$$x(t) = S_2(t) + w(t).$$

* the o/p of the top correlator is

$$x_1 = \int_0^{T_b} x(t)\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} [S_2(t) + w(t)]\, \phi_1(t)\, dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\,dt$$

$$x_1 = s_{21} + w_1$$

$S_{21} = 0$ bcz $\underline{\phi_1(t)\,\phi_2(t)}$ orthonormal fn.

$$\boxed{x_1 = 0 + w_1} \longleftarrow ①$$

* Mean is given by

$$E[x_1] = E[w_1] = 0 + 0 = 0.$$

* Variance of x_1

$$Var[x_1] = var[0 + w_1] = 0 + \frac{No}{2}$$

$$\boxed{Var[x_1] = \frac{No}{2}}$$

* The output of the bottom correlator is

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt$$

$$x_2 = \int_0^{T_b} [s_2(t) + w(t)]\,\phi_2(t)\,dt$$

$$\boxed{x_2 = s_{22} + w_2} \longleftarrow ②$$

$$S_{22} = \int_0^{T_b} s_2(t)\,\phi_2(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_2(t)\,\phi_2(t)\,dt$$

$$= \sqrt{E_b} \int_0^{T_b} \underbrace{\phi_2^2(t)\,dt}_{\text{unit energy}}$$

$$\therefore \boxed{x_2 = \sqrt{E_b} + W_2} \longleftarrow ②$$

Mean of x_2 is

$$E[x_2] = \sqrt{E_b}$$

$$E[x_2] = E[\sqrt{E_b}] + E[\underset{0}{W_2}]$$

$$= \sqrt{E_b}$$

* Variance of x_2 is $Var[x_2] = \underbrace{Var[\sqrt{E_b}]}_{0} + Var[w_2]$

$$\boxed{Var[x_2] = \frac{N_0}{2}}$$

* Let us find the mean and variance of random variable

$$y = x_1 - x_2$$

Mean $E[y] = E[x_1] - E[x_2]$

$$= 0 - \sqrt{E_b}$$

$$\therefore \boxed{E[y] = -\sqrt{E_b}}$$

* $Var[y] = Var[x_1] + Var[x_2]$

$$= \frac{N_0}{2} + \frac{N_0}{2}$$

$$\boxed{Var[y] = N_0}$$

Note:- The variance of the random variable 'y' is independent of which binary symbol was transmitted.

* Conditional pdf when symbol '0' transmitted is given by

$$f_y(y/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

$$\mu = -\sqrt{E_b} \text{ and } \sigma^2 = N_0$$

$$f_y(y/0) = \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(l+\sqrt{E_b})}{2N_0}}$$

$$\boxed{f_y(y/0) = \frac{1}{\sqrt{2\pi N_0}} e^{-\left(\frac{l+\sqrt{E_b}}{\sqrt{2N_0}}\right)^2}} \leftarrow ③$$

* Let $P_e(0)$ denotes the conditional probability of deciding in favour of Symbol '1' when Symbol '0' is transmitted.

∴ Region Z_1: $0 \leq x \leq \infty$

$$P_e(0) = \int_0^{\infty} f_y(y|0)\, dy = \int_0^{\infty} \frac{1}{\sqrt{2\pi N_0}} e^{-\left(\frac{y+\sqrt{E_b}}{\sqrt{2N_0}}\right)^2} dy$$

let $Z = \frac{y+\sqrt{E_b}}{\sqrt{2N_0}}$

$dz = \frac{dy}{\sqrt{2N_0}} + 0$

$$\boxed{dy = dz(\sqrt{2N_0})}$$

$\underline{\underline{LL}}$

when $y = \infty \Rightarrow Z = \infty$

when $y = 0 \Rightarrow Z = \sqrt{\frac{E_b}{2N_0}}$

$$\therefore P_e(0) = \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi N_0}} e^{-Z^2} dz(\sqrt{2N_0})$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} e^{-Z^2} dz \longleftarrow ④$$

Complementary error function

$$\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-u^2} du \longleftarrow ⑤$$

Comparing eq^n (4) and (5)

$$\boxed{P_e(0) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

lly we can calculate probability of error for 2nd kind

$$P_e(1) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

* if Symbol 0's and 1's are equiprobable then

$$p(0) = p(1) = 1/2$$

* The average probability of error

$$P_e = p(0)\,P_e(0) + p(1)\,P_e(1)$$

$$P_e = \frac{1}{2}\left[\frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)\right] + \frac{1}{2}\left[\frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)\right]$$

$$\therefore\ \boxed{P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

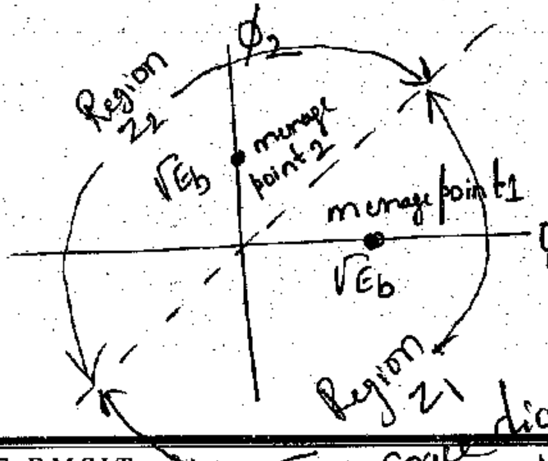


fig:- Signal Space diagram for coherent binary FSK System problem 1.

A binary FSK Syltem transmits data at a rate of 2Mbps over an AWGN channel. the noise power Spectral density $\frac{N_0}{2} = 10^{-20}$ W/Hz. Determine the probability Error P_e for coherent detection of FSK. assume amplitude of the received Signal 1μW. [Note erf (2.2819) = 0.9995

Soln:- $R_b = 2$ Mbps , $\frac{N_0}{2} = 10^{-20}$ W/Hz $\Rightarrow N_0 = 2\times10^{-20}$ W/Hz

$A_C = 1\times10^{-6}$ Volt

$$T_b = \frac{1}{R_b} = \frac{1}{2\times10^{6}} = 0.5\times10^{-6}\ \text{sec}$$

* $E_b = P\cdot T_b$

$$P_{avg} = \frac{A_c^2}{2} = \frac{(1\times10^{-6})^2}{2} = \underline{\underline{5\times10^{-13}}}\ \text{W} = \underline{\underline{0.5\ \text{pW}}}$$

$$E_b = 5\times10^{-13}\times\frac{1}{2\times10^{6}} = \underline{\underline{2.083\times10^{-19}}}\ \text{Joules}$$

* W.k.T $P_{e,FSK} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{2.083\times10^{-19}}{2\times2\times10^{-20}}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{5.2075}\right)$$

$$= \frac{1}{2}\,\text{erfc}(2.2819)$$

$$= \frac{1}{2}\left[1-\text{erf}(2.2819)\right] = \frac{1}{2}\left[1-0.9995\right]$$

$$\boxed{P_e = 2.05\times10^{-4}}$$

problem 2

Binary data are transmitted at a rate of 10^6 bit per Second over a microwave link. Assuming channel noise is AWGN with Zero mean and PSD at the receiver input is 10^{-6} W/Hz. Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent BFSK. note:- $\text{erfc}(2.8) = 2\times10^{-4}$.

soln:-

$$R_b = 10^6 \text{ bps}$$

$$T_b = \frac{1}{R_b} = 1\times10^{-6} \text{ sec}$$

$$\frac{N_0}{2} = 10^{-6} \text{ W/Hz} \implies N_0 = 2\times10^{-6} \text{ W/Hz}. \ P_e \leq 10^{-4}$$

$$P_{e,FSK} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$10^{-4} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\left.\begin{aligned}\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) &= 2\times10^{-4} \\ \text{given} \quad \text{erfc}(2.8) &= 2\times10^{-4}\end{aligned}\right\} \text{comparing}$$

$$\implies \sqrt{\frac{E_b}{2N_0}} = 2.8 \implies \frac{E_b}{2N_0} = 7.84$$

$$E_b = 2\times7.84\times 2\times10^{-6}$$

$$\boxed{E_b = 31.36\times10^{-6}} \text{ Joules}$$

$$* \ E_b = P T_b$$

$$P = \frac{E_b}{T_b} = E_b R_b$$

$$= 31.36\times10^{-6}\times 1\times10^{6}$$

$$\boxed{P = 31.36 \text{ Watts}}$$

Introduction to QPSK

- W.K.T
  In BPSK transmitt's only one bit per Symbol.
  (Even in ASK . FSK also)
- the phase shift's used are 0° and 180° [i.e 0 (or) π]
- But in Qpsk, transmitts 2 bits per Symbol which helps in Conserving the Bandwidth.
- So, these two bits Can have "four" combinations

00
01
10
11

∴ To transmitt two bits / Symbol we need 4 phases (i.e name Quadrature phase shift keying).

- ∴ divide 360° phase into 4 phases

$$\frac{360°}{4} = 90° \quad (\pi/2)$$

So here we have a Separation of phaseangle by 90° in 4-phase's.

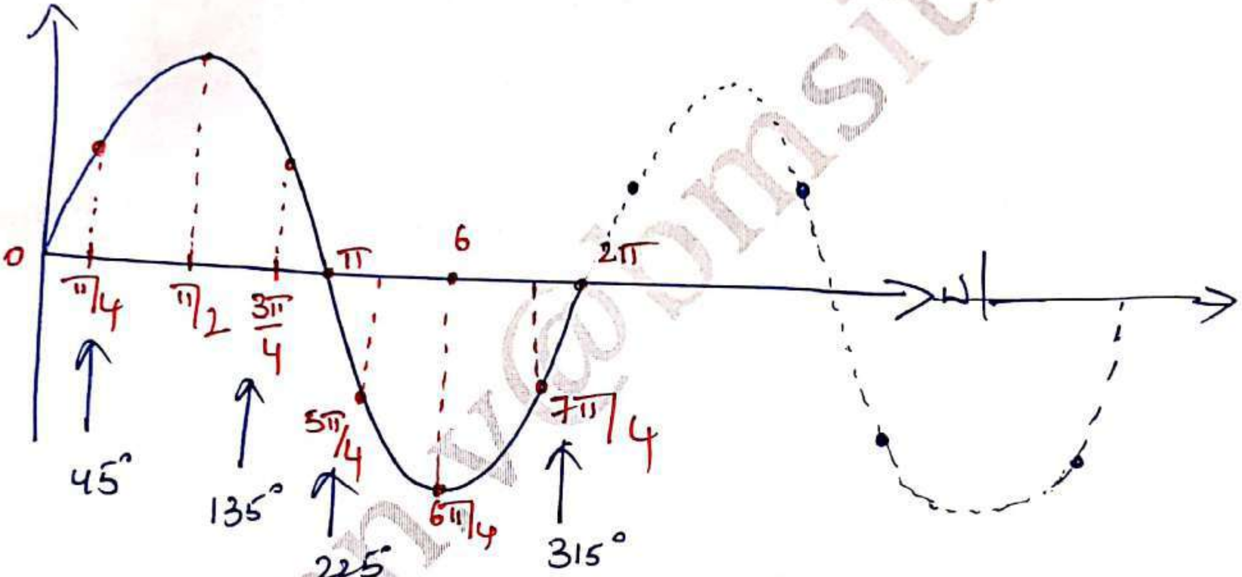
* In Qpsk we use following phases

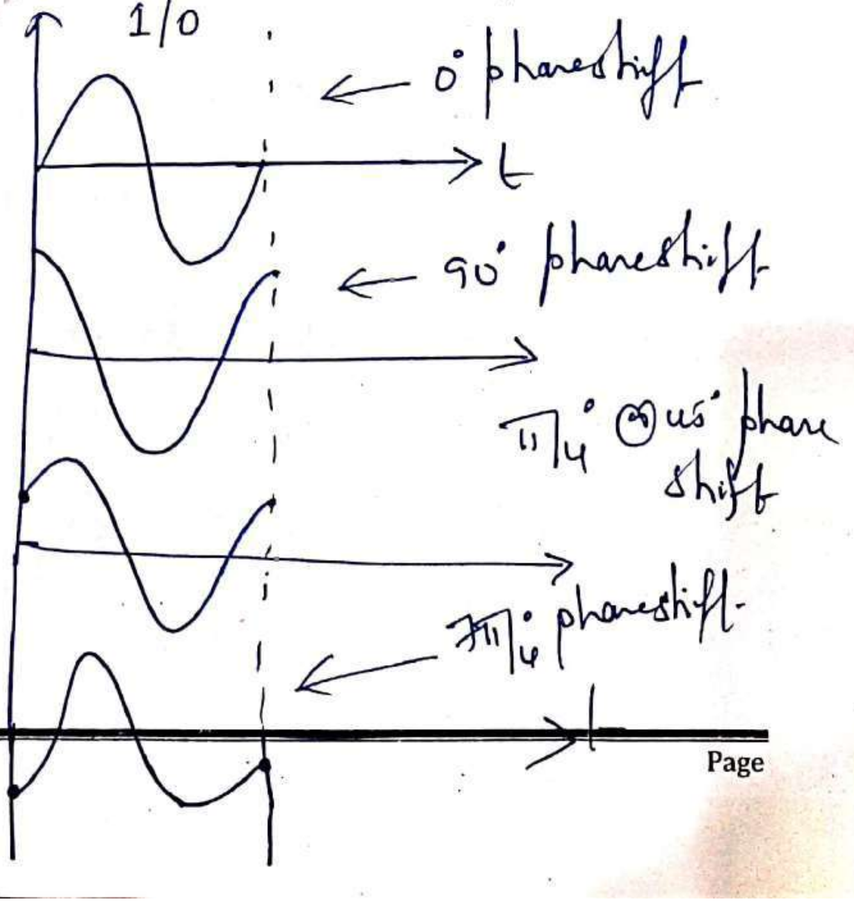
45° → π/4

45° + 90' → 135° ⇒ 3π/4

135° + 90° → 225° ⇒ 5π/4.

225 + 90° = 315° ⇒ 7π/4.





* Qpsk represents 00, 01, 10 and 11 ∴ transmission of 2 bits in a single symbol reduces the signal rate. This reduces the frequency of the carrier needed for transmission and the transmission channel Bandwidth (Bw) is reduced. (Bw) ↓.

Mathematical Expression

$$S'(t) = A_c \cos\left[2\pi f_c t + \phi(t)\right]$$

where $\phi(t) = (2m-1)\pi/4$

$m = 1, 2, 3, 4.$

using identity,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

$$S(t) = A_c \cos\omega_c t \cos\phi(t) - A_c \sin\omega_c t \sin\phi(t).$$

$$= A_c \cos\left[\omega_c t + \phi(t)\right]$$

$$s(t) = A_c \cos 2\pi f_c t \cos\phi(t) - A_c \sin 2\pi f_c t \sin\phi(t)$$

w.k.t

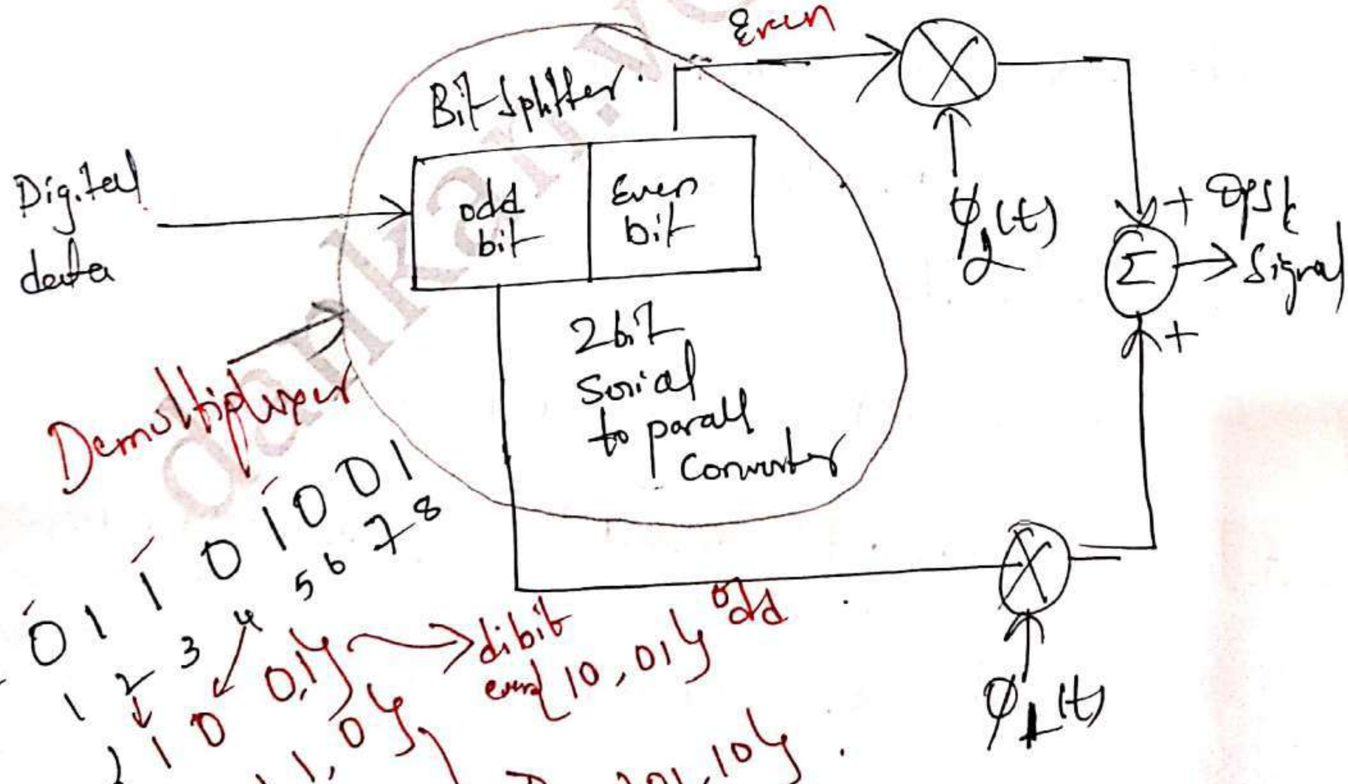
$$A_c = \sqrt{\frac{E}{T}} \quad \cancel{E_b} \xrightarrow{\text{here}} \boxed{E = 2E_b}$$

$$s(t) = \underbrace{\sqrt{\frac{2E}{T}} \cos 2\pi f_c t}_{\phi_1(t) \text{ basis fn}} \cos\phi(t) - \underbrace{\sqrt{\frac{2E}{T}} \sin 2\pi f_c t}_{\phi_2(t) \text{ basis fn}} \sin\phi(t)$$

$\phi_1(t)$ basis fn $= \sqrt{\frac{2}{T}} \cos 2\pi f_c t$

$\phi_2(t)$ basis fn $= \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$$s(t) = \sqrt{E}\phi_1(t) \cos\phi(t) - \sqrt{E}\phi_2(t) \sin\phi(t).$$



$b_i = 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1$
$\quad\ \ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8$

$b_{ei} = \{1\,0\ \ 0,1\}$ → dibit even $\{10, 01\}$

$b_{oi} = \{0\,1\ 1,0\}$ → dibit odd $\{01, 10\}$.

QPSK.

Module-3

With a neat block diagram, explain the generation and coherent detection of QPSK signals. **(06 Marks) June-July 2018.**

- Quadrature phase shift keying (QPSK) is a form of PSK, in which "two bits are modulated at once."

- it selects one of four possible Carrier phase shifts [$\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$].

- Qpsk allows the signal to carry twice as much information as ordinary psk using the same "Bandwidth".

- Qpsk is used for sattellite transmision for MPEG-2 video, cable modem and cellular phone system.

- A Qpsk signal can be represented in time domain as

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\pi/4\right]; & 0 \le t \le T \\ 0 & : ow \end{cases} \quad \leftarrow ①$$

where

$i = 1, 2, 3, 4$ and

E - Signal Energy per symbol.

T - Symbol duration.

* There are four message points and associated signal vector are defined by

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \theta_i\right] & ; 0 \le t \le T \\ 0 & ; ow \end{cases} \longleftarrow \textcircled{2}$$

where $\theta_i = (2i-1)\pi/4$

eq^n ② can be written as

dibit	i	$\theta_i = (2i-1)\pi/4$
10	1	$\pi/4$
00	2	$3\pi/4$
01	3	$5\pi/4$
11	4	$7\pi/4$

$$S_1(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \pi/4\right);$$
dibit 10

$$S_2(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 3\pi/4\right);$$
dibit 00

$$S_3(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 5\pi/4\right) \; ; \text{ dibit } 01$$

$$S_4(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 7\pi/4\right) \; ; \text{ dibit } 11.$$

using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

eq^n ① can be re-written as

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos 2\pi f_c t \cdot \cos\left[(2i-1)\pi/4\right] - \sqrt{\frac{2E}{T}} \sin 2\pi f_c t \sin\left[(2i-1)\pi/4\right]; & 0 \le t \le T \\ 0 \; ; \quad ow \end{cases} \longleftarrow \textcircled{3}$$

* from equations ③, we observe that there are two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$ defined by

$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \; ; \; 0 \le t \le T \quad \longleftarrow ④$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t) \; ; \; 0 \le t \le T$$

* The Elements of the signal vector, namely s_{i1} and s_{i2}

i	$S_i(t)$	I/p dibit	phase of Qpsk signal	Co-ordinates of message points	
				s_{i1}	s_{i2}
1	$S_1(t)$	10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
2	$S_2(t)$	00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
3	$S_3(t)$	01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
4	$S_4(t)$	11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

* The above table represented in the form of signal space diagram shown in fig c.

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QPSK Transmitter :-

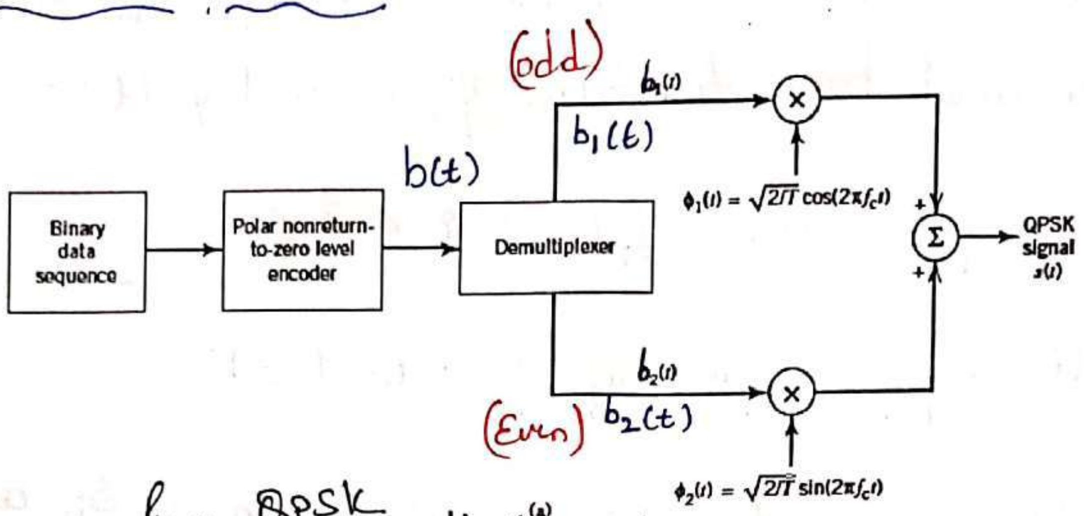


fig a. QPSK Transmitter (a)

* The input binary Sequence b(t) represented in polar form is divided into odd $[b_1(t)]$ and Even $[b_2(t)]$ numbered bits by using Demultiplexer. They are denoted as $b_1(t)$ and $b_2(t)$.
* These two Sequences, phase modulated by two carrier Signal of same frequency but Quadrature in phase.
* Since Each Carrier two bits, the Signaling rate decreased ∴ BW required is half of the BW required compared to BPSK.

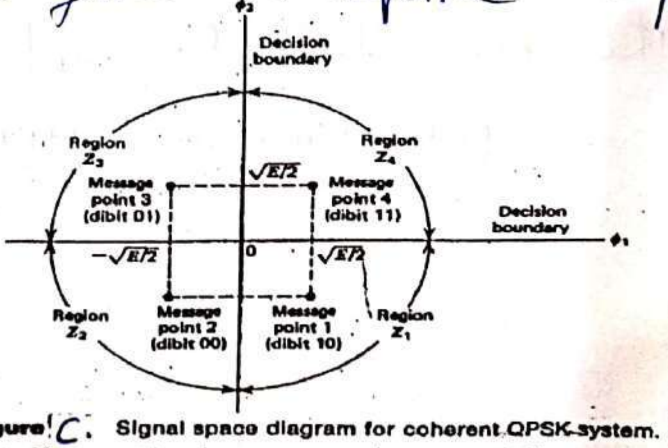


Figure C. Signal space diagram for coherent QPSK system.

Qpsk Receiver :-

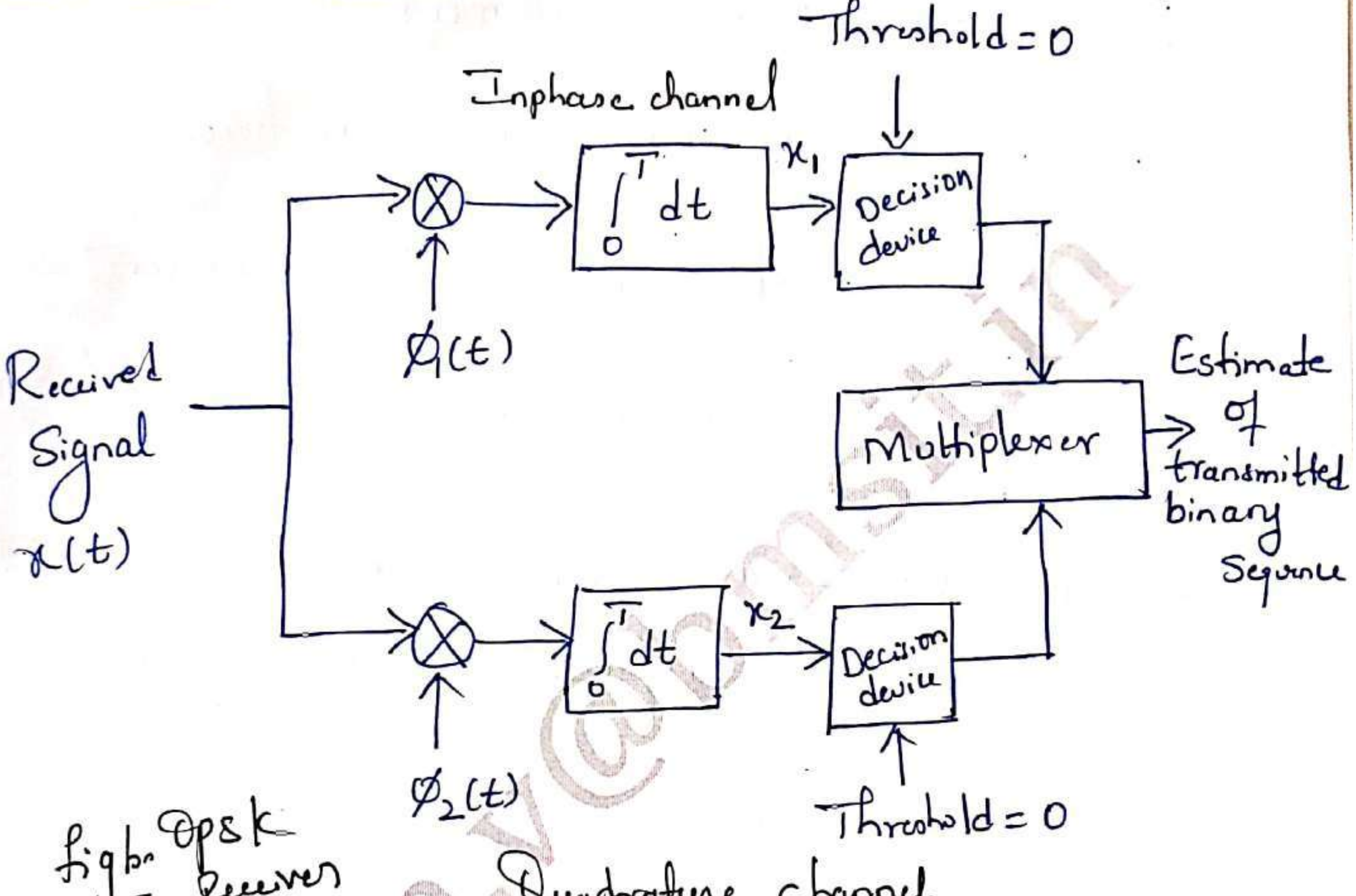


fig: Qpsk Receiver

* The Qpsk receiver consists of a pair of Correlator with Locally generated Carrier Signal $\phi_1(t)$ and $\phi_2(t)$.

* The output of the two Correlators are x_1 and x_2 are Compared with a threshold 0.

if $x_1 > 0$ => decision is made in favour of Symbol 1.

if $x_1 < 0$ => decision is made in favour of Symbol 0.

if $x_2 > 0$ => decision is made in favour of Symbol 1.

if $x_2 < 0$ => decision is made in favour of Symbol 0.

* The two output's are Combined in a Multiplexer to to reproduce original binary Sequence.

Probability of Error in QPSK

* The Signal points S_1, S_2, S_3 and S_4 are Located Symmetrically in two dimensional Signal Space diagram as shown in fig c.

* Computing probability Error for one message point is remains Same for other three points.

* Consider transmission of Symbol $S_4(t)$ then Received Signal $x(t)$ will be

$$x(t) = S_4(t) + \omega(t) \quad : 0 \le t \le T$$

* The Samples x_1 and x_2 are computed as follows

$$x_1 = \int_0^T x(t)\,\phi_1(t)\,dt = \int_0^T [S_4(t) + \omega(t)]\,\phi_1(t)\,dt$$

$$x_1 = \int_0^T S_4(t)\,\phi_1(t)\,dt + \int_0^T \omega(t)\,\phi_1(t)\,dt$$

$$x_1 = S_{41} + W_1$$

where $S_{41} = \sqrt{\frac{E}{2}} = S_{42}$

$$\boxed{x_1 = \sqrt{\frac{E}{2}} + W_1}$$

* lly. $x_2 = \int_0^T x(t)\,\phi_2(t)\,dt = \int_0^T [S_4^{(t)} + \omega(t)]\,\phi_2(t)\,dt$

$$x_2 = S_{42} + W_2$$

$$\boxed{x_2 = \sqrt{\frac{E}{2}} + W_2}$$

- * X_1 and X_2 are guassian random variables with mean $\mu = \sqrt{E/2}$
- * W_1 and W_2 are also guassian random variable with variance $\sigma^2 = N_0/2$.
- * When signal $S_4(t)$ is transmitted the received signal point lies in the decision region Z_4.

  if $x_1 > 0$ and $x_2 > 0$ ⇒ Leading to Correct decision.
  (Symbol-1) (symbol-1)

- * The conditional pdf is given by

$$f_{x_1}(x_1 | S_4(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left[\frac{(x_1-\mu)^2}{2\sigma^2}\right]}$$

$$f_{x_2}(x_2 | S_4(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left[\frac{(x_2-\mu)^2}{2\sigma^2}\right]}$$

use $\mu = \sqrt{\frac{E_b}{2}}$ and $\sigma^2 = \frac{N_0}{2}$

$$f_{x_1}(x_1 | S_4(t)) = \frac{1}{\sqrt{2\pi \cdot \frac{N_0}{2}}} e^{-\left[\frac{\left(x_1 - \sqrt{\frac{E}{2}}\right)^2}{2 \cdot N_0/2}\right]}$$

$$f_{X_2}\left(x_2 \middle| S_4(t)\right) = \frac{1}{\sqrt{\cancel{2}\pi \cdot \frac{N_0}{\cancel{2}}}} e^{-\left[\frac{\left(x_2-\sqrt{\frac{E}{2}}\right)^2}{\cancel{2}N_0/\cancel{2}}\right]}$$

$$\boxed{f_{X_1}\left(x_1 \middle| S_4(t)\right) = \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_1-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2}} \leftarrow ①$$

$$\boxed{f_{X_2}\left(x_2 \middle| S_4(t)\right) = \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_2-\sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2}} \leftarrow ②$$

* Let us assume $S_4(t)$ is transmitted. if the received signal 'x' should fall in region Z_4 i.e both x_1 and x_2 should be +ve.

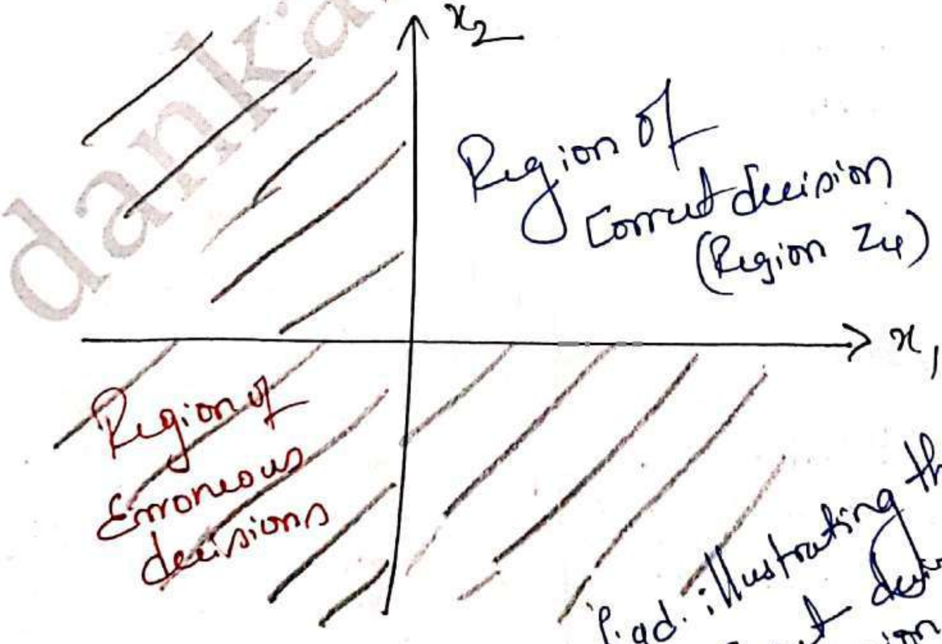


fig d: illustrating the region of Correct decision and the region of Erroneous decision, given $S_4(t)$ is transmitted.

* probability of correct decision 'P_c' is equal to the product of conditional probabilities of Events $x_1 > 0$ & $x_2 > 0$ both given that $S_4(t)$ was transmitted.

In Region Z_4:- $$\boxed{\begin{matrix} 0 \le x_1 \le \infty \\ 0 \le x_2 \le \infty \end{matrix}}$$

$$P_c = \int_0^{\infty} f_{X_1}(x_1|S_4(t))\, dx_1 \times \int_0^{\infty} f_{X_2}(x_2|S_4(t))\, dx_2 \longrightarrow ③$$

$$P_c = \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2} dx_1 \cdot \int_0^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\left[\frac{x_2 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}\right]^2} dx_2$$

let $Z = \frac{x_1 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}$

$dz = \frac{dx_1 - 0}{\sqrt{N_0}}$

$\boxed{dx_1 = \sqrt{N_0}\, dz}$

UL $x_1 = \infty$ then $\boxed{Z = \infty}$

LL. $x_1 = 0$, $Z = \frac{0 - \sqrt{E/2}}{\sqrt{N_0}}$

$\boxed{Z = -\sqrt{\frac{E}{2N_0}}}$

$Z = \frac{x_2 - \sqrt{E/2}}{\sqrt{N_0}}$

$dz = \frac{dx_2 - 0}{\sqrt{N_0}}$

$\boxed{dx_2 = \sqrt{N_0} \cdot dz}$

UL. $x_2 = \infty$ then $\boxed{Z = \infty}$

LL:- $x_2 = 0$, $Z = \frac{x_2 - \sqrt{\frac{E}{2}}}{\sqrt{N_0}}$

$Z = \frac{0 - \sqrt{E/2}}{\sqrt{N_0}}$ $\boxed{Z = -\sqrt{\frac{E}{2N_0}}}$

$$P_c = \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi \cancel{N_0}}} e^{-z^2} \cancel{\sqrt{N_0}}\, dz \cdot \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi \cancel{N_0}}} e^{-z^2} \cancel{\sqrt{N_0}}\, dz$$

$$P_c = \int_{-\sqrt{E/2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz \cdot \int_{-\sqrt{E/2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} e^{-z^2} dz = \left[\frac{1}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} e^{-z^2} dz\right]^2 \leftarrow ④$$

from the definition of the Complementary errorfn. we have

$$\frac{1}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} e^{-z^2} dz = 1 - \frac{1}{2} erfc\left(\sqrt{\frac{E}{2N_0}}\right) \leftarrow ⑤$$

using eqn ③ in eqn ④

$$\therefore \; P_c = \left[1 - \frac{1}{2} erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]^2$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a = 1 \quad b = \frac{1}{2} erfc\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$P_c = 1 + \frac{1}{4} erfc^2\left(\sqrt{\frac{E}{2N_0}}\right) - erfc\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$\boxed{P_c = 1 + \frac{1}{4} erfc^2\left(\sqrt{\frac{E}{2N_0}}\right) - erfc\left(\sqrt{\frac{E}{2N_0}}\right)}$$

Thus average probability of symbol Error $P_e = 1 - P_c$

$$P_e = 1 - \left[1 + \frac{1}{4} erfc^2\left(\sqrt{\frac{E}{2N_0}}\right) - erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

$$\boxed{P_e = erfc\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4} erfc^2\left(\sqrt{\frac{E}{2N_0}}\right)}$$

* In the region Z_4: $\left(\frac{E}{2N_0}\right) >> 1$, hence we can ignore second term

$$P_e \simeq erfc\left(\sqrt{\frac{E}{2N_0}}\right)$$

and in QPSK two bits are transmitted, thus

$$E = 2E_b$$

$$\therefore \boxed{P_{e\,(QPSK)} \simeq erfc\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

P_e in terms of Q function

$$\boxed{P_{e\,QPSK} \simeq 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

Note: use

$$\boxed{Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right)}$$

Sketch the QPSK waveform for the sequence 01101000. **(06 Marks) June-July 2019.**

Soln:

$b_i = \{0\ 1\ 1\ 0\ 1\ 0\ 0\ 0\}$

$\quad\ \ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8$

$b_{oi} = \{0\ 1\ 1\ 0\}$

$b_{ei} = \{1\ 0\ 0\ 0\}$

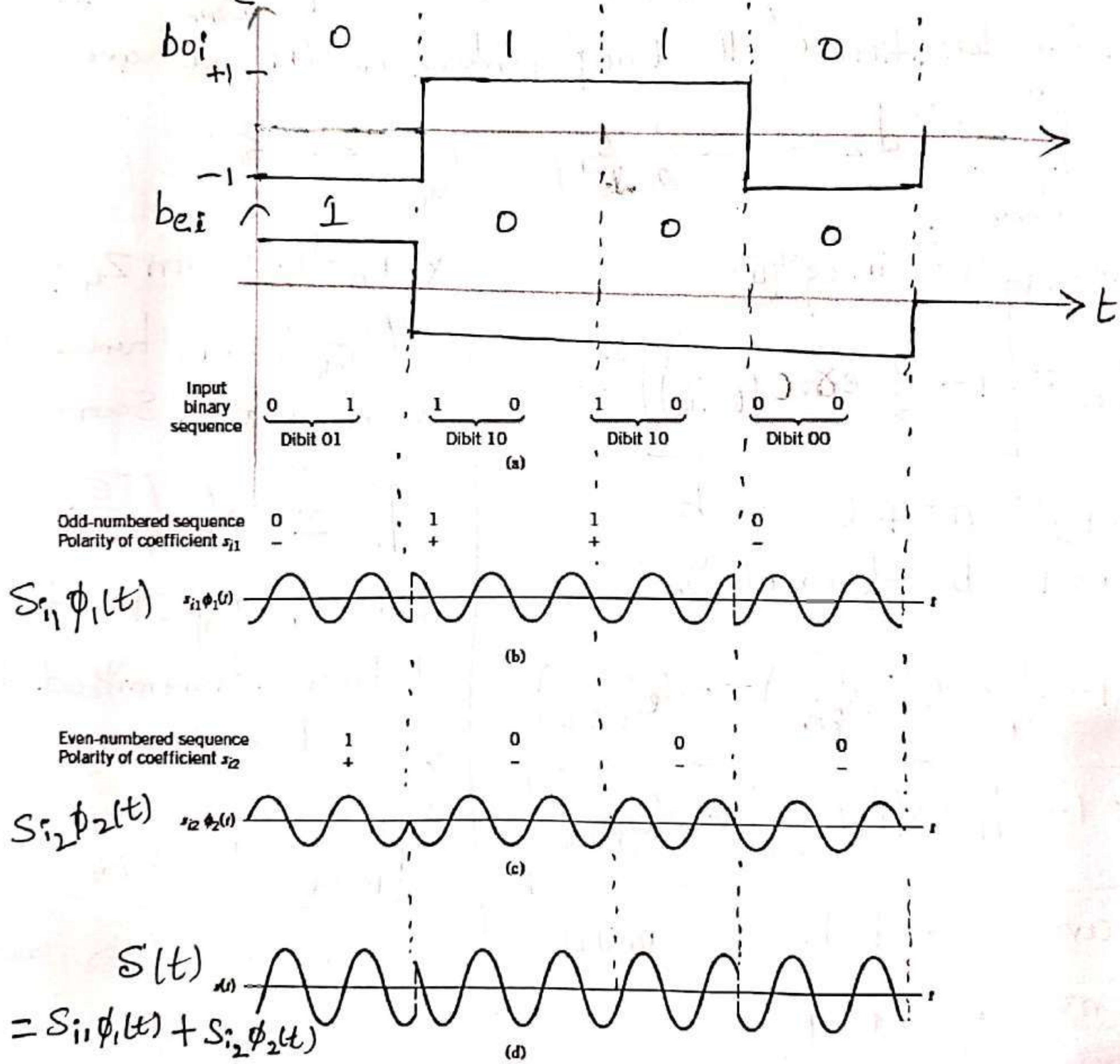


Figure **(a) Input binary sequence. (b) Odd-numbered dibits of input sequence and associated binary PSK signal. (c) Even-numbered dibits of input sequence and associated binary PSK signal. (d) QPSK waveform defined as $s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$.**

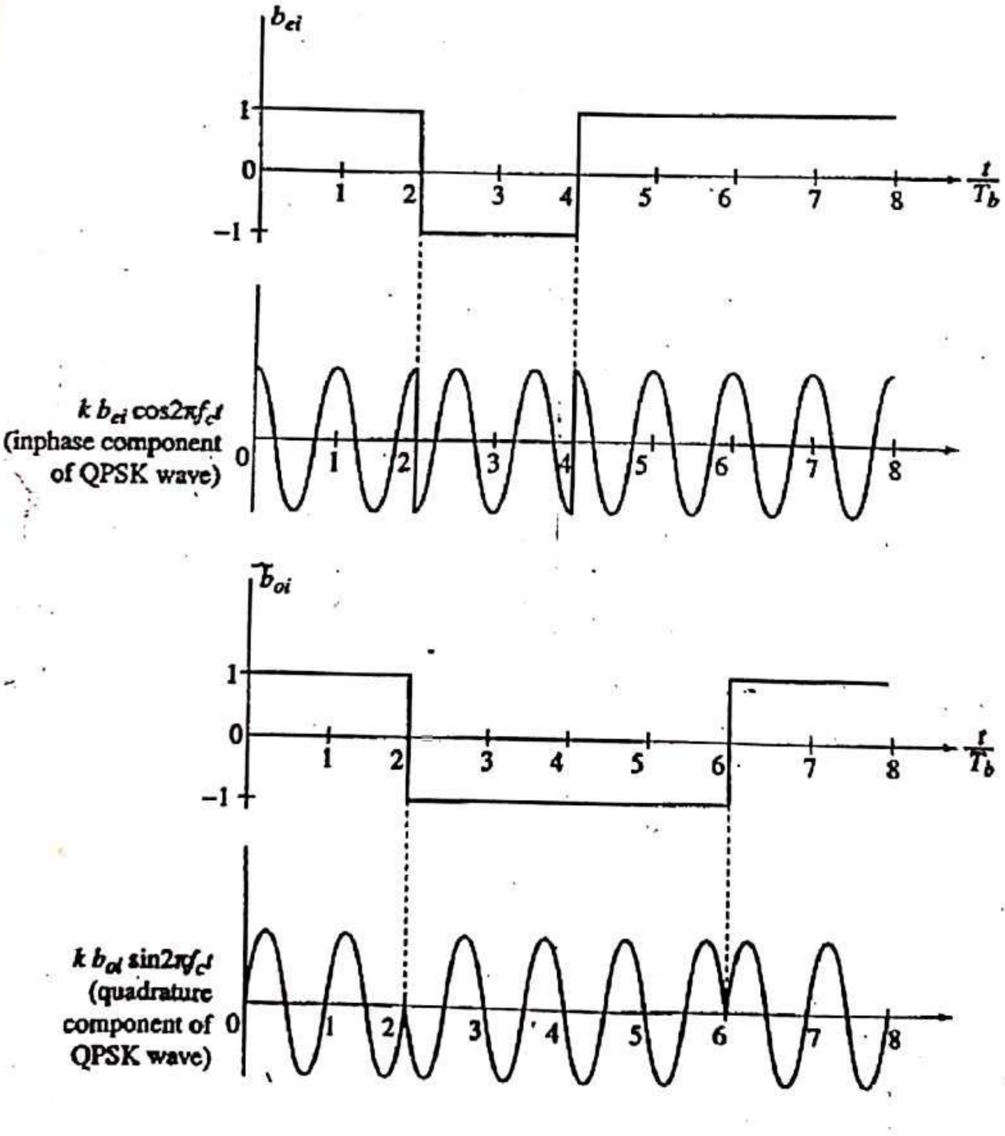
Sketch the inphase and quadrature components of a QPSK signal for the binary sequence 1 1 0 0 1 0 1 1 1. Assume carrier frequency f_c to be equal to $1/T_b$. Choose any convenient basis functions.

Solution: For the present context, $\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t$ and $\sqrt{\frac{2}{T_b}}\sin 2\pi f_c t$ are chosen as basis functions.

We have

$$\{bi\} = \{1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 1\}$$
$$\{b_{ei}\} = \{1 \quad 0 \quad 1 \quad 1 \quad 1\}$$
$$\{b_{oi}\} = \{1 \quad 0 \quad 0 \quad 1\}$$

The sketches of $\{b_{ei}\}$, $\{b_{oi}\}$ and quadrature components of QPSK signal are shown in Figure . Note that, we have taken, $f_c = 1/T_b$.



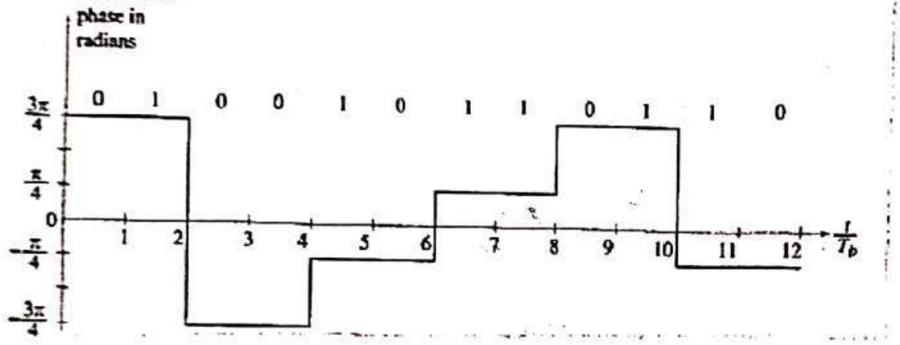
The input binary sequence to a QPSK modulator is

$$\{b_k\} = \{0\,1\,0\,0\,1\,0\,1\,1\,0\,1\,1\,0\}.$$

Sketch the transmitted phase of the carrier as a function of time.

Solution: The transmitted phase for each dibit is as follows.

Dibit	Transmitted phase
11	$\frac{\pi}{4}$ rad
01	$\frac{3\pi}{4}$ rad
00	$-\frac{3\pi}{4}$ rad
10	$-\frac{\pi}{4}$ rad



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Mary PSK :-

- Need for M-ary psk
- what are the difference b/w psk and M-ary psk.

Defn:- By using psk, we are limited in the usage of symbols and number of bits so in order to increase the capacity of channels we are going for "M-ary PSK".

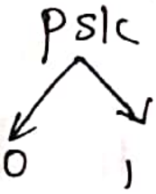
i. PSK (or) 2psk (or) BPSK.

M - No. of symbols
N - No. of bits

$$\boxed{M = 2^N}$$

∴ In psk

$M = 2$, ⇒ { $N = 0$ (or) 1; no. of bits are 1.



only two symbol's

- BW required is More in PSK.
- per symbol only one bit is txtted. (i.e either 0 (or) 1)
- aim BW↓ R_b↑ increase the data rate.

ii. 4-psk (QPSK)

$M = 4$. $M = 2^2 = 2^N$.

∴ $N = 2$ (no. of bits per symbol is 2).

00
01
10
11

} no. of bits = 2.
Capacity increases twice compared to psk.

8-PSK

$M = 8 = 2^3 = 2^N$

$\therefore \boxed{N=3}$ 3 bits per symbol.

000
001
010
011
100
101
110
111

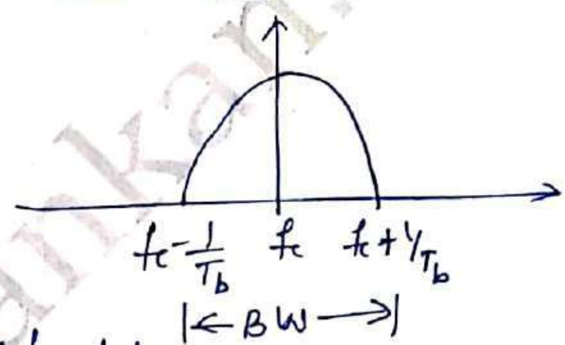
@ Recieverend By decoding Each Symbol we get '3' bits.

∴ compared to psk, 8psk increases the capacity of data transmission "3 times".

Now Current technology uses 64 QAM. but in 5G uses => 256 QAM. (latest).

Band width Variation :-

for psk



Txtted
1 bit / symbol.

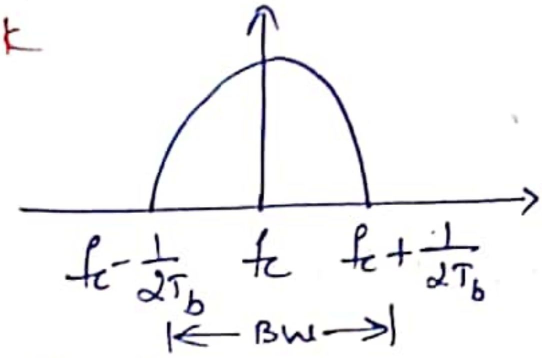
BW = U.f - L.f

$BW = f_c + \frac{1}{T_b} - f_c + \frac{1}{T_b}$

$$\boxed{BW = \frac{2}{T_b} = 2R_b}$$

Note:- $R_b = \frac{1}{T_b}$ ie bit rate Hz/sec.

4-psk | qpsk



two bit's (2 bit)/Symbol.

$$BW = f_c + \frac{1}{2T_b} - f_c + \frac{1}{2T_b}$$

$$BW = 2\left(\frac{1}{2T_b}\right) = \frac{1}{T_b}$$

$$\boxed{BW = \frac{1}{T_b} = R_b} \; Hz.$$

obs:- $(BW)_{psk} = \frac{2}{T_b}$

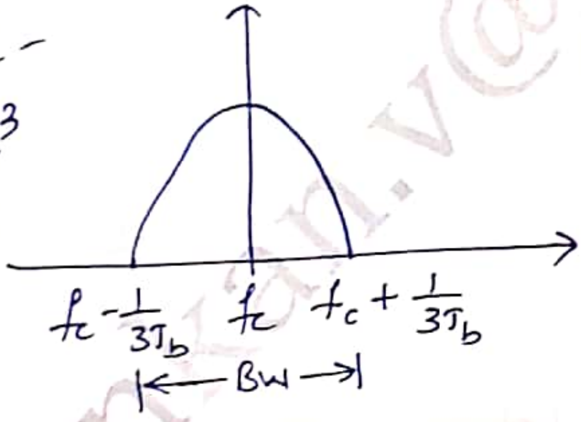
$(BW)_{qpsk} = \frac{1}{T_b}$

$\Rightarrow$ As $M\uparrow \Rightarrow BW\downarrow \Rightarrow R_b\downarrow$

8psk :-

$M = 8 = 2^3$

$N=3$



$$BW = f_c + \frac{1}{3T_b} - f_c + \frac{1}{3T_b}$$

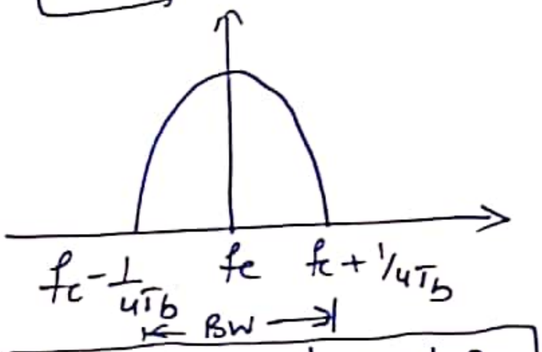
$$\boxed{BW = \frac{2}{3T_b}} \; H_z = \frac{2}{3} R_b$$

ⓐ $\boxed{BW = \frac{2}{3} R_b}$

16-psk

$M = 16 = 2^N = 2^4$

$\boxed{N=4}$



$$\boxed{BW = \frac{2}{4T_b} = \frac{1}{2T_b} = \frac{1}{2} R_b}$$

obs:- $BW\downarrow$

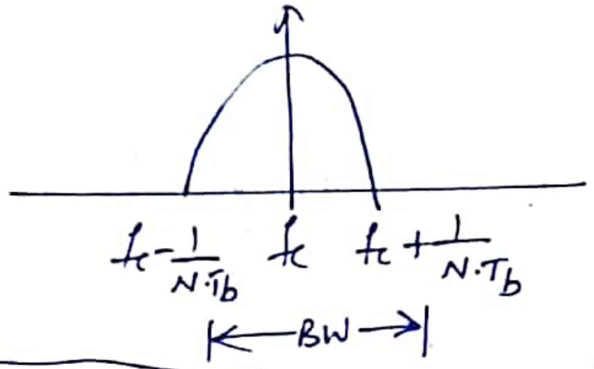
(data rate) $R_b \therefore \downarrow$

$Cp\uparrow$

In general :- M-ary psk

$$\boxed{M = 2^{N}} \leftarrow \text{no of bits per symbol.}$$

ary



$$\therefore \boxed{BW = \frac{2}{N \cdot T_b} = \frac{2}{N} R_b}$$

from the eqn $M = 2^{N}$ $\log_2$ on both side

$$\log_2 M = \log_2 2^{N}$$

$$\log_2 M = \cancelto{1}{(\log_2 2)} \cdot N$$

$$\therefore \boxed{N = \log_2 M}$$

$$\therefore \boxed{(BW)_{M\text{-ary psk}} = \frac{2R_b}{\log_2 M}}$$

phase shift's in M-ary psk

$$\text{phase shift} = \frac{2\pi}{\text{No. of symbols}}$$

$$\boxed{M = 2^N}$$

In psk :- Txt is only one bit | symbol.

ie $\left.\begin{matrix}0\\1\end{matrix}\right\} \Rightarrow$ no. of symbols = 2. $\Rightarrow 0°, 180° \Rightarrow$ 2 phases

M = 2 and N = 1

$$\therefore \text{phaseshift} = \frac{2\pi}{2} = \pi$$

(ie 0° and 180° are two phaseshift's in psk).

In Qpsk (4-psk) :- $M = 4 = 2^N$

$\boxed{N = 2} \Rightarrow$ 2 bits | symbol.

4 symbol's 00 ← Each symbol having two bit's

$\left.\begin{matrix}00\\01\\10\\11\end{matrix}\right\} \Rightarrow$ total 4 distinct symbol's

$$\text{phase shift} = \frac{2\pi}{4} = \pi/2$$

$\Rightarrow$ 0°, 90°, 180°, 270° 4 phases.

Illy for

M-ary psk $\text{phaseshift} = \frac{2\pi}{\text{No. of symbols}} = \frac{2\pi}{M}$

and duration of Each symbol will be $\boxed{T_s = NT_b}$

where N - no of bits.

i.e. $T_s = T_b \rightarrow$ Bpsk

$T_s = 2T_b \rightarrow$ Qpsk

$T_s = 3T_b \rightarrow$ 8 psk

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Define bandwidth efficiency. Tabulate and comment on the bandwidth efficiency of M-ary PSK signals for different values of M. (06 Marks) Dec 2018-Jan 2019.

Soln:- w.k.t the Bw of a M-ary PSK is given by

$$\boxed{B.W = \frac{2R_b}{\log_2 M}} \text{ Hz}$$

where $R_b = \frac{1}{T_b}$: bitrate
M- no.of symbols.

Bandwidth efficiency of M-ary PSK signal is the ratio of bit rate by Bandwidth.

i.e $$\boxed{\rho = \frac{R_b}{B.W} = \frac{\log_2 M}{2}}$$

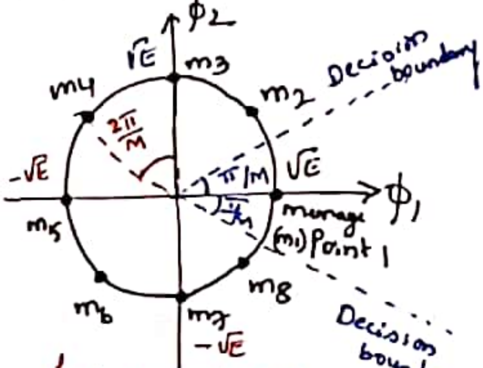


fig:- Signal Space diagram (M-ary PSK)

Table Bandwidth efficiency of *M*-ary PSK signals

M	2	4	8	16	32	64
ρ (bit/(s/Hz))	0.5	1	1.5	2	2.5	3

obs:- i. as M ↑ B.W efficiency (ρ) ↑

ii. if M↑ Results Bw ↓ and increase in datarate ↑.

iii. As the no. of states (i.e M) in M-ary PSK is increased, the Bandwidth efficiency (ρ) is improved at the expense of "Error peformance".

Remedy

* To Ensure NO degradation in Error performance we have to increase (E_b/N_0) to compensate for the increase in M.

Problem :- In digital CW Communication System, the bit rate of NRZ data stream is 1mbps and carrier frequency is 100 MHz. Find the Symbol rate of transmission and B.W requirement of the channel in the following cases of different techniques used.

i. Bpsk system.
ii. Qpsk System.
iii. 16-ary psk system.

Soln:- Given bit rate $f_b = 1 mbps = 1\times10^6$.

hence $T_b = \frac{1}{f_b} = \frac{1}{1\times10^6} = 1\times10^{-6} = 1 \mu sec.$

i. Bpsk System:-

$$BW = \frac{2}{T_b} = \frac{2}{1\times10^{-6}} = 2\times10^6 = 2MHz.$$

In Bpsk, one bit is considered as one Symbol.

$$\therefore T_s = 1 . T_b = 1\times10^{-6}$$

$$\therefore \text{Symbol rate} = \frac{1}{T_s} = \frac{1}{1\times10^{-6}} = \underline{\underline{1\times10^6 \text{ Symbols/sec}}}$$

$$\text{bit rate} = 1\times10^6 \text{ bits/sec}$$

ii. QPSK System

$$BW = \frac{1}{T_b} = f_b = 1 \times 10^6 = 1 MHz.$$

In QPSK two bits / symbol are transmitted.

$$\therefore \quad T_s = 2T_b = 2 \times 10^{-6} = 2 \mu sec.$$

$$\text{Symbol rate} = \frac{1}{T_s} = \frac{1}{2 \times 10^{-6}} = 500 \times 10^3 \text{ Symbols/sec}$$

$$= 500 \times 10^3 = \underline{\underline{0.5 \times 10^6}} \text{ bits/sec.}$$

iii. 16-PSK System.

$$BW = \frac{1}{2T_b} = \frac{1}{2 \times 1 \times 10^{-6}} = 0.5 \times 10^6$$

$$= \underline{\underline{500 kHz}}.$$

In 16 psk ⇒ 4 bits / Symbol

$$\therefore \quad T_s = 4 \cdot T_b$$

$$= 4 \times 1 \times 10^{-6} = 4 \mu sec$$

$$\therefore \text{Symbol rate} = \frac{1}{T_s} = \frac{1}{4 \times 10^{-6}} = 250 \times 10^3 \text{ Symbols/sec}$$

$$= \underline{\underline{0.25 \times 10^6}} \text{ bits/sec}$$

M-ary psk :-

one bit

M = 2 ⇒ Bpsk 0|1 phase 0, π

M = 4 ⇒ Qpsk 00, 01, 10, 11 } two bits per symbol and 4 phases

if M = 8 ⇒ 8 psk

8-psk

↓

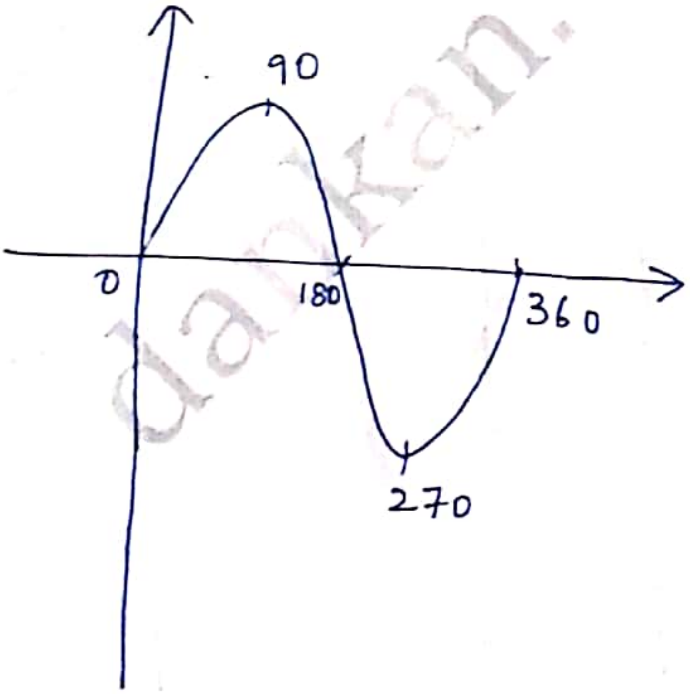
2^3 phase shift keying

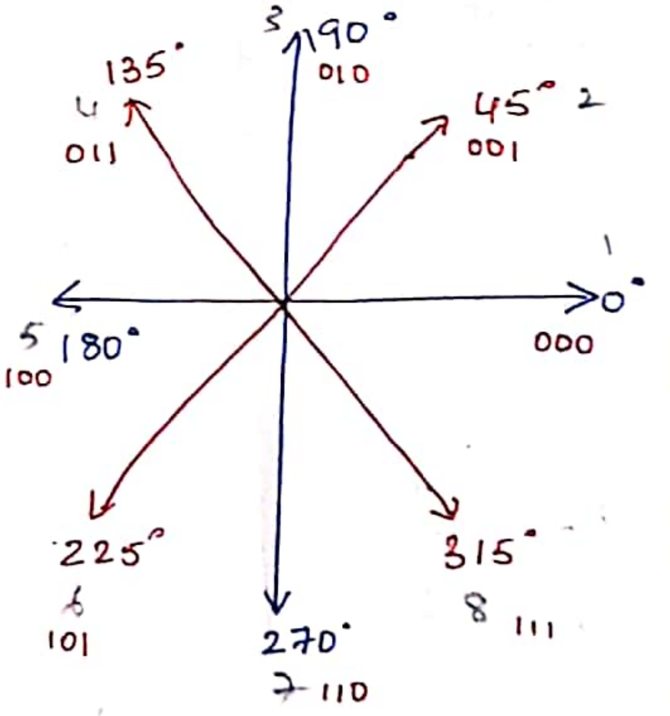
$2^{③} = 8$ → 3 bit/symbol. (i.e Transmission is 3 bits/symbol).

and total 8 phases.

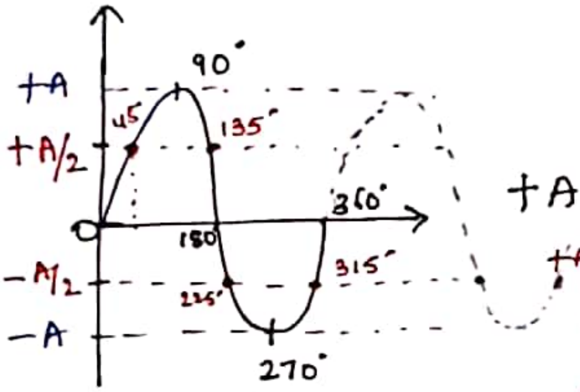
phasor diagram :-

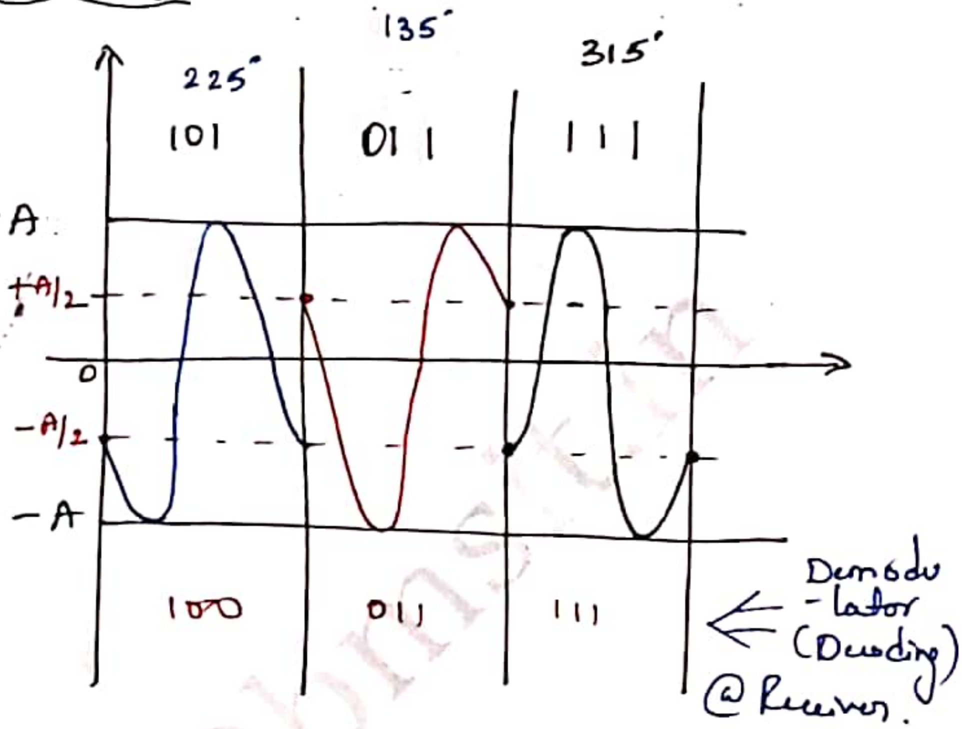
Constellation diagram (3bits)

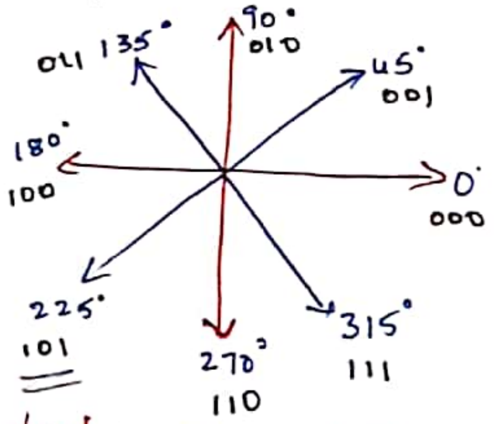


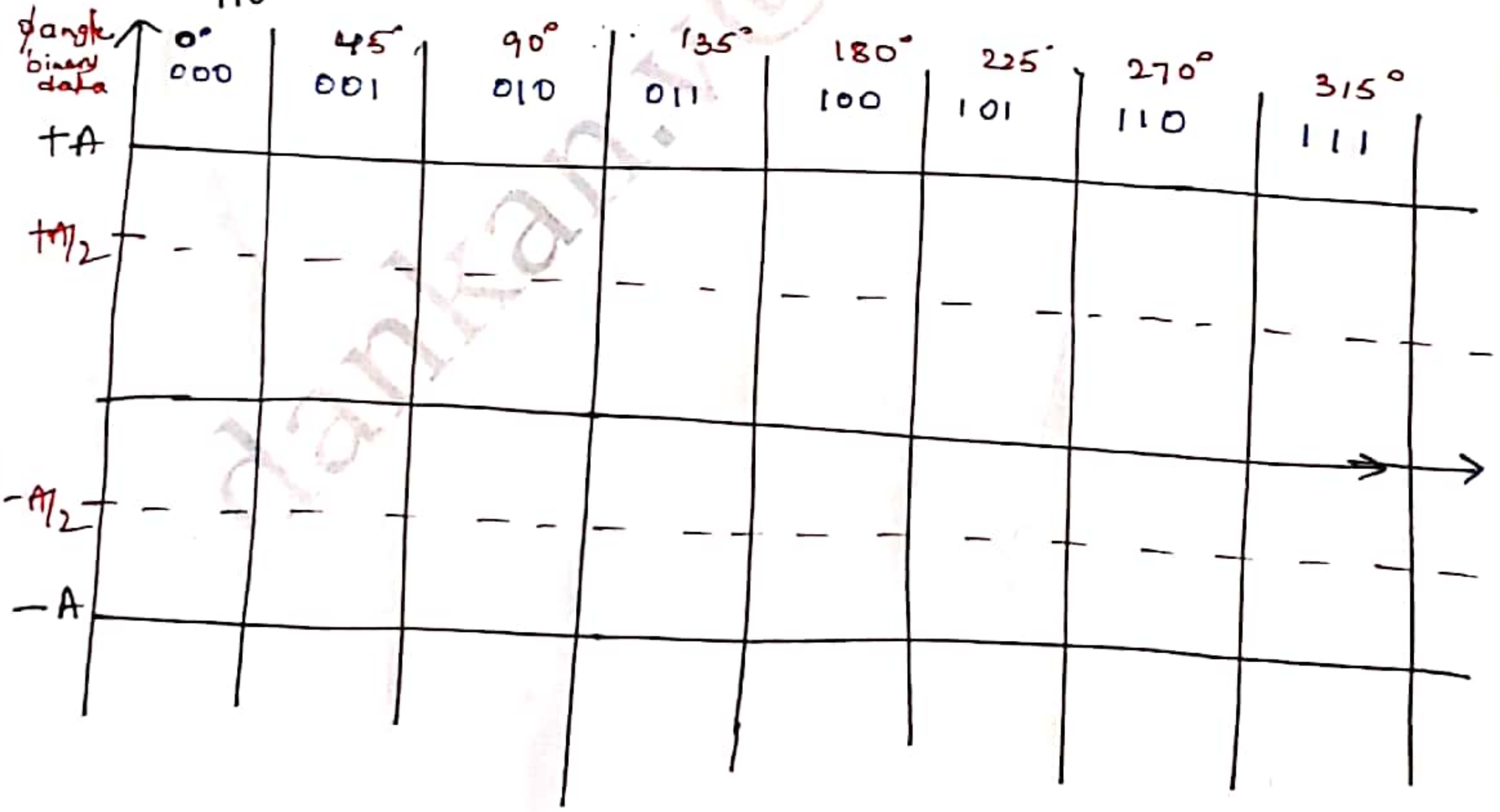


How to draw 8-psk signal









Problem.

Assume that you are required to transmitt $f_b = 90$mbps (ie bit rate) in an authorised bandwidth of 20MHz. which modulation techniques would you consider? Explain why?

Soln:- for this transmission, M-ary psk modulation method can be used.

$$(BW)_{M\text{-ary psk}} = \frac{2}{N \cdot T_b}$$

given $f_b = 90\times10^6$ Hz

$T_b = \frac{1}{90\times10^6}$ sec.

and $BW = 20$ M Hz

$= 20\times10^6$ Hz.

for a transmission

$$BW \le \frac{2}{NT_b}$$

$$20\times10^6 \le \frac{2}{N\times\left(\frac{1}{90\times10^6}\right)}$$

$$\cancel{2}\cancel{0}\times\cancel{10^6} \le \frac{9\cancel{0}\times\cancel{10^6}\times\cancel{2}}{N}$$

$$\boxed{N \le 9} \Rightarrow \boxed{N = 9}$$

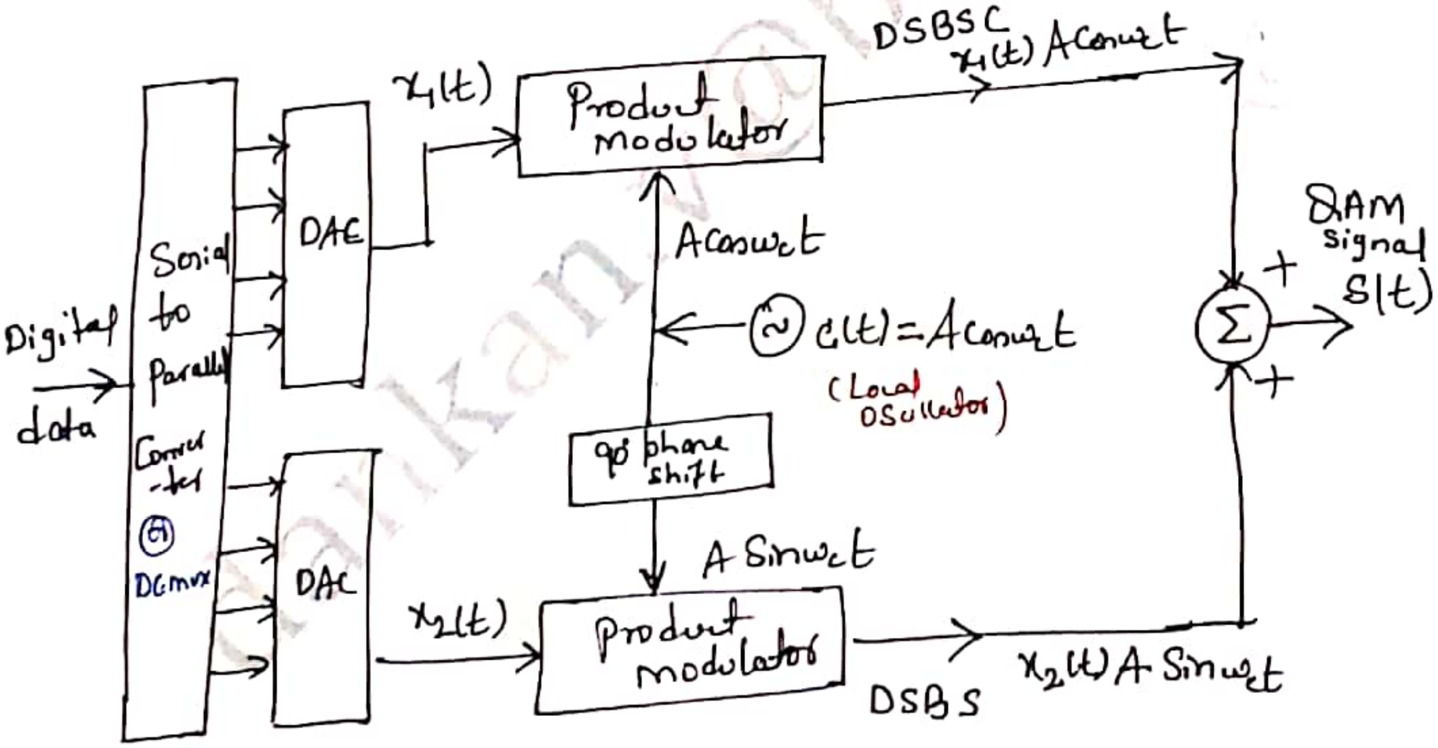
$\therefore\ M = 2^N = 2^9 = 512 \Rightarrow$ 512-ary psk

This shows that 512-ary psk can transmitt the 90mbps signal in the BW of 20MHz.

Quadrature Amplitude Modulation:- (QAM)

- We are utalizing QAM in both Digital and Analog modulation.
- QAM is a Combination of ASK and PSK in Digital modulation.
- QAM is a Combination of Amplitude and phase mod/n in Analog mod/n Technique.

QAM Modulation / Transmitter:-

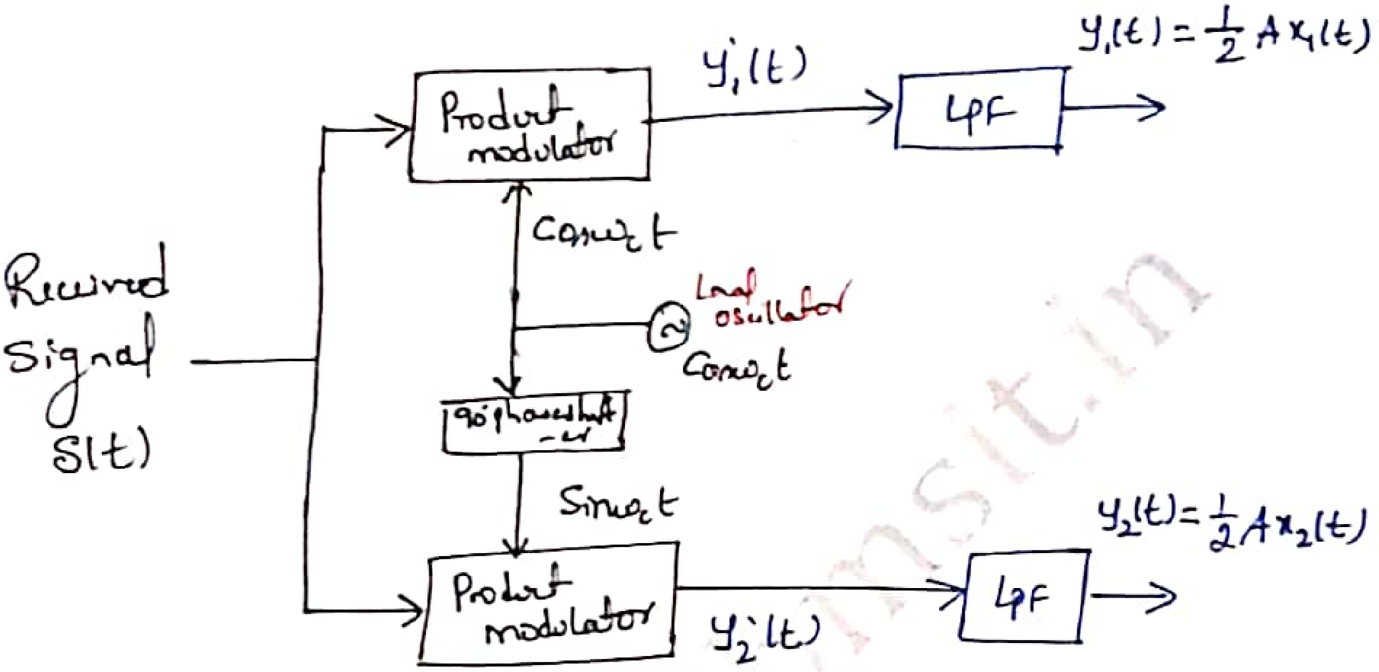


$$S(t) = x_1(t)A\cos\omega_c t + x_2(t)A\sin\omega_c t$$

- Since we are transmitting two DSBSC Signals over a Same channel ∴ we are utalizing channel BW effectively.

(QAM Effective utalization of channel BW).

QAM Demodulator | Receiver :-



$$y_1'(t) = S(t)\cos\omega_c t$$

$$= [x_1(t)A\cos\omega_c t + x_2(t)A\sin\omega_c t]\cos\omega_c t$$

$$= x_1(t)A\cos^2\omega_c t + x_2(t)A\sin\omega_c t\cos\omega_c t \qquad \boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$= x_1(t)A\left[\frac{1+\cos 2\omega_c t}{2}\right] + \frac{x_2(t)A}{2}\left[\sin 2\omega_c t\right] \qquad \boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

$$y_1'(t) = \overset{\checkmark}{\frac{x_1(t)A}{2}} + \overset{\times\ HPC}{\frac{x_1(t)}{2}A\cos\omega_c t} + \overset{\times\ HPC}{\frac{x_2(t)A}{2}\sin 2\omega_c t}$$

$$y_1'(t) \longrightarrow \boxed{LPF} \longrightarrow \frac{x_1(t)A}{2}$$

Illy $y_2(t)$.

↳ eliminates high frequency component (HPC)

* Quadrature Amplitude Modulation (QAM) is also called as Bandwidth Conservation Scheme.

* In QAM, two Carriers are used in quadrature (phase shift of 90° degrees).

* QAM can be used, either with analog signals (or) digital signals. if the message signal is analog in nature, then we use it with Amplitude mod^n (AM) and phase Mod^n (PM):
but when the message signal is in digital form, then ASK and PSK is used.

* Using QAM, it is possible to transmit two analog message signals (or) two digital bit sequences (two symbols) by modulating Carrier wave's amplitude using Analog mod^n technique (AM) (or) digital Mod^n technique.

* In Quadrature Amplitude Modulation, the two carrier waves are used, that are out of phase by 90°.
∴ this Scheme is known as quadrature amplitude modulation, and these Carriers are known as Quadrature Carriers.

* These Modulated waves are summed at the output of the transmitter,
∴ the final waveform is the Combination of both PSK and ASK in digital case (or) PM and AM for analog case.

QAM Transmitter

* The transmitting two message signals $x_1(t)$ and $x_2(t)$ are analog in nature.

* input to the first product Modulator is $x_1(t)$ and Carrier wave, while the input to Second product modulator is $x_2(t)$ and Carrier wave with a phase shift of 90°.

* QAM ∴ Signal

$$S(t) = x_1(t) A \cos w_c t + x_2(t) A \sin w_c t$$

* observe that QAM is the combination of both ASK and PSK. (as phase change of carrier wave takes place).

* QAM scheme Enables two Modulated signals to occupy the Same transmission channel and allow the separation of these two message signals at the receiver output it is also known as "Bandwidth Conservation Scheme".

* With this scheme we are Simultaneously transmitting two amplitude modulated waves / two amplitude shift keying signals over single channel.

QAM Receiver :-

* The multiplexed Signal is applied to the inputs of both the product Modulators.

* outputs of both the product modulators are passed over a LPF. * o/p of LPF is the message signal.

* The oscillators used to produce carrier waves at the transmitting and receiving ends must be in coherence. i.e the carriers at the transmitting and receiving ends must be in Same phase.

QAM

Quadrature Amplitude Modulation.

- it is a combination of psk + ASK. (noise affect is more)
- high data rate } advantages
- BW conservation }
- noise affect in more drawback.

8-QAM :- (+A, (0°, 90°, 180°, 270°)

2 Amplitude × 4 phase

= 8 QAM.

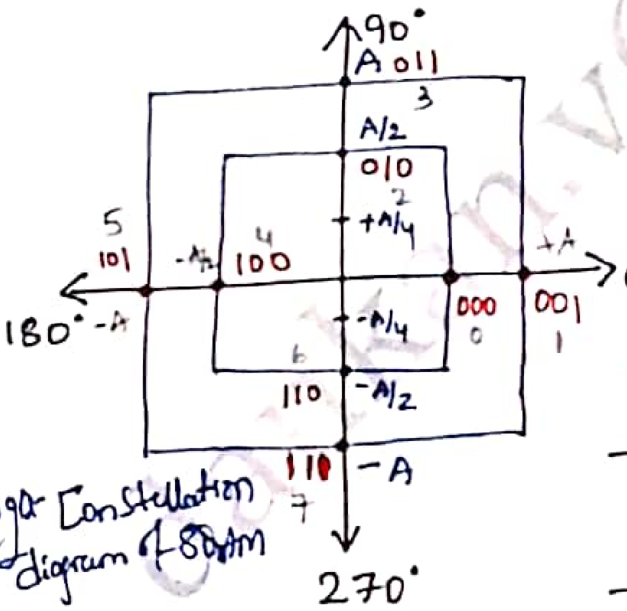
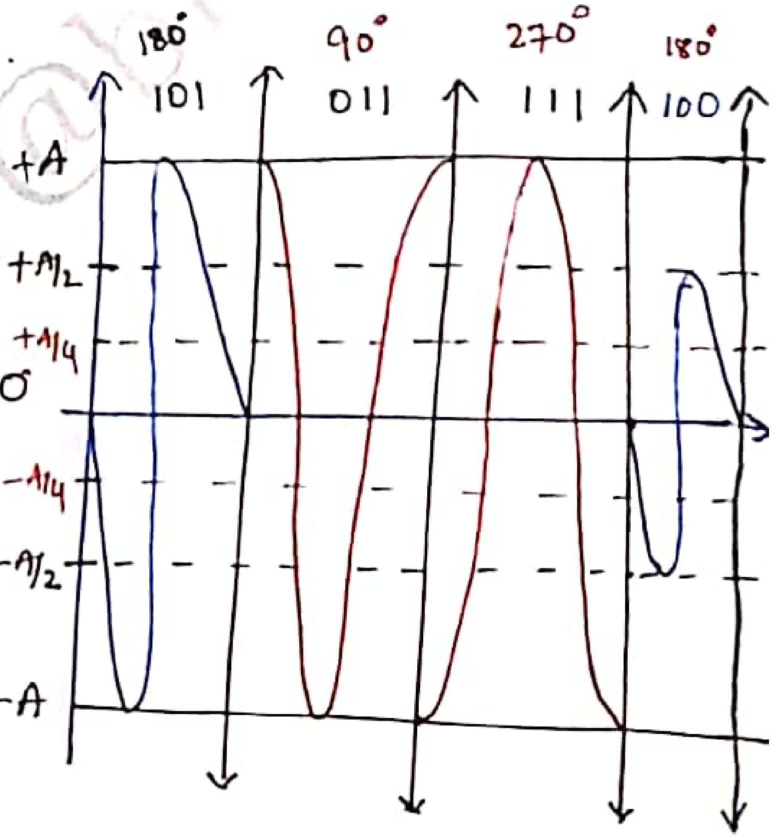


fig a Constellation diagram of 8QAM



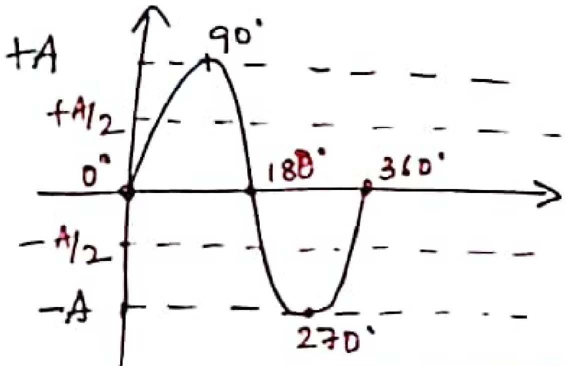
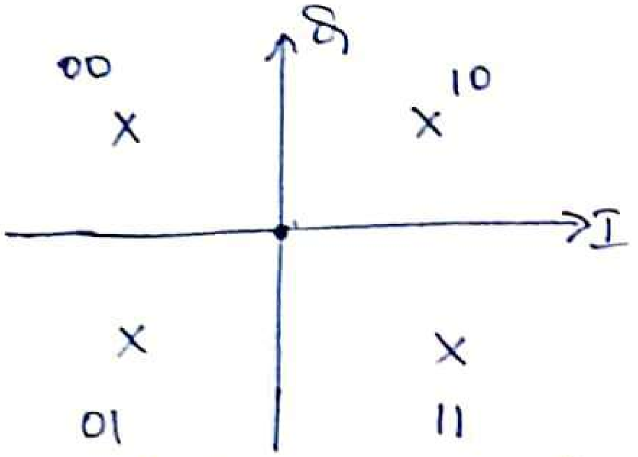


fig b. phasor diagram.

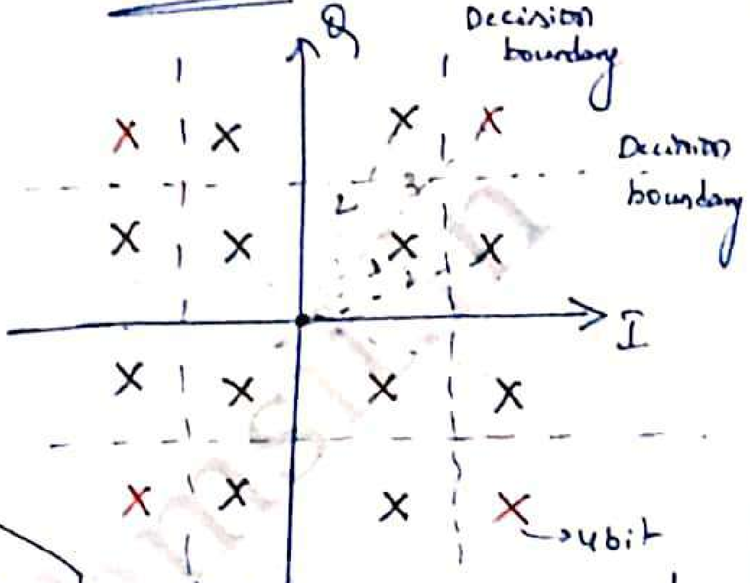
Constellation diagram

4 QAM

= 1 amplitude × 4 phase



16QAM



→ distance is same (ie) Amplitude is (constant) fixed w.r.t Center.

→ only phase changes.

* distance is varying w.r.t Center point

* different amplitude levels.

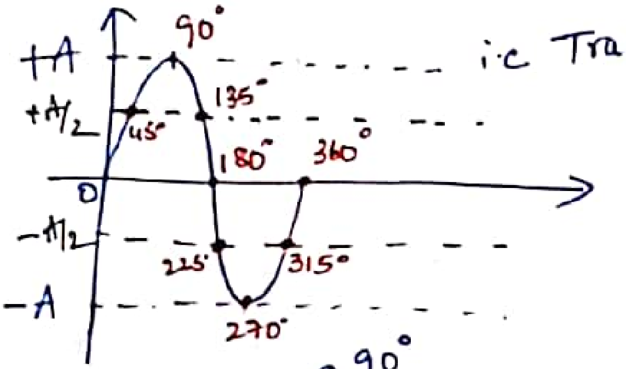
16 QAM

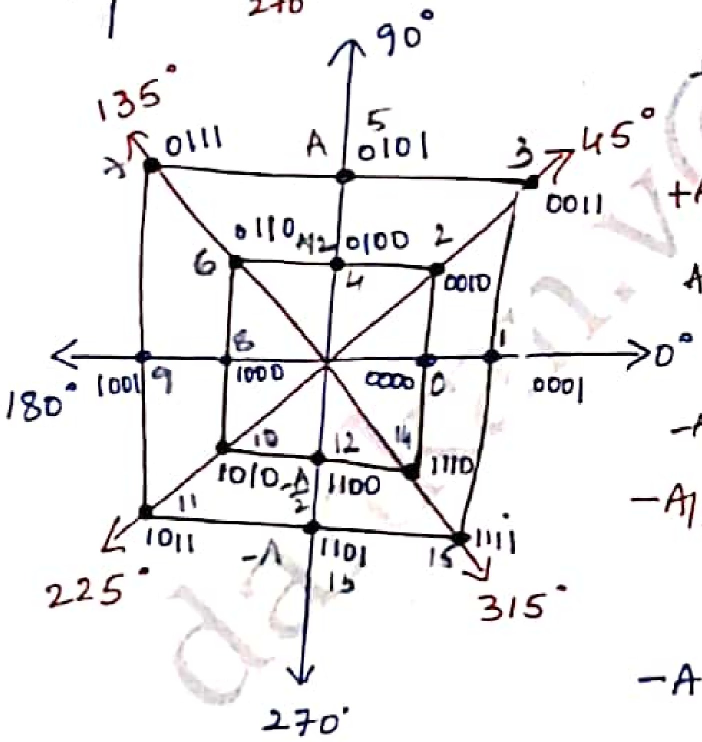
= 2 Amplitude × 8 phases } combination of ASK & PSK.

= 16 QAM

= 2^{4} QAM. ⇒ 4 bits/symbol.

i.e Transmission achieved 4 bits/symbol.





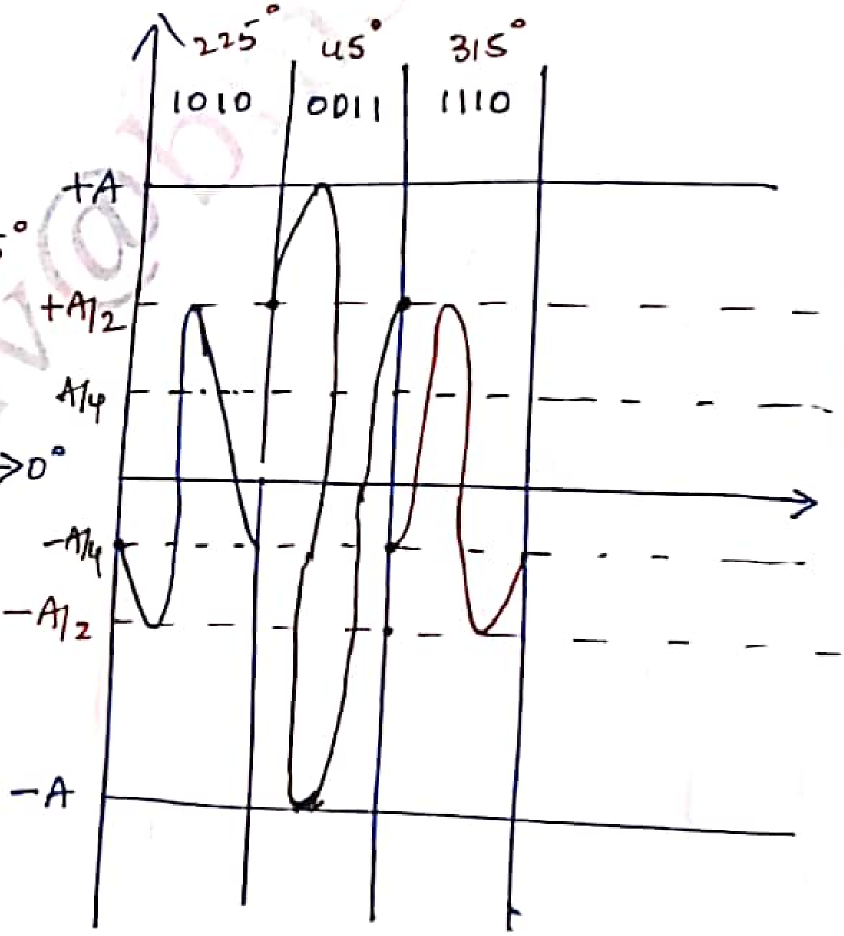


fig:- Constellation diagram of 16 QAM.

⇒ Ily 64 QAM
↓
8 amp × 8 phase = 64 QAM
(or) 4 amp × 16 phase = 64 QAM.

⇒ data rate ↑
speed ↑

← 128 QAM
8 × 16 phases = 128 QAM

Comparision of QAM and PSK Mod^n

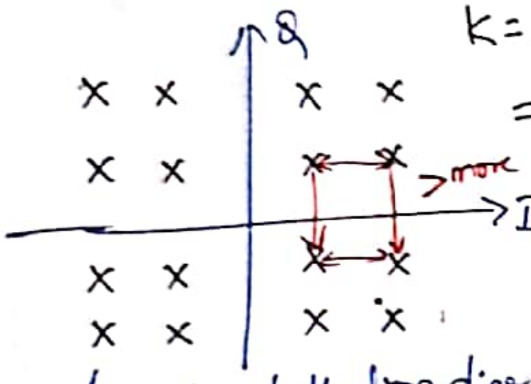
Comparision w.r.t
- i. Band width Efficiency.
- ii. Noise immunity.
- iii. Transmitted power.
- iv. Bits/symbol.
- v. operating frequency (@ what operating frequency which scheme is suitable).

BW efficiency:- Both of the schemes represents 16-symbol's for the same frequency. (∴ both are equally good)

bits/symbols:- $K = \log_2 m$ Both are same

relevance.

16 QAM



$K = \log_2 M$
$= 4$ bits/symbol.

fig:- constallation diagram

* Both Amplitude & phase changes

16 PSK

By increasing radius we can increase distance b/w constallation points.

fig: Constallation diagram

* Amplitude is constant (only phase changes)

Each quadrant there are 4 constallation points with same amplitude.

parameter	16-QAM	(M-ary) 16-PSK
BW Efficiency	✓	✓
bits/symbol.	✓	✓
Noise immunity	✓	X
Transmitting power	✓	X
Operating (GHz) frequency	X	✓

→ Both are equally good

→ — 11 —

Noise immunity ∝ depends on distance b/w the constallation points. (QAm is better compared to PSK)

Noise immunity:-

- * Noise immunity of digital signals is digital data can be recovered without any error as long as the distortion and noise are within limits.
- * Noise immunity depends on the hamming distance (ie the distance b/w the constellation points).
- * more the hamming distance better the noise immunity.
- * QAM has a better Noise immunity compared to M-ary PSK.

the distance b/w the constellation points in psk can be increased by increasing the radius. this causes the change in amplitude ∴ it results / requires more transmitting power.

- * at Lower transmitting power QAM has better a noise immunity
- * at a fixed Txmitting power QAM has better noise immunity compared to PSK

Transmitting power:- for Same Noise immunity 16-PSK requires 1.6dB higher power compared to 16QAM. (ie under given Noise immunity QAM requires less transmitting power compared to PSK).

operating frequency:- at extreamly High frequency (ie GHz range) QAM (circuit complexity is more since it uses both (ASK & PSK) ∴ at very high frequency M-ary PSK is suitable in terms of cost reduction. Since in psk Amplitude is fixed only phase is varied ∴ Cost of implementation is less compared to QAM.

What is the advantage of M-ary QAM over M-ary PSK system? Obtain the constellation of QAM for M = 4 and draw signal space diagram. (04 Marks) Dec 2018-Jan 2019.

soln:- advantages:-

i. Noise immunity:- at a fixed transmitting power QAM has a better noise immunity compared to M-ary PSK.

ii. Transmitting power:- For a same noise immunity 16-PSK requires 1.6dB higher power compared to 16QAM. so under given noise immunity QAM requires Less transmitting power compared to M-ary psk.

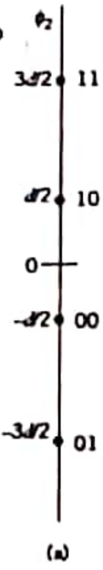
drawback:- at extreamly high frequency ie (in the range of GHz) QAM is not preferred b/z Cost of implementation becomes more. Compared to M-ary PSK.

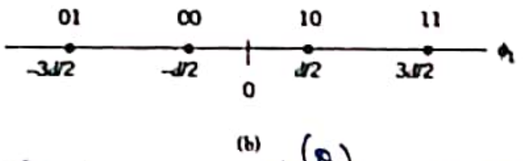
Similarities:-

i. B.W efficiency:- Both ~~the~~ techniques are Equally Good interms of B.W efficiency. (Since both uses M-symbols under given frequency.

ii. bits/symbol:- Both techniques have Same bits/symbol transmission.

ie $\boxed{K = \log_2 M}$ bits/symbol.

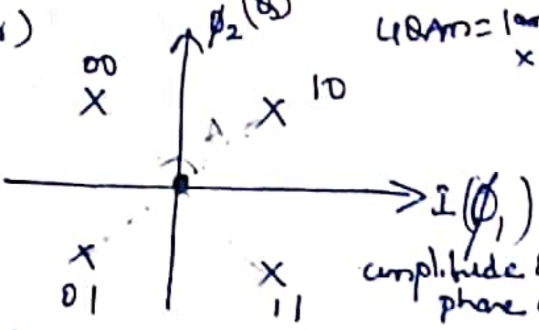




Figure

The two orthogonal constellations of the 4-ary PAM. (a) Vertically oriented constellation. (b) Horizontally oriented constellation. As mentioned in the text, we move top-down along the ϕ_2-axis and from left to right along the ϕ_1-axis.

(or)



4QAM = 1amplitude × 4phase

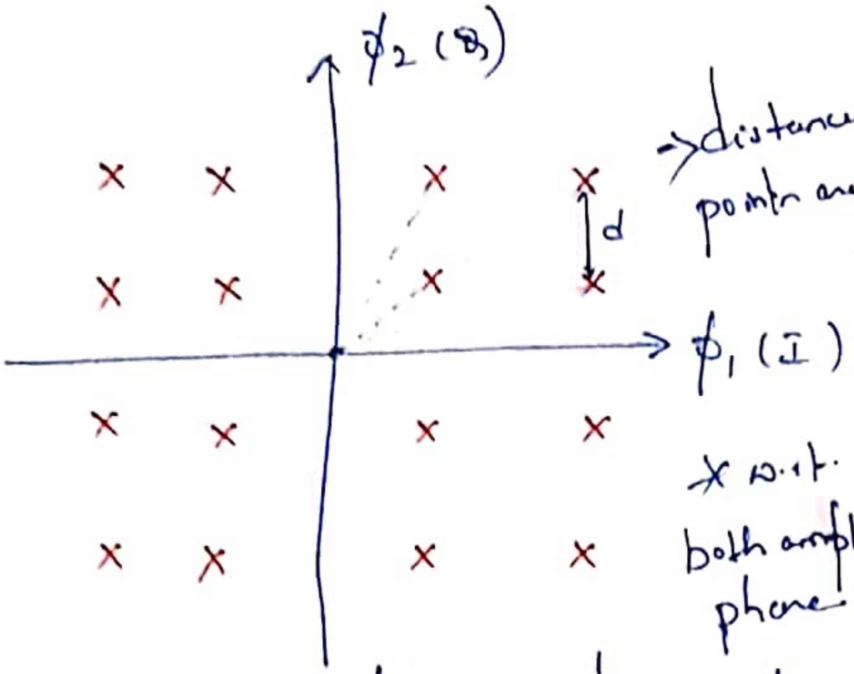
amplitude is fixed phase changes.

fig c:- 4QAM Constellation diagram.

Obtain the constellation of QAM for M=16 and Draw the signal Space Diagram. **(04 Marks) June-July 2019.**

16QAM = 2amplitudes × 8phase .= 16QAM.

soln:



→distance b/w Constellation points are called hamming distance.

x w.r.t. Center point both amplitude and phase changes.

fig a. Constellation diagram of 16QAM.

*> No.of symbol's (M) and No.of bits (N) per symbol are related as $\boxed{N = \log_2 M}$

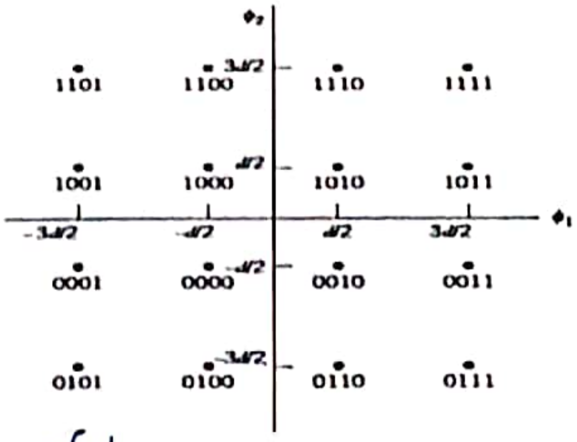


fig b

fig b (a) Signal-space diagram of M-ary QAM for M = 16; the message points in each quadrant are identified with Gray-encoded quadbits.

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Topic 3: Non coherent orthogonal modulation techniques

With a neat diagram, explain the non coherent detection of binary frequency shift keying technique. **(04Marks) June-July 2018.**

With neat diagram and expressions, explain binary FSK generation and noncoherent detection scheme. **(06 Marks) Dec 2018-Jan 2019.**

Soln:- Generation of BFSK Signal :-

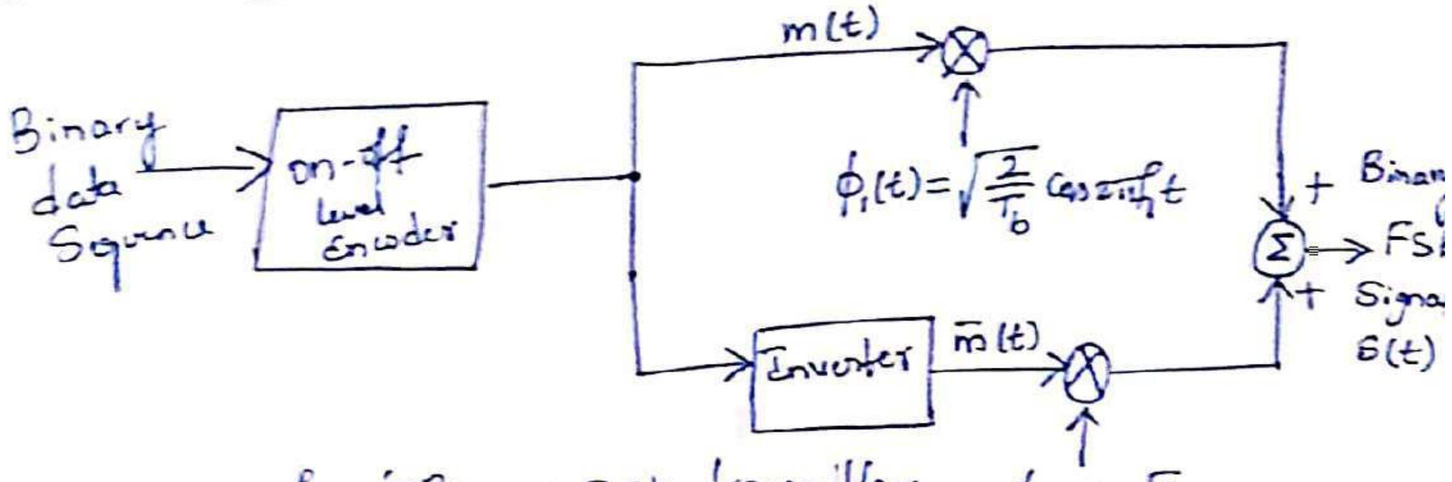


fig a:- Binary FSK transmitter

* Fig a. shows the scheme for generation of BFSK Signal.

* Input binary data is unipolar NRZ format applied to the multiplier-1 along with Carrier Signal $\phi_1(t)$.

i.e $S_1(t) = m(t)\cdot\phi_1(t)$

* Input m(t) is applied to multiplier-2 through inverter along with another Carrier Signal $\phi_2(t)$.

i.e $S_2(t) = \overline{m}(t)\cdot\phi_2(t)$.

* When Symbol 1 is transmitted. output from upper path i.e $S_1(t)$ and no output from the Lower path i.e

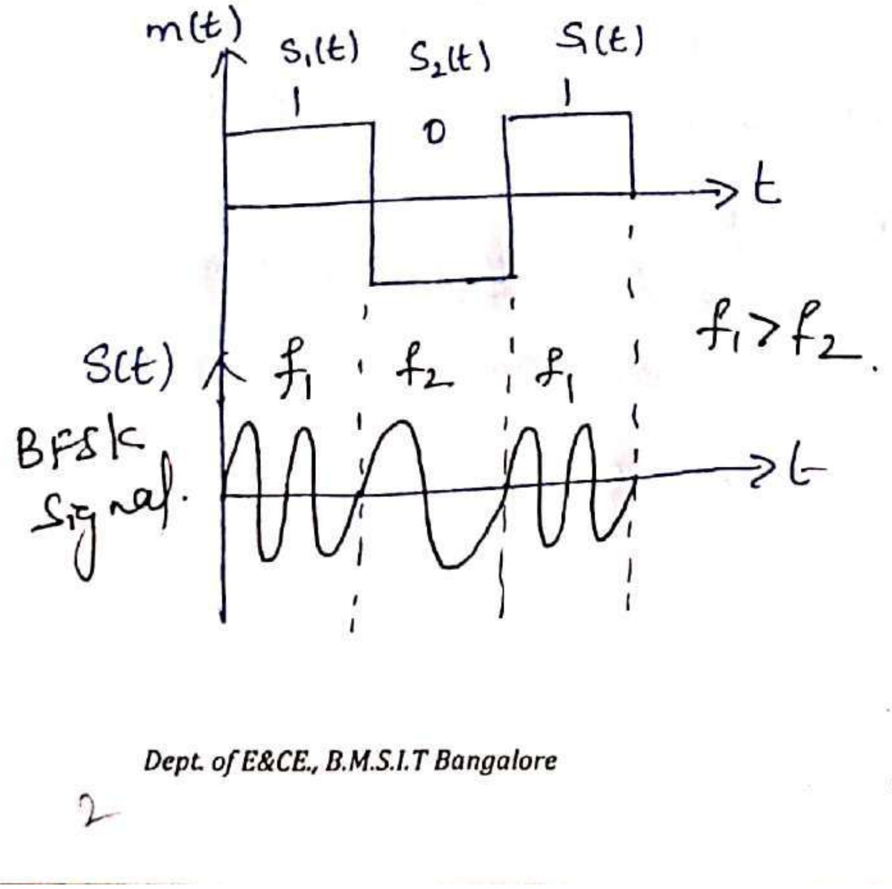
$S_2(t) = 0$.

Thus frequency 'f_1' is transmitted for Symbol '1'.

* when Symbol '0' is transmitted, output appears from Low-prpath i.e '$S_2(t)$' and no output from the upper path i.e $S_1(t) = 0$.

Thus frequency 'f_2' is transmitted for Symbol 0.

* frequency $f_1 > f_2$ is chosen and the phase of the FSK remains continuous ∴ FSK is also known as Continuous phase FSK.



Non-coherent detection of BFSK

- The Receiver Consists of a pair of matched filters followed by Envelope detectors as shown in figb.
- The filter in the upper path of the receiver is matched to $\cos(2\pi f_1 t)$ and the filter in the lower path is matched to $\cos(2\pi f_2 t)$ for the Signaling interval $0 \le t \le T_b$.
- The resulting Envelope detector outputs are Sampled at $t = T_b$ and their values are Compared.
- The Envelope samples of the upper and lower paths in figb are l_1 and l_2
- The receiver decides in favour of Symbol 1 if $l_1 > l_2$. and in favour of Symbol 0 if $l_1 < l_2$.
- If $l_1 = l_2$ the receiver simply guesses randomly in favour of Symbol 1 (or) 0.

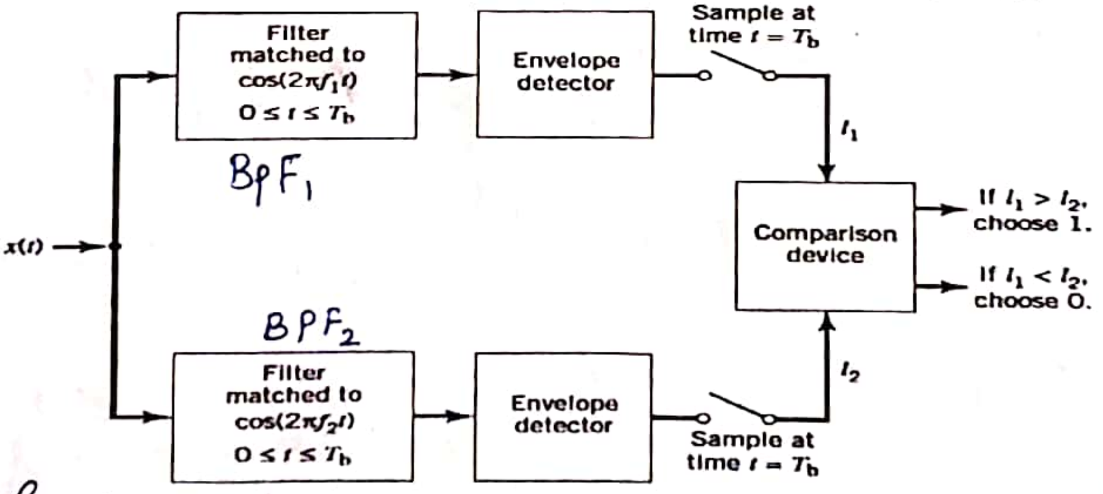


fig b. Noncoherent receiver for the detection of binary FSK signals.

- Error probability of non-coherent Binary FSK is

$$\boxed{P_e = \frac{1}{2} \exp\left(\frac{-E_b}{2N_0}\right)}$$

E_b - Signal energy per bit

Problem :-

An FSK System transmits binary data at the rate of 2×10^{6} bps. During the source of transmission, AWGN of Zero mean and two sided PSD 10^{-20} W/Hz is added to the Signal. The amplitude of the received Sinusoidal wave for digit 1 (or) 0 is $1\mu V$. Determine the average probability of Symbol Error assuming non-coherent detection. (04 Marks). June/July 2019.

Soln:- Give as $f_b = 2\times10^{6}$ bps $\Rightarrow T_b = \frac{1}{f_b} = 0.5\times10^{-6}$ sec $= 0.5\mu$sec.

Using Noncoherent detection

two sided PSD $\frac{N_0}{2} = 10^{-20}$ W/Hz.

$N_0 = 2\times10^{-20}$ W/Hz.

$A = 1\mu V$

$$(P_e)_{\substack{\text{Noncoherent}\\ \text{FSK}}} = \frac{1}{2}e^{-\left(\frac{E_b}{2N_0}\right)}$$

$E_b = ?$ $\quad P_{avg} = \frac{A^2}{2}$ and $P_{avg} = \frac{E_b}{T_b}$

$$\Rightarrow \frac{E_b}{T_b} = \frac{A^2}{2} \Rightarrow \boxed{E_b = \frac{A^2 T_b}{2}} \text{ Joules}$$

$$E_b = \frac{(1\times10^{-6})^2\times(0.5\times10^{-6})}{2} = 2.5\times10^{-19} \text{ Joules.}$$

Prob. of Error

$$P_e = \frac{1}{2}e^{-\left[\frac{2.5\times10^{-19}}{2\times2\times10^{-20}}\right]} = \frac{1}{2}e^{-6.25} = \frac{1}{2}\times1.9304\times10^{-3}$$

$$\boxed{\underset{\text{(Non-coherent FSK)}}{P_e} = 0.965227\times10^{-3}} \leftarrow ⓐ$$

b. prob. of Error using Coherent FSK.

$$P_{e\,(\text{Coherent FSK})} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{2.5\times10^{-19}}{2\times2\times10^{-20}}}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(\sqrt{6.25}\right)$$

$$= \frac{1}{2}\,\text{erfc}\left(2.5\right)$$

$$= \frac{1}{2}\left[1-\text{erf}(2.5)\right]$$

Note:- $\text{erf}(2.5) = 0.99959$

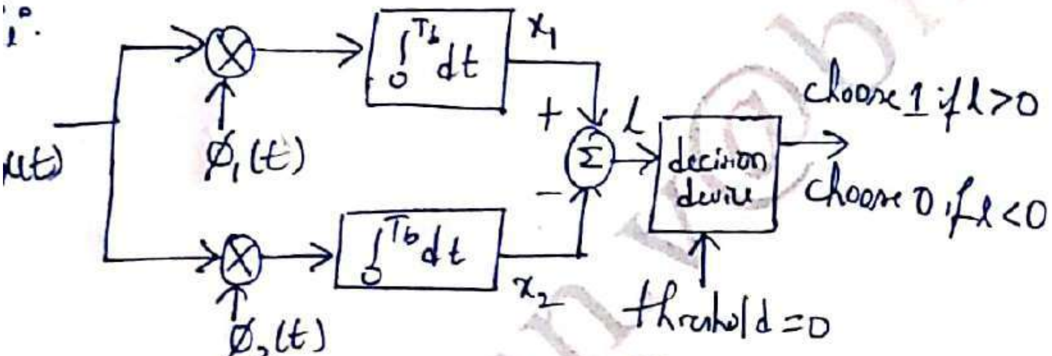
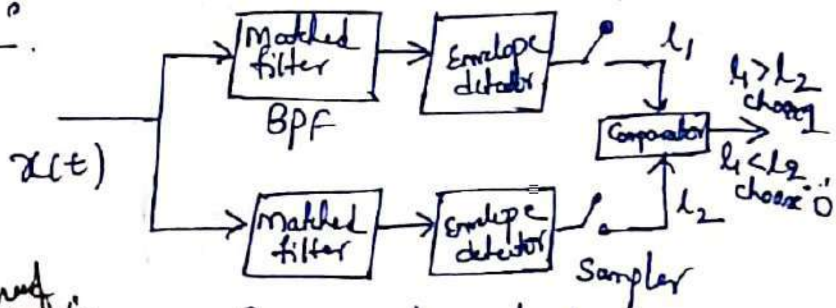
$$= \frac{1}{2}\left[1-0.99959\right]$$

$$= \frac{1}{2}\left(4.1\times10^{-4}\right)$$

$$P_{e\,(\text{coherent FSK})} = 2.05\times10^{-4} = \underline{\underline{0.205\times10^{-3}}} \leftarrow ⓑ$$

Note:- From equation ⓐ and ⓑ probability of Error of Non-coherent FSK is more compared to Coherent FSK.

Comparision of Detection of FSK Signal using Coherent and Non-coherent technique

Coherent detection of FSK	Non-coherent detection of FSK.
i. also called Synchronous detection.	i. also called Envelope detection.
ii.  fig a. coherent detection.	ii.  fig. Non-coherent detection.
iii. Advantage :- Simple drawback :- Costly (bcz we need two oscillators at Receiver end)	iii. Advantage :- Low cost.
iv. $P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$	iv. $P_e = \frac{1}{2} e^{\left(-\frac{E_b}{2N_0}\right)}$
v. P_e is Less for a given E_b & $\frac{N_0}{2}$.	v. P_e is more for a given E_b & $N_0/2$.

What is DPSK → it is the non coherent detection of PSK.

Why DPSK → In PSK Non-coherent detection is not possible ∴ DPSK.

the o/p of PSK system is

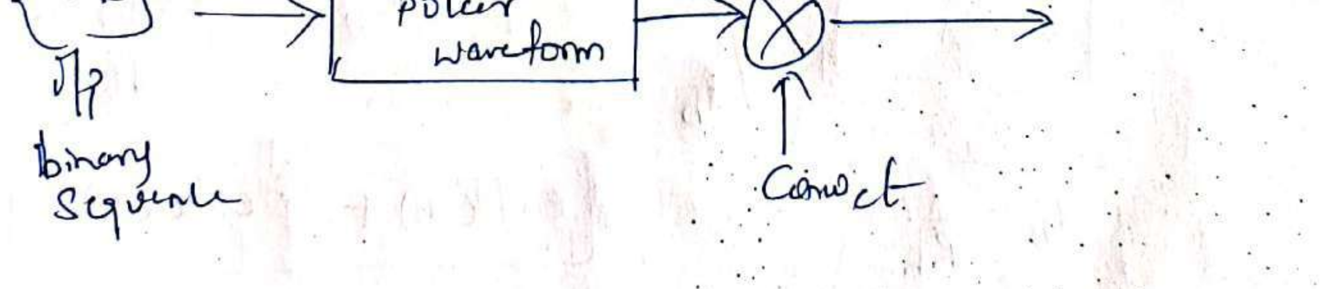
$$\boxed{\pm a_k} \cos\omega_c t$$

$\to +a_k \to$ Symbol 1
$-a_k \to$ Symbol 0.

But the non-coherent methods such as

i. Envelope detector $\sqrt{(\pm a_k)^2} = +ve$
∴ not possible $\{+ \to 1, - \to 0\}$

ii. Square law detector $(\pm a_k)^2 = +ve$
not possible.



DPSK (Differential phase shift keying)

- → Basics of DPSK
- → DPSK transmitter
- → DPSK wave form generation.
- → DPSK Receiver
  - i. → Non-coherent Detection
  - ii. → optimal Receiver (coherent Detection).
- → probability of Error of DPSK.
- → Advantages of DPSK
- → Disadvantages of DPSK.

Basics of DPSK

* it is not possible to have non coherent detection of psk signal.

BCZ :- Non-coherent methods such as

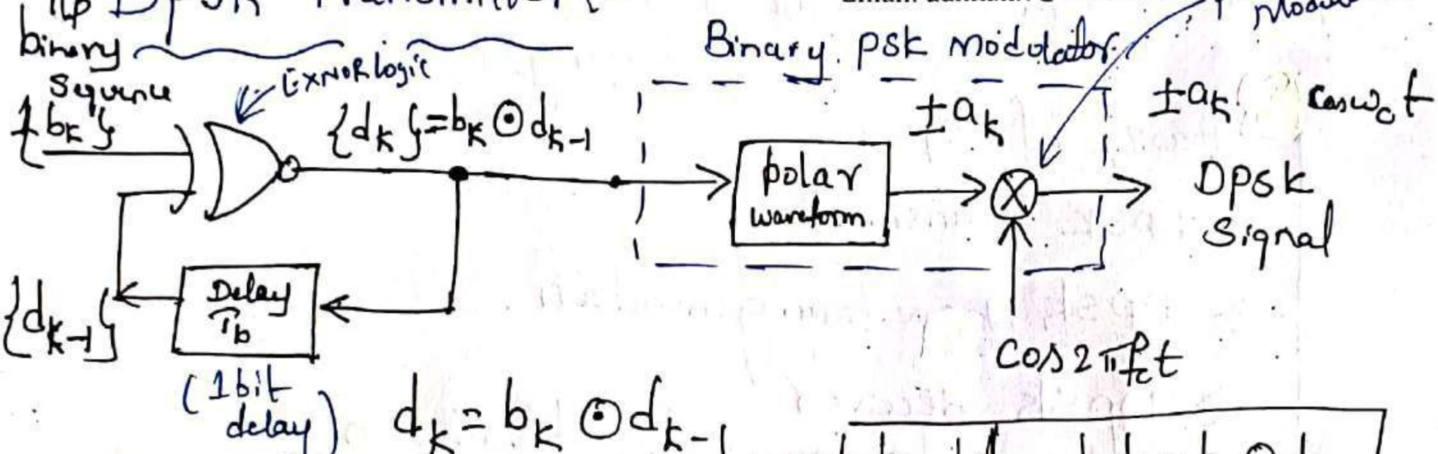
Envelope detector o/p = $\sqrt{\ }$

Square Law detector o/p = $(\ )^2$.

* To detect non coherent detection of phase we use DPSK.

* it reduces the cost of circuit.

DPSK Transmitter:-



$$d_k = b_k \odot d_{k-1}$$

$$d_k = \overline{b_k}\,\overline{d_{k-1}} + b_k d_{k-1}$$

b_k	d_{k-1}	$d_k = b_k \odot d_{k-1}$
0	0	1
0	1	0
1	0	0
1	1	1

DPSK waveform:-

- To send symbol 1, the phase of the DPSK signal remains unchanged.
- To send symbol 0, the phase of the DPSK signal is shifted by 180°.

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
Differentially Encoded sequence $\{d_k\}$	{reference} 1	1	0	1	1	0	1	1	1
Transmitted phase (radians)	0	0	π	0	0	π	0	0	0

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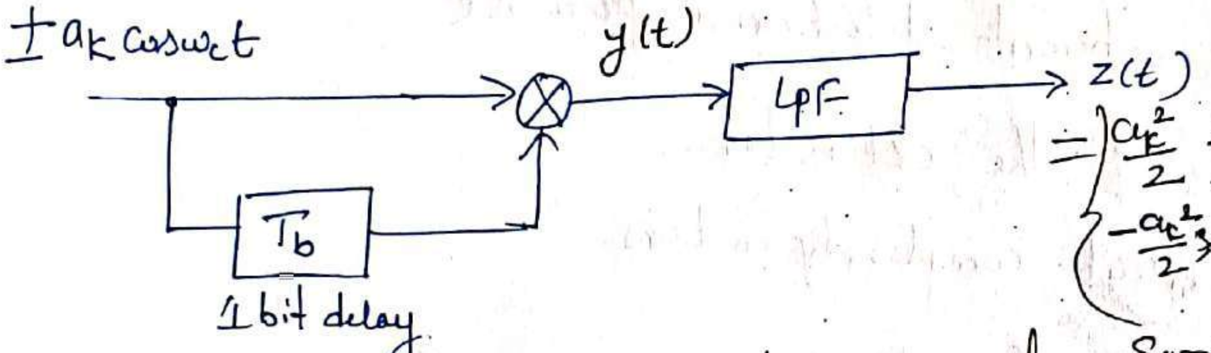
DPSK Receiver :-



$$z(t) = \begin{cases} \frac{a_k^2}{2} \text{ favours symbol 1} \\ -\frac{a_k^2}{2} \text{ favours symbol 0.} \end{cases}$$

Case i [Received signal (i/p) and delayed signal are Same polarity]

$$y(t) = (a_k \cos\omega_c t)(a_k \cos\omega_c t)$$
$$= a_k^2 \cos^2 \omega_c t$$
$$= a_k^2 \left[\frac{1+\cos 2\omega_c t}{2}\right]$$
$$= \frac{a_k^2}{2} + \frac{a_k^2}{2} \cos 2\omega_c t \xrightarrow{LPF} \frac{a_k^2}{2} \rightarrow \text{Symbol } 1$$

$\times$ HFC

Case ii [Received / i/p signal and delayed signal both are in opposite polarity]

i.e $$y(t) = (a_k \cos\omega_c t)(-a_k \cos\omega_c t)$$
$$= -a_k^2 \cos^2 \omega_c t$$
$$= -a_k^2 \left[\frac{1+\cos 2\omega_c t}{2}\right]$$
$$= -\frac{a_k^2}{2} - \frac{a_k^2}{2} \cos 2\omega_c t \xrightarrow{LPF} -\frac{a_k^2}{2} \rightarrow \text{Symbol } 0$$

$\times$ HFC

Advantages:-

* Non-coherent detection is possible
* Cost of the ckt is less.
* Circuit complexity is less.

Drawback of DPSK:-

* Noisy.

Decoding of of DPSK Signal

i/p Sequence {bk}		1	0	0	1	0	0	1	1
$\{d_{k-1}\}$		1	1	0	1	1	0	1	1
Differentially Ecoded Sequence $\{d_k\}$	① reference	1	0	1	1	0	1	1	1
phase of DPSK	0	0	π	0	0	π	0	0	0
phase of shifted Data		0	0	π	0	0	π	0	0
phase comparision o/p		+	−	−	+	−	−	+	+
Detected Binary Sequence		1	0	0	1	0	0	1	1

Probability of Error of DPSK

- ✓ BER v/s Pe comparison.
- ✓ table incomplete
- ✓ DPSK problems merge
- ✓ errorfn table

$$\boxed{P_{e(DPSK)} = \frac{1}{2} e^{\left(-E_b/N_0\right)}}$$

"Non-coherent" detection.

Bit

E_b - Energy (Joules)

$\left(\frac{N_0}{2}\right)$ - Noise density (W/Hz)

$$\boxed{P_{e(PSK)} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

"coherent" detection

$$P_{avg} = \frac{E_b}{T_b} \text{ Joules/sec}$$

T_b = bit duration (sec)

$$P_{avg} = \frac{A^2}{2}$$

$$\frac{A^2}{2} = E_b/T_b$$

$$\therefore \boxed{E_b = \frac{A^2 \cdot T_b}{2}} \text{ Joules}$$

$$\boxed{T_b = \frac{1}{R_b}}$$

$$\boxed{\operatorname{erf}(x) + \operatorname{erfc}(x) = 1}$$

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Explain the generation and optimum detection of differential phase – shift keying with neat block diagram. **(06 Marks) Dec 2018-Jan 2019./ (8 Marks) Dec 2019-Jan 2020.**

Generation of DPSK :-

- DPSK Eliminates the need for a coherent reference Signal at the receiver by combining two basic operations at the transmitter.
  i. Differential Encoding of the input binary wave and
  ii. phase-shift keying.

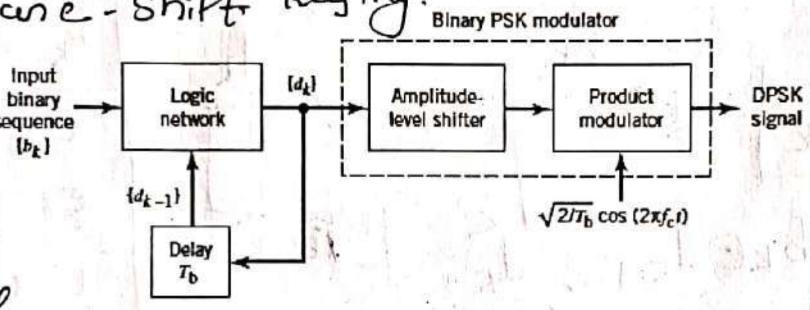


fig a :- Block diagram of a DPSK transmitter.

Hence the name DPSK.

- For symbol '1', a carrier signal with 0° phase shift is transmitted.
- For symbol '0', a carrier signal with 180° phase shift is transmitted.
- The differential Encoding process at the transmitter starts with an arbitrary bit. Serving as reference, and there-after the differentially Encoded sequence $\{d_k\}$ is generated by using the Logic equation.

$$\boxed{d_k = b_k \odot d_{k-1}}$$

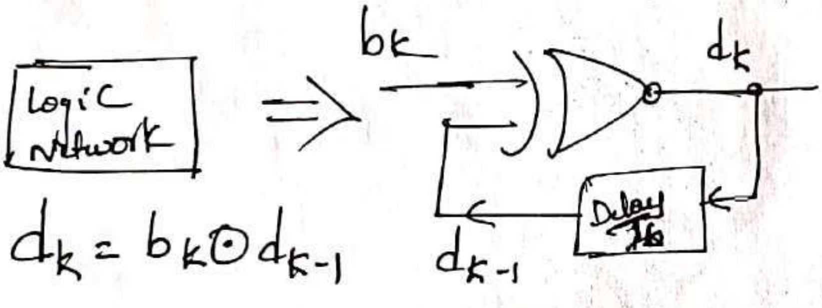
$\odot$ Ex-NOR operation.

$d_k = b_k \odot d_{k-1}$

$$\boxed{d_k = b_k d_{k-1} + \bar{b}_k \bar{d}_{k-1}}$$

where b_k is the input binary digit at time kT_b. $k \in Z+$.

d_{k-1} is the previous value of the differentially Encoded digit.



$d_k = b_k \odot d_{k-1}$

b_k	d_{k-1}	$d_k = b_k \odot d_{k-1}$
0	0	1
0	1	0
1	0	0
1	1	1

DPSK Receivers:-

methods:- i. Non-coherent DPSK receiver.

ii. optimum detection (coherent receiver) of DPSK.

i. Non-coherent Receiver

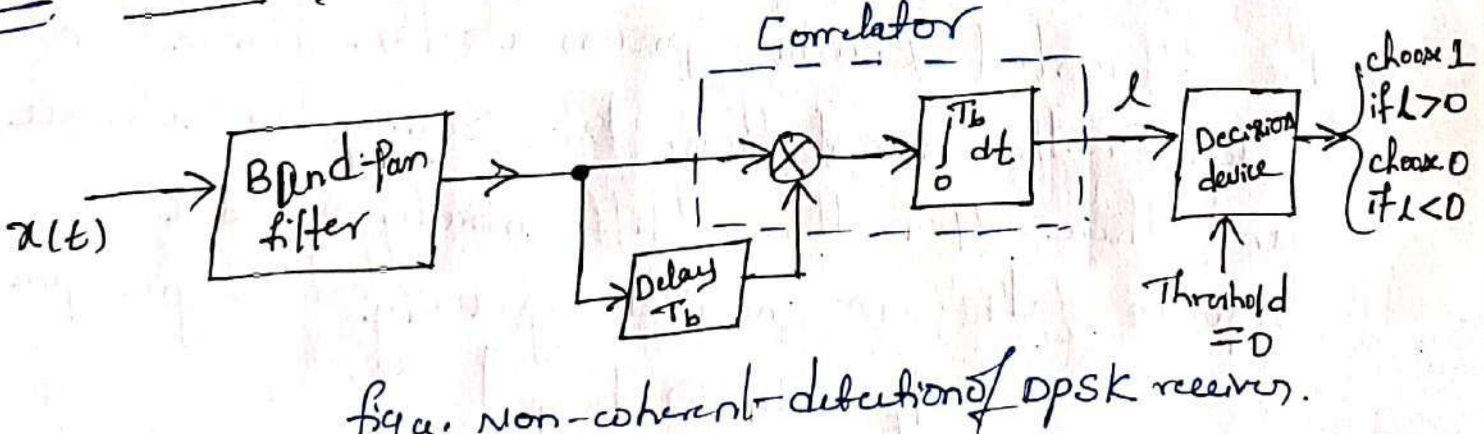


fig a. Non-coherent detection of DPSK receiver.

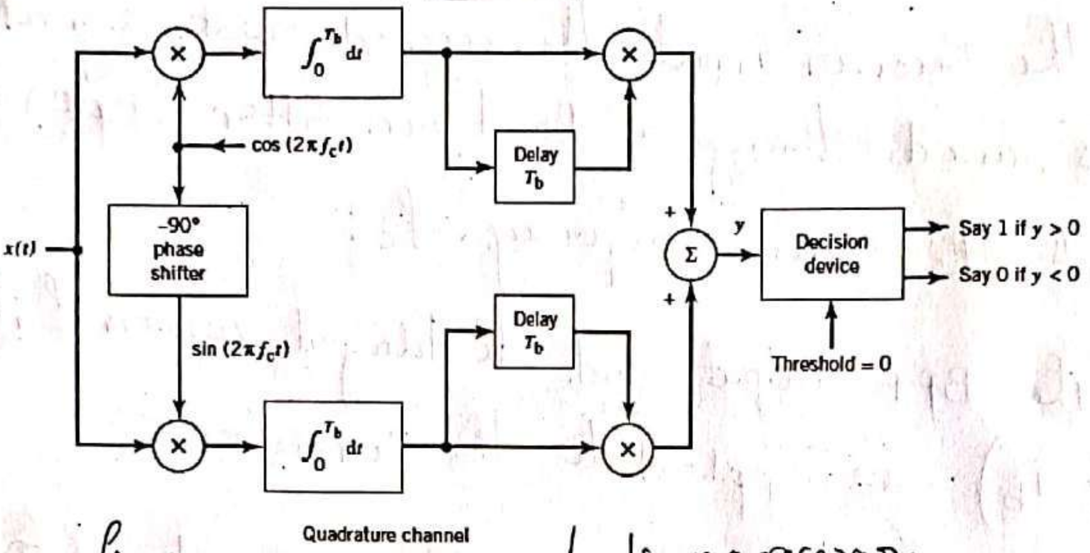
* at the Receiver input, the received DPSK signal is passed through a Band pass filter (BPF) centered at the carrier frequency f_c.

* The BPF output and a delayed version of it (delayed $= T_b$) are applied to the correlator.

* The output of the correlator 'l' is compared with zero threshold.

* if $l > 0$ => Receiver decides in favour of symbol '1'.
* if $l < 0$ => Receiver decides in favour of symbol '0'.

ii. Optimum Receiver (or) Coherent Detection of DPSK :-



figa:- Block diagram of a DPSK receiver. / optimum receiver.

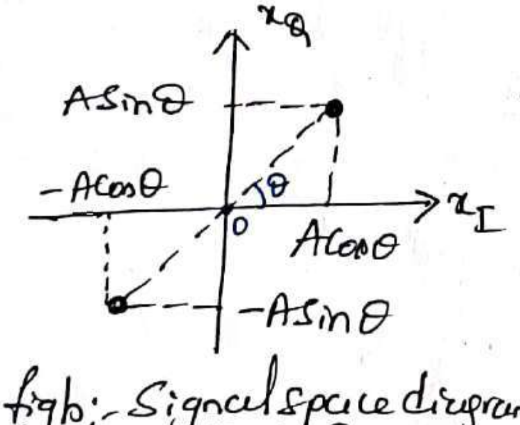
* A Signal-space diagram where the received signal points over the two bit interval $0 \leq t \leq 2T_b$ are defined by $(A\cos\theta, A\sin\theta)$ and $(-A\cos\theta, -A\sin\theta)$ where A denotes Carrier amplitude and θ - Carrier phase.

* The geometry of possible Signal is shown in figb. for two bit interval $0 \leq t \leq 2T_b$, the receiver measures the co-ordinates x_{I_0}, x_{Q_0} first, at time $t = T_b$ and then measures x_{I_1}, x_{Q_1} at time $t = 2T_b$.

* Decision is based on the inner product.

* the inner product $x_0^T x_1$.

$$x_0^T x_1 = \begin{bmatrix} x_{I_0} & x_{Q_0} \end{bmatrix} \begin{bmatrix} x_{I_1} \\ x_{Q_1} \end{bmatrix}$$



figb:- Signal space diagram of DPSK receiver

Decision Rule :-

if $x_{I_0} x_{I_1} + x_{Q_0} x_{Q_1} > 0 \Rightarrow$ Symbol '1'

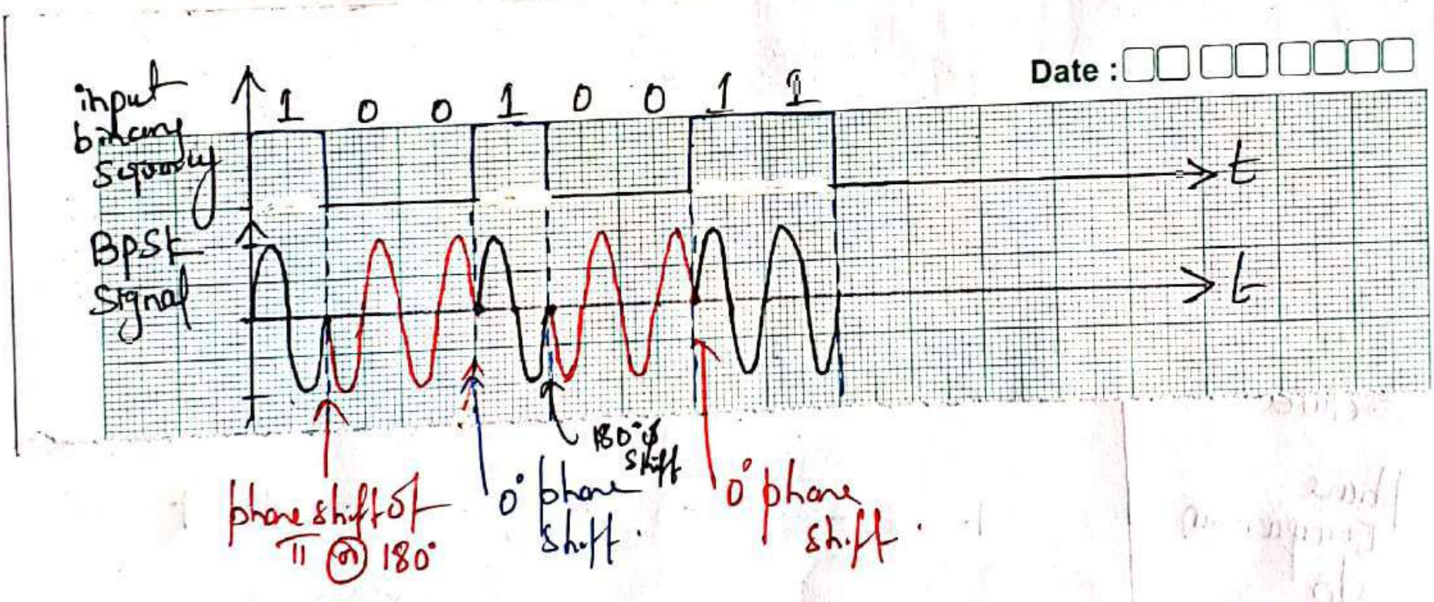
if $x_{I_0} x_{I_1} + x_{Q_0} x_{Q_1} < 0 \Rightarrow$ Symbol '0'.

For the binary sequence given by 10010011, illustrate the operation DPSK. **(02 Marks) June-July 2018.**

Given the binary data **10010011,** draw the BPSK and DPSK waveform. **(06 Marks) Dec 2019-Jan 2020.**

Soln:- 8. BPSK signal for the binary data 10010011.

- for symbol '1' is represented by transmitting carrier wave with 0° phase shift.
- for symbol '0' is represented by transmitting carrier wave with 180° (or) π phase shift.



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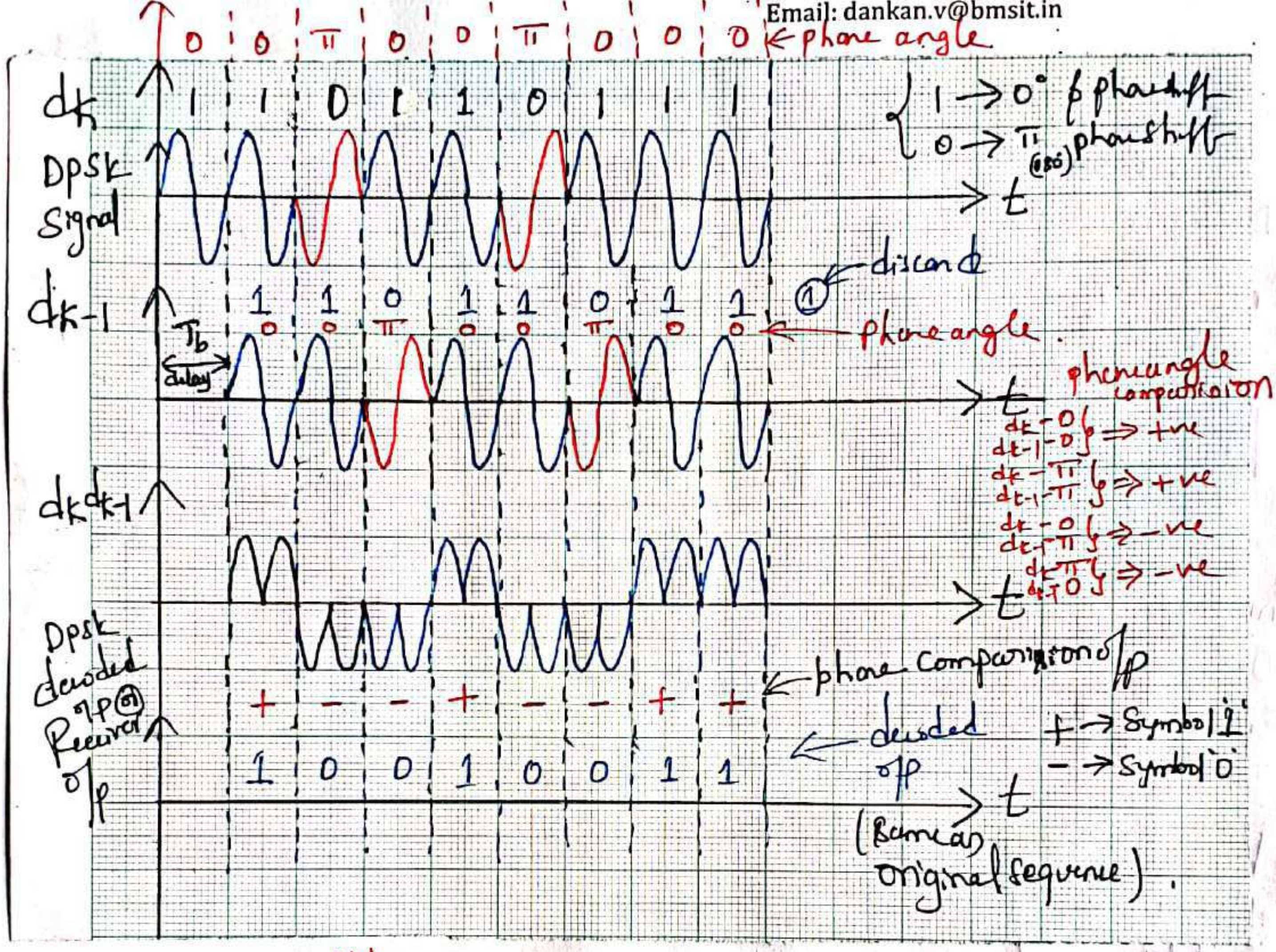
Ex. DPSK Waveform.

Binary data $\{b_k\}$		1	0	0	1	0	0	1	1
	⊙ EXNOR	⊙	⊙	⊙	⊙	⊙	⊙	⊙	
Differentially Encoded data $\cdot\{d_k\}$	reference bit 1	1	0	1	1	0	1	1	1
phase of DPSK	0	0	π	0	0	π	0	0	0
shifted Differentially Encoded data $\{d_{k-1}\}$		1	1	0	1	1	0	1	1
phase of shifted data		0	0	π	0	0	π	0	0
phase comparision o/p		+	−	−	+	−	−	+	+
Detected Binary Sequence		1	0	0	1	0	0	1	1

Note:- + → Symbol '1'
− → Symbol '0'.



Note:- Received bit

d_k	0	π	0	π
delayed bit d_{k-1}	0	π	π	0
phase comparision o/p	+	+	−	−
dpsk decoded o/p	1	1	0	0

2nd Method (or) Assume Reference bit '0'

Binary data $\{b_k\}$		1	0	0	1	0	0	1	1
		⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙
Differentially Encoded data $\{d_k\}$	(assume) reference bit 0	0	1	0	0	1	0	0	0
phase of DPSK	π	π	0	π	π	0	π	π	π
Shifted Differentially Encoded Data $\{d_{k-1}\}$		0	0	1	0	0	1	0	0
phase of shifted data		π	π	0	π	π	0	π	π
phase Comparision o/p		+	-	-	+	-	-	+	+
Detected Binary Sequence $\{b_k\}$		1	0	0	1	0	0	1	1

Note:- + → symbol '1'
- → Symbol '0'

The binary sequence **1100100010** is applied to the DPSK transmitter.

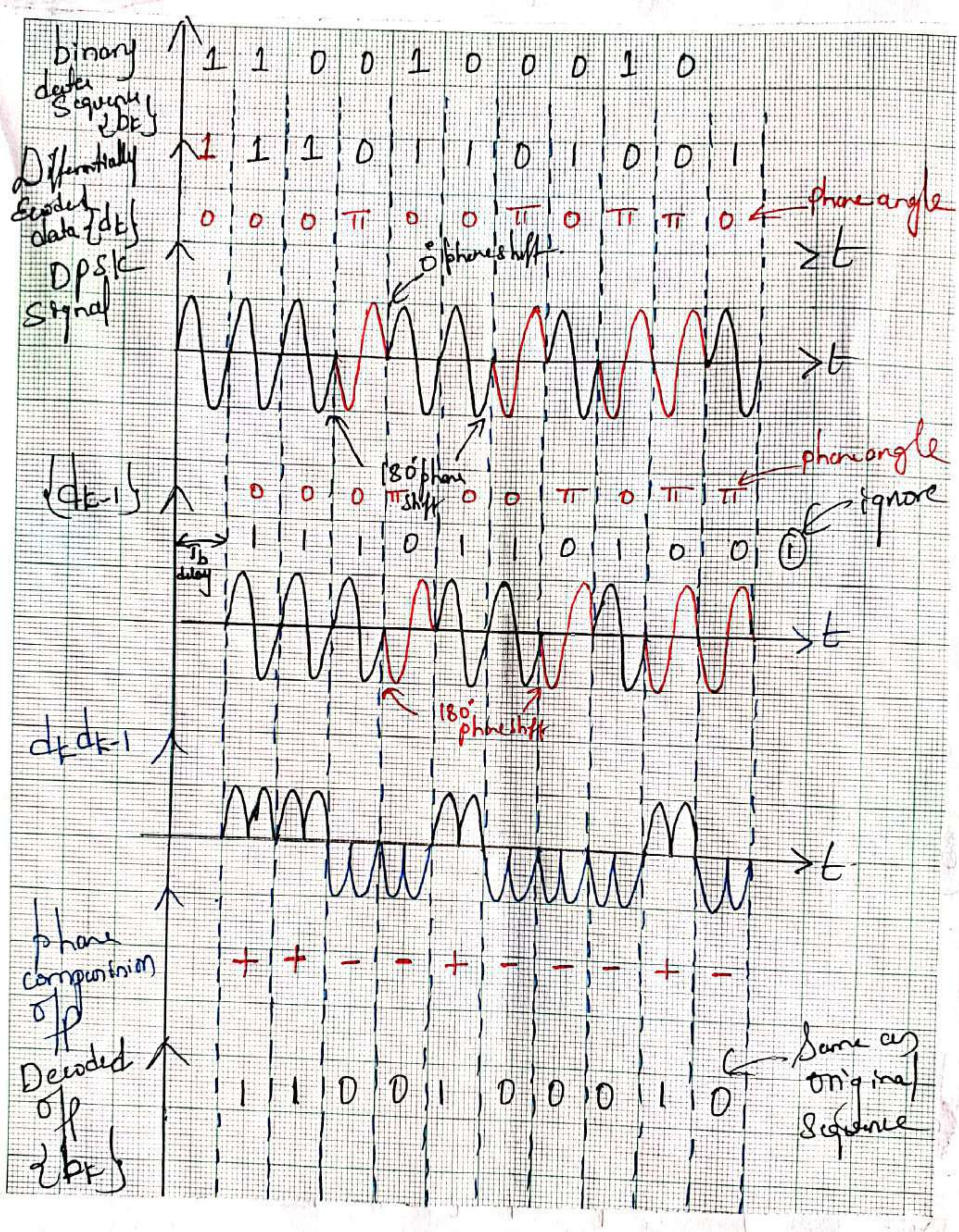
i. Sketch the resulting wave form at the transmitter output.
ii. Applying this waveform to the DPSK receiver, show that in the absence of noise, the original binary sequence is reconstructed at the receiver output.

(06Marks) June-July 2019.

soln:- (i)

Binary Data $\{b_k\}$		1	1	0	0	1	0	0	0	1	0
Differentially Encoded data $\{d_k\}$	reference bit (assume) 1	1	1	0	1	1	0	1	0	0	1
phase of DPSK	0	0	0	π	0	0	π	0	π	π	0
Shifted Differentially Encoded data $\{d_{k-1}\}$		1	1	1	0	1	1	0	1	0	0
(ii) Decoding phase of Shifted Data		0	0	0	π	0	0	π	0	π	π
phase Comparison output		+	+	−	−	+	−	−	−	+	−
Detected output $\{b_k\}$		1	1	0	0	1	0	0	0	1	0

Note:- + → Symbol '1'
− → Symbol '0'


binary data Sequence {bk}
1 1 0 0 1 0 0 0 1 0
Differentially Encoded data {dk}
1 1 1 0 1 1 0 1 0 0 1
0 0 0 π 0 0 π 0 π π 0
phase angle
DPSK Signal
0° phase shift
t
t
180° phase shift
{dk-1}
0 0 0 π 0 0 π 0 π π
phase angle
ignore
Tb delay
1 1 1 0 1 1 0 1 0 0 ①
t
180° phase shift
dk dk-1
t
phase comparision o/p
+ + − − + − − − + −
Decoded o/p {bk}
1 1 0 0 1 0 0 0 1 0
Same as original Sequence

Problem.

Binary data are transmitted over a microwave Link at the rate of 10^6 bits/sec and the power spectral density of the noise at the receiver input is 10^{-10} watts/Hz. Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for the following cases.

i. Binary PSK using coherent detection.

ii. DPSK.

note:- take $\text{erfc}(2.63) = 2\times10^{-4}$, $Q(3.7) = 10^{-4}$.

(6 Marks) Dec 2018 / Jan 2019.

Soln:- i. using BPSK (coherent detection)

$$P_{e(BPSK)} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \leq 10^{-4} \leftarrow ①$$

given $Q(3.7) = 10^{-4} \leftarrow ②$

Comparing eqn ① and eqn ②

$$\sqrt{\frac{2E_b}{N_0}} = 3.7$$

$$\frac{2E_b}{N_0} = (3.7)^2$$

$$\frac{E_b}{(N_0/2)} = (3.7)^2$$

$$E_b = \left(\frac{N_0}{2}\right)(3.7)^2$$

Given

$P_e \leq 10^{-4}$.

$T_b = \frac{1}{f_b} = \frac{1}{10^6}$

$= 1\,\mu sec$.

$\frac{N_0}{2} = 10^{-10}$ W/Hz.

$$E_b = (3.7)^2 \times 10^{-10}$$

$$E_b = 13.69 \times 10^{-10}$$

$$\Rightarrow \boxed{E_b = 1.369 \times 10^{-9}} \text{ Joules}$$

$$P_{avg} = \frac{E_b}{T_b} = \frac{1.369 \times 10^{-9}}{10^{-6}} = 1.369 \times 10^{-3} \text{ watts}$$

$$\boxed{P_{avg(BPSK)} = 1.369 \text{ m watts}} \leftarrow ⓐ$$

ii) using DPSK (non-coherent detection)

$$P_{e(DPSK)} = \frac{1}{2} e^{(-E_b/N_0)}$$

given $P_{e(DPSK)} = 10^{-4}$

$$10^{-4} = \frac{1}{2} e^{-E_b/N_0}$$

$$e^{-E_b/N_0} = 2 \times 10^{-4}$$

By solving using calci $\Rightarrow$ $\frac{E_b}{N_0} = 8.5171$

$$E_b = 8.5171 \times N_0$$

$$E_b = 8.5171 \times 2 \times 10^{-10}$$

$$\boxed{E_b = 1.70343 \times 10^{-9} \text{ Joules}}$$

$$P_{avg} = \frac{E_b}{T_b} = \frac{1.7034 \times 10^{-9}}{10^{-6}}$$

$$P_{avg(DPSK)} = 1.7034 \times 10^{-3}$$

$$\boxed{P_{avg(DPSK)} = 1.7034 \text{ mW}} \leftarrow ⓑ$$

Note:- i. under given P_e BPSK consumes less transmission power compared to DPSK.

Problem :- In a BPSK System binary 1 is represented by $S_1(t) = A\cos\omega_c t$ and binary 0 by $S_2(t) = -A\cos\omega_c t$ is transmitted over a AWGN channel at 0.2msec. The Carrier amplitude at the receiver is 1mVolt. Noise PSD is 10^{-11} w/Hz. Find the average Probability of Error for i. Coherent PSK detection.

ii. Non-coherent DPSK detection.

Note: $\text{erfc}(2.236) = 4.1\times10^{-4}$

soln :-

$T_b = 0.2\text{msec} = 0.2\times10^{-3}$ sec.

$A = 1\text{m volt} = 1\times10^{-3}$ volt.

$\frac{N_0}{2} = 10^{-11}$ w/Hz. $\Rightarrow N_0 = 2\times10^{-11}$ w/Hz

i. $P_{e(BPSK)} = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$

$= \frac{1}{2}\text{erfc}\left(\sqrt{\frac{0.1\times10^{-9}}{2\times10^{-11}}}\right)$

$= \frac{1}{2}\text{erfc}(\sqrt{5})$

$= \frac{1}{2}\text{erfc}(2.236)$

$= \frac{1}{2}\times4.1\times10^{-4}$

$\boxed{P_{e(BPSK)} = 0.205\times10^{-3}}$ ← ⓐ

$E_b = P.T_b$

$= \frac{A^2}{2}.T_b$

$E_b = \frac{(10^{-3})^2}{\cancel{2}}\times(\overset{0.1}{\cancel{0.2}}\times10^{-3})$

$E_b = 10^{-10}$ Joules

$\boxed{E_b = 0.1\times10^{-9}\text{ joules}}$

Note:- $\text{erfc}(2.236) = 4.1\times10^{-4}$

ii) $$P_{e(DPSK) \atop (Noncoherent)} = \frac{1}{2} e^{-E_b/N_0}$$

$$= \frac{1}{2} e^{-\left[\frac{0.1 \times 10^{-9}}{2 \times 10^{-11}}\right]}$$

$$= \frac{1}{2} e^{-[5]}$$

$$= \frac{1}{2} e^{-5} = \frac{1}{2} \times 6.7379 \times 10^{-3}$$

$$\boxed{P_{e \atop (non-coherent\ DPSK)} = 3.36897 \times 10^{-3}} \leftarrow ⓑ$$

Comparing eqⁿ ⓐ and eqⁿ ⓑ

Note:- i.e. under given (E_b/N_0), the

$$P_{e(non-coherent \atop DPSK)} > P_{e(BPSK)}$$

Advantages of Dpsk

i. Non-coherent Detection is possible.

ii. ie Dpsk doesnot need Carrier at its receiver. hence the Complicated Circuitry for generation of local carrier is avoided.

iii. Cost of the circuit is Less.

iv. Circuit complexity is Less.

v. the B.W requirement of Dpsk is reduced compared to Bpsk.

i.e
$$\boxed{\begin{array}{l} BW_{(Bpsk)} = \frac{2}{T_b} = 2f_b \\ \hline BW_{(Dpsk)} = \frac{1}{T_b} = f_b \end{array}} \begin{array}{l} Hz \\ Hz. \end{array} \Rightarrow (BW)_{Dpsk} = \frac{1}{2}(BW)_{Bpsk}$$

Drawbacks of Dpsk

i. The probability of Error (or) bit Error rate of Dpsk is higher compared to that of Bpsk.

ii. Under given (E_b/N_0), Bpsk consumes Less transmission power avg. compared to Dpsk.

iii. Since Dpsk uses two Successive bits for its reception, Error in the first bit creates Error in the Second bit. hence Error propogation of Dpsk is more. (Whereas in psk single bit can go in error Since detection of each bit is independent.

iv. Noise interference in Dpsk is more.

Parameter	Bpsk	BFsk	Dpsk.	Qpsk	M-ary Psk
1. Modulation of	phase $(0^\circ, \pi)$	Frequency f_1, f_2	phase $(0^\circ, \pi)$	phase $(0^\circ, 90^\circ, 180^\circ, 270^\circ)$	phase
2. Bit per Symbol	One	one	One	two	N
3. No. of possible symbols	TWO $\{0, 1\}$	TWO $\{0, 1\}$	TWO $\{0, 1\}$	four $\{00, 01, 10, 11\}$	$M = 2^N$
4. Minimum B.W	$2f_b = \frac{2}{T_b}$	$4f_b = \frac{4}{T_b}$	$f_b = \frac{1}{T_b}$	$f_b = \frac{1}{T_b}$	$\frac{2f_b}{N} = \frac{2}{NT_b}$ $= \frac{2R_b}{\log_2 M}$
5. Symbol duration	$T_s = T_b$	$T_s = T_b$	$T_s = 2T_b$	$T_s = 2T_b$	$T_s = NT_b$
6. Detection type	Coherent	Coherent & Non coherent	Non coherent	coherent	Coherent
7. probability of Error	$P_{e\,(BPSK)} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$ $= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_{e\,(BFSK)} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$ $= Q\left(\sqrt{\frac{E_b}{N_0}}\right)$	$P_{e\,(\text{Non coherent DPSK})} = \frac{1}{2}e^{(-E_b/N_0)}$ ↑ exponential fn.	$P_{e\,(QPSK)} = \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$ $= 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$	$P_e = \mathrm{erfc}\left[\sqrt{\frac{E_s}{N_0}}\sin\frac{\pi}{M}\right]$ $= 2Q\left[\sqrt{\frac{E_s}{N_0}}\sin\frac{\pi}{M}\right]$

$$\boxed{\text{i.e. } P_{e\,(M\text{-ary PSK})} = \mathrm{erfc}\left(\sqrt{\frac{E_s}{N_0}}\sin\frac{\pi}{M}\right) = 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right]}$$

Note:- i. $Q(x) = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$

ii. $\mathrm{erf}(x) + \mathrm{erfc}(x) = 1$.

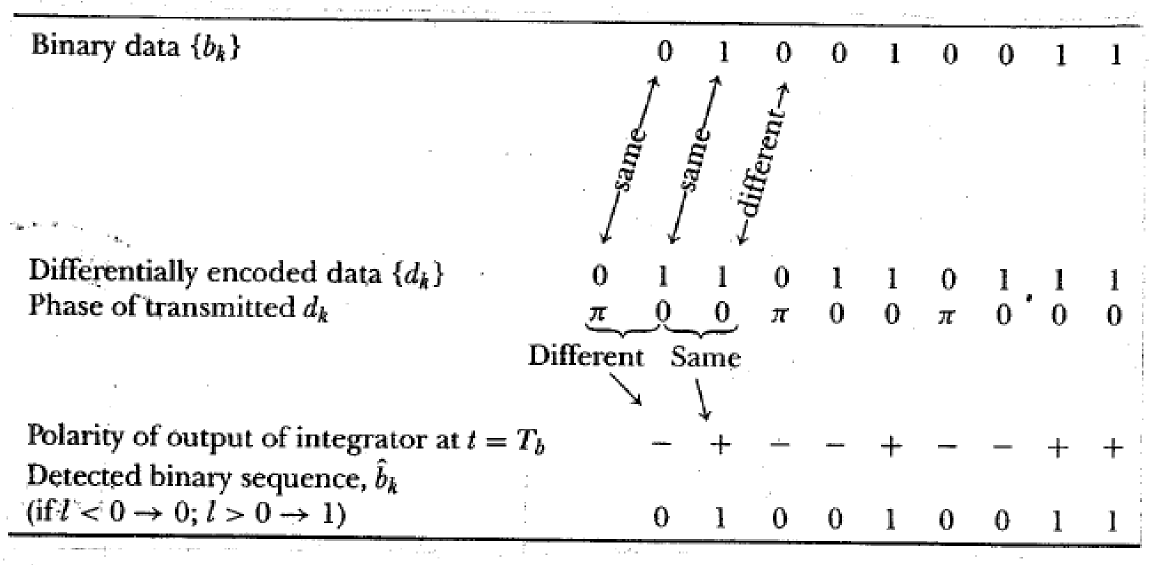
iii. $\frac{N_0}{2}$ = Noise density (or) Both sided Noise density.

Problem1:

A binary data $\{b_k\}$=$\{010010011\}$ is to be transmitted using DPSK. Choose d_1=0 and determine:

- **i. The differential encoded sequence $\{d_k\}$ and phase of the transmitted DPSK signal.**
- **ii. Polarity of the integrator output at $t=T_b$ of the DPSK receiver.**
- **iii. Decision rule and detected binary sequence.**

Soln:



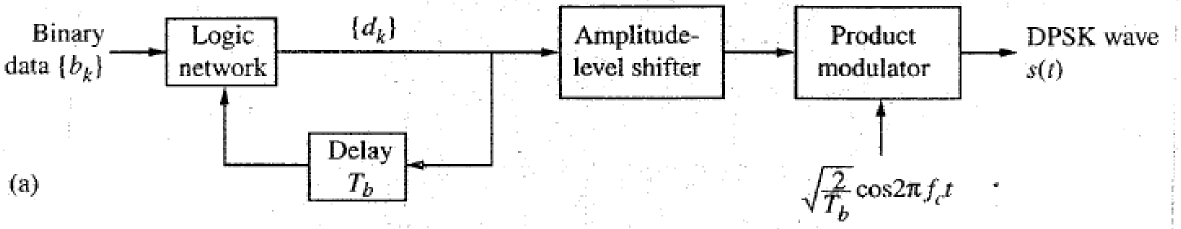
Binary data $\{b_k\}$		0	1	0	0	1	0	0	1	1
Differentially encoded data $\{d_k\}$	0	1	1	0	1	1	0	1	1	1
Phase of transmitted d_k	π	0	0	π	0	0	π	0	0	0
Polarity of output of integrator at $t = T_b$		−	+	−	−	+	−	−	+	+
Detected binary sequence, $\hat{b}_k$ (if $l < 0 \rightarrow 0$; $l > 0 \rightarrow 1$)		0	1	0	0	1	0	0	1	1

Problem 2.

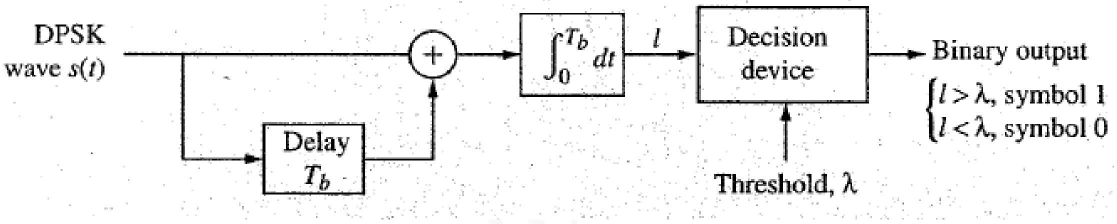
DPSK transmitter shown in Figure (a). The channel introduces a phase reversal of 180°.

(a) Sketch the transmitted DPSK waveform assuming an initial bit of 1. What is the effect of changing initial bit to 0?

(b) Assuming the channel is noise free, show that the DPSK detector in the receiver shown in Figure (b) produces the original binary sequence, despite the 180° phase reversal in the channel. For demonstration, take DPSK waveform with $d_{k-1} = 1$.



(a)



(b)

Soln:

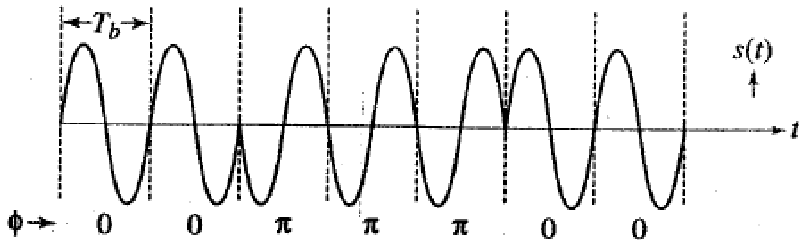
Solution: (a) The logic used for generating DPSK sequence $\{d_k\}$ is

$$d_k = d_{k-1}b_k + \overline{d_{k-1}}\,\overline{b_k}$$

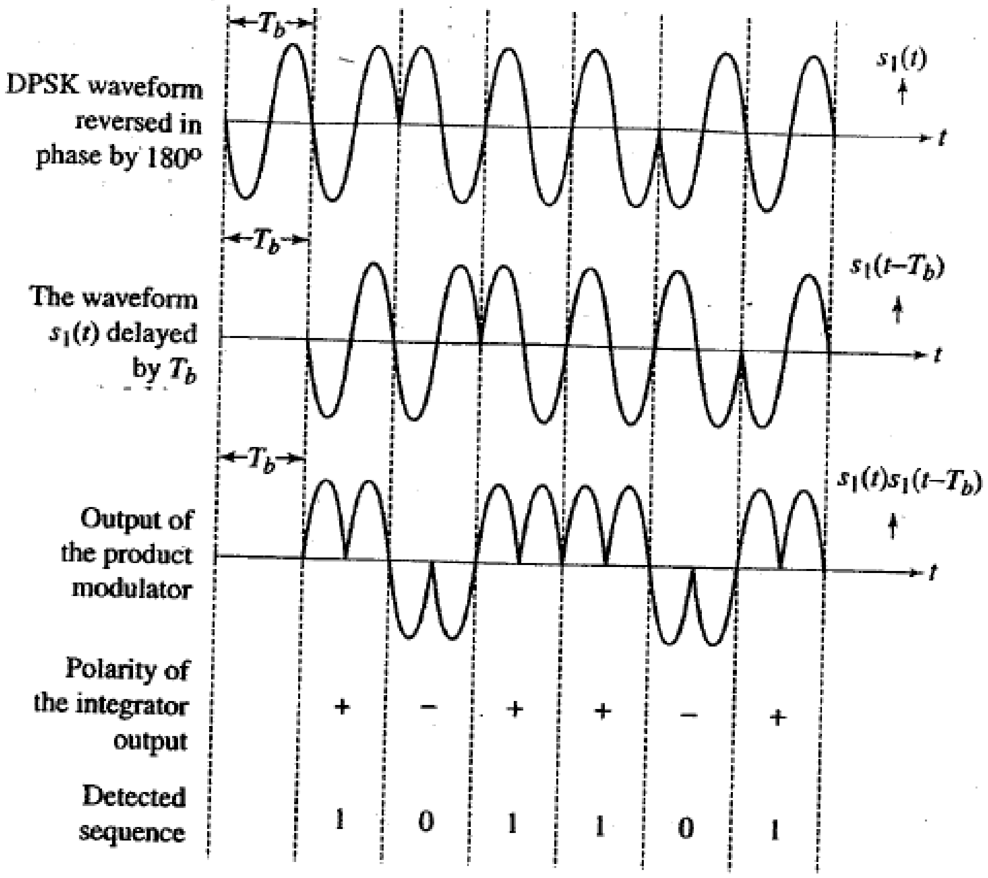
where the symbol + stands for logical OR operation.

(a) Initial bit, $d_{k-1} = 1$

$k \rightarrow$	−1	0	1	2	3	4	5
Binary data $\{b_k\}$		1	0	1	1	0	1
Differentially encoded data $\{d_k\}$	1	1	0	0	0	1	1
Phase of the DPSK wave (rad)	0	0	π	π	π	0	0



(b)


T_b
DPSK waveform reversed in phase by 180°
$s_1(t)$
t
T_b
The waveform $s_1(t)$ delayed by T_b
$s_1(t-T_b)$
t
T_b
Output of the product modulator
$s_1(t)s_1(t-T_b)$
t
Polarity of the integrator output
+
−
+
+
−
+
Detected sequence
1
0
1
1
0
1

Problem 3.

Binary data is transmitted over AWGN channel using BPSK at a rate of 1 Mbps. It is desired to have average probability of error $P_e \leq 10^{-4}$. *Noise PSD is* $N_0/2 = 10^{-12}$ *Watt/Hz. Determine the avearge carrier power required at receiver input if the detector is of coherent type. [Assume erfc (3.5) = 0.00025].*

VTU : Aug.-08, June-15, Marks 4, June-16, Marks 8

Sol. : Given data

$$T_b = \frac{1}{1 \times 10^6} \text{ sec}$$

$$P_e \leq 10^{-4}$$

$$\frac{N_o}{2} = 10^{-12} \text{ W/Hz} \quad \Rightarrow \quad N_o = 2 \times 10^{-12}$$

$$\text{erfc}\,(3.5) = 0.00025$$

Carrier power :

For BPSK,
$$P_e = \frac{1}{2} erfc\sqrt{\frac{E}{N_o}} = \frac{1}{2} erfc\sqrt{\frac{PT_b}{N_o}}$$

Putting values in above equation,

$$10^{-4} \leq \frac{1}{2} erfc\sqrt{\frac{P \times \frac{1}{1 \times 10^6}}{2 \times 10^{-12}}}$$

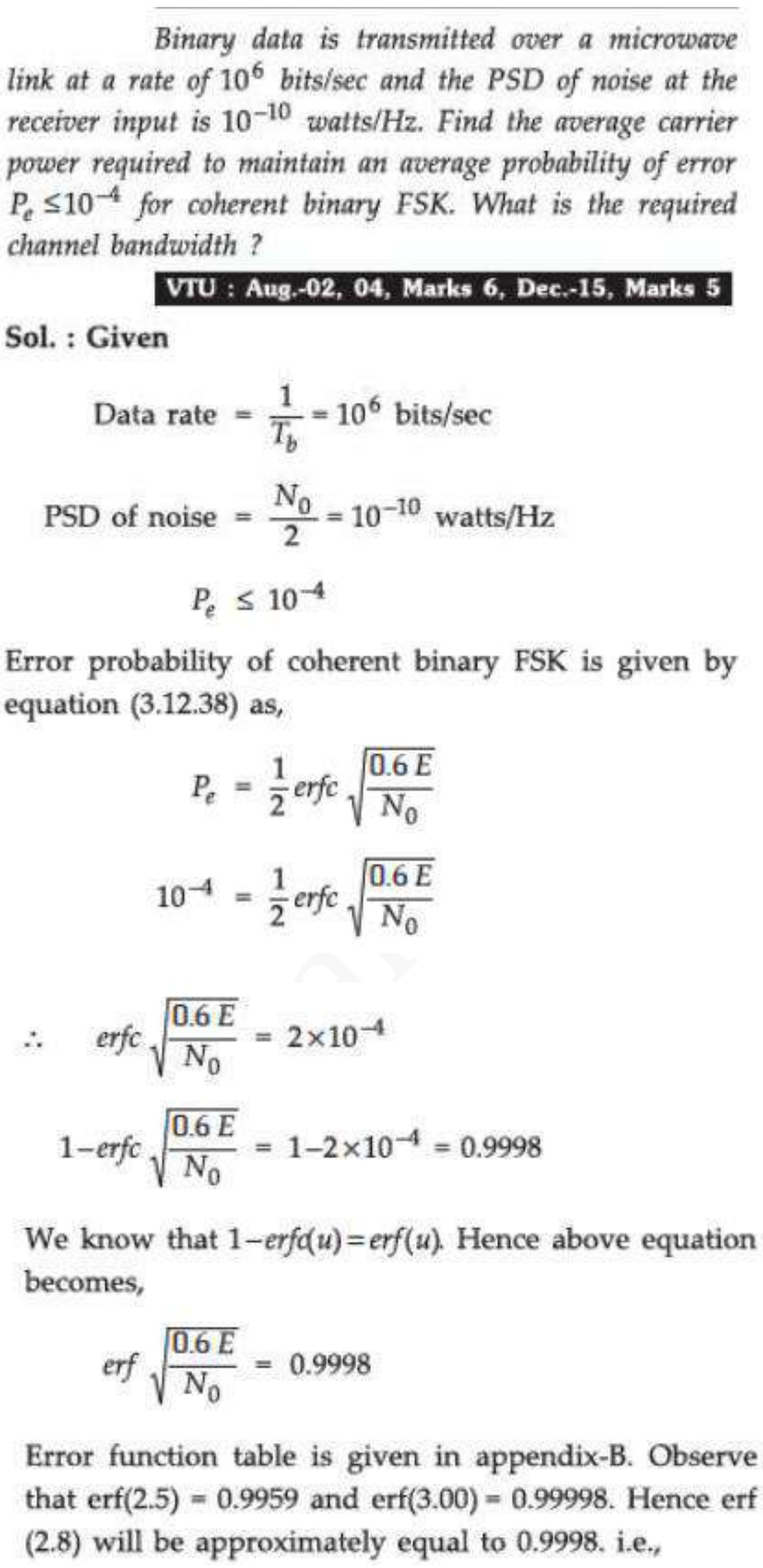
$$2 \times 10^{-4} \leq erfc\sqrt{\frac{P}{2 \times 10^{-6}}} \quad \text{i.e.} \quad 0.002 \leq erfc\sqrt{\frac{P}{2 \times 10^{-6}}}$$

It is given that erfc (3.5) = 0.00025, then above equation becomes,

$$\text{erfc}\,(3.5) \leq erfc\sqrt{\frac{P}{2 \times 10^{-6}}}$$

i.e.
$$3.5 \leq \sqrt{\frac{P}{2 \times 10^{-6}}} \quad \Rightarrow \quad P \geq 2.45 \times 10^{-5} \text{ W}$$

Problem 4.



Binary data is transmitted over a microwave link at a rate of 10^6 bits/sec and the PSD of noise at the receiver input is 10^{-10} watts/Hz. Find the average carrier power required to maintain an average probability of error $P_e \leq 10^{-4}$ for coherent binary FSK. What is the required channel bandwidth ?

VTU : Aug.-02, 04, Marks 6, Dec.-15, Marks 5

Sol. : Given

$$\text{Data rate} = \frac{1}{T_b} = 10^6 \text{ bits/sec}$$

$$\text{PSD of noise} = \frac{N_0}{2} = 10^{-10} \text{ watts/Hz}$$

$$P_e \leq 10^{-4}$$

Error probability of coherent binary FSK is given by equation (3.12.38) as,

$$P_e = \frac{1}{2} erfc \sqrt{\frac{0.6\, E}{N_0}}$$

$$10^{-4} = \frac{1}{2} erfc \sqrt{\frac{0.6\, E}{N_0}}$$

$$\therefore \quad erfc \sqrt{\frac{0.6\, E}{N_0}} = 2 \times 10^{-4}$$

$$1 - erfc \sqrt{\frac{0.6\, E}{N_0}} = 1 - 2 \times 10^{-4} = 0.9998$$

We know that $1 - erfc(u) = erf(u)$. Hence above equation becomes,

$$erf \sqrt{\frac{0.6\, E}{N_0}} = 0.9998$$

Error function table is given in appendix-B. Observe that erf(2.5) = 0.9959 and erf(3.00) = 0.99998. Hence erf (2.8) will be approximately equal to 0.9998. i.e.,

$$\sqrt{\frac{0.6\,E}{N_0}} = 2.8$$

$$\therefore \quad \frac{0.6\,E}{N_0} = (2.8)^2 = 7.84$$

We know that

$$\frac{N_0}{2} = 10^{-10}$$

$$\therefore \quad N_0 = 2\times10^{-10}$$

$$\therefore \quad \frac{0.6\,E}{2\times10^{-10}} = 7.84$$

$$\therefore \quad \mathrm{E} = \frac{7.84\times2\times10^{-10}}{0.6}$$

$$= 2.61\times10^{-9} \text{ Joules}$$

Energy of one bit is given as,

$$\mathrm{E} = P\,T_b$$

$$\therefore \quad \mathrm{P} = \frac{E}{T_b}$$

$$= 2.61\times10^{-9}\times10^{6}$$

$$= 2.61 \text{ mW}$$

Therefore to achieve $P_e \leq 10^{-4}$, average power of the carrier must, $P \geq 2.61$ mW.

The channel bandwidth is approximately equal to bit rate i.e.,

$$B_T = \frac{1}{T_b} = 10^6 \text{ bits/sec} = 1 \text{ MHz.}$$

Problem 5.

Compare the average power requirements of binary noncoherant ASK, coherent PSK, DPSK and noncoherent FSK signalling schemes operating at a data rate of 1000 bits/sec. Over a bandpass channel having a bandwidth of 3000 Hz, $\eta/2 = 10^{-10}$ W / Hz, $P_e = 10^{-5}$.

VTU : Aug.-05, Marks 10

Sol. : $T_b = \frac{1}{1000}$ sec = 0.001 sec

$\frac{N_0}{2} = 10^{-10}$ W/Hz $\Rightarrow N_0 = 2\times10^{-10}$

W/Hz

$P_e = 10^{-5}$

i) Noncoherent ASK

$$P_e = \frac{1}{2}e^{-E_b/2N_0}$$

$$10^{-5} = \frac{1}{2}e^{-E_b/(2\times2\times10^{-10})}$$

$\therefore \quad E_b = 4.328 \times 10^{-9}$

$\therefore \quad PT_b = 4.328 \times 10^{-9}$ since $E_b = PT_b$

$\therefore \quad P = \frac{4.328\times10^{-9}}{T_b} = \frac{4328\times10^{-9}}{(1/1000)}$

$= 4.328 \times 10^{-6}$ W

Coherent PSK

$$P_e = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_o}}$$

$$10^{-5} = \frac{1}{2} erfc\sqrt{\frac{PT_b}{N_o}} \qquad \text{since } E_b = PT_b$$

$$= \frac{1}{2} erfc\sqrt{\frac{P\times 1/1000}{2\times 10^{-10}}}$$

$$= \frac{1}{2} erfc\sqrt{\frac{P}{2\times 10^{-7}}}$$

$$\therefore \qquad 2 \times 10^{-5} = erfc\sqrt{\frac{P}{2\times 10^{-7}}}$$

$$\therefore \quad 1 - 2 \times 10^{-5} = 1 - erfc\sqrt{\frac{P}{2\times 10^{-7}}}$$

$$0.99998 = erf\sqrt{\frac{P}{2\times 10^{-7}}}$$

From error function tables erf (3) = 0.99998. Hence above equation becomes,

$$\sqrt{\frac{P}{2\times 10^{-7}}} = 3 \quad \Rightarrow \quad P = 1.8 \times 10^{-6} \text{ W}$$

iii) Noncoherent binary FSK

$$P_e = \frac{1}{2} e^{-E_b/2N_o}$$

This is same as that of noncoherent ASK. Hence power requirement will also be the same i.e.

$$P = 4.328 \times 10^{-6} \text{ W}$$

iv) DPSK

$$P_e = \frac{1}{2}e^{-E_b/N_o}$$

$$= \frac{1}{2}\, e^{-PT_b/N_o}$$

Putting values in above equation

$$10^{-5} = \frac{1}{2}\, e^{-\left(P\times\frac{1}{1000}\right)\Big/(2\times10^{-10})}$$

$$\therefore \qquad 2\times10^{-5} = e^{-P/(2\times10^{-7})}$$

$$\therefore \qquad P = 2.164\times10^{-6}$$

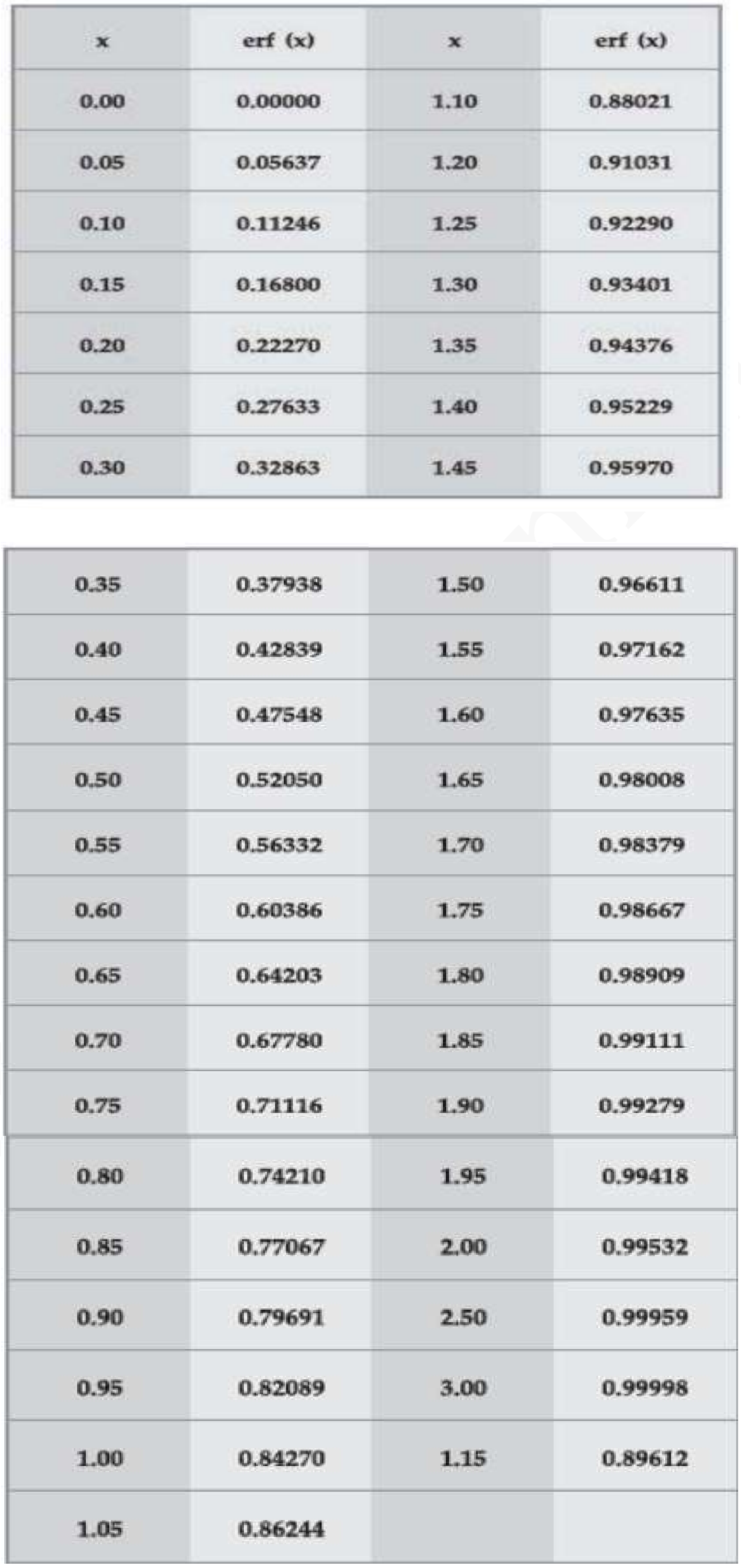
Following table lists the average power requirements.

Sr. No.	Digital modulation technique	Power requirement, μW
1.	Noncoherent ASK	4.328
2.	Coherent PSK	1.8
3.	Noncoherent FSK	4.328
4.	DPSK	2.164

Table : Average power requirements

Above results shows that coherent PSK requires lowest power.

Table: Error function erf(x) and Complementary error function erfc(x)



x	erf (x)	x	erf (x)
0.00	0.00000	1.10	0.88021
0.05	0.05637	1.20	0.91031
0.10	0.11246	1.25	0.92290
0.15	0.16800	1.30	0.93401
0.20	0.22270	1.35	0.94376
0.25	0.27633	1.40	0.95229
0.30	0.32863	1.45	0.95970
0.35	0.37938	1.50	0.96611
0.40	0.42839	1.55	0.97162
0.45	0.47548	1.60	0.97635
0.50	0.52050	1.65	0.98008
0.55	0.56332	1.70	0.98379
0.60	0.60386	1.75	0.98667
0.65	0.64203	1.80	0.98909
0.70	0.67780	1.85	0.99111
0.75	0.71116	1.90	0.99279
0.80	0.74210	1.95	0.99418
0.85	0.77067	2.00	0.99532
0.90	0.79691	2.50	0.99959
0.95	0.82089	3.00	0.99998
1.00	0.84270	1.15	0.89612
1.05	0.86244		

The error function, denoted by *erf* (*u*) is defined as,

$$erf\,(u) = \frac{2}{\sqrt{\pi}} \int_{o}^{u} e^{-z^2}\,dz$$

And the complementary error function, *erfc*(*u*) is defined as,

$$erfc\,(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-z^2}\,dz$$

And, $erfc(u) = 1-erf\,(u)$

$$erf\,(-u) = -erf\,\,(u))$$

The relationship between Q-function and error function.

We know that Q(u) is given as,

$$\mathrm{Q(u)} = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} e^{-x^2/2}\,dx$$

$$\mathrm{Q(u)} = \frac{1}{2} erfc\left(\frac{u}{\sqrt{2}}\right)$$

and

$$\mathrm{erfc\ (u)} = 2\,\mathrm{Q}\left(\sqrt{2}\,u\right)$$

Comparison of Digital Modulation :

Following table lists the bit error rates :

Sr. No.	Name of the scheme	Bit error rate (P_e)
1.	Binary ASK	$\frac{1}{2}\, erfc \sqrt{\frac{E}{4N_0}}$
2.	Binary coherent PSK	$\frac{1}{2}\, erfc \sqrt{\frac{E}{N_0}}$
3.	Binary coherent FSK	$\frac{1}{2}\, erfc \sqrt{\frac{E_b}{2N_0}}$
4.	Binary noncoherent FSK	$\frac{1}{2}\, e^{-\frac{E}{2N_0}}$
5.	Binary DPSK	$\frac{1}{2}\, e^{-\frac{E_b}{N_0}}$
6.	QPSK	$erfc \sqrt{\frac{E_b}{2N_0}}$

Table Bit error rates of digital modulation systems

Fig. shows the plot of BER as a function of $\frac{E_b}{N_0}$.

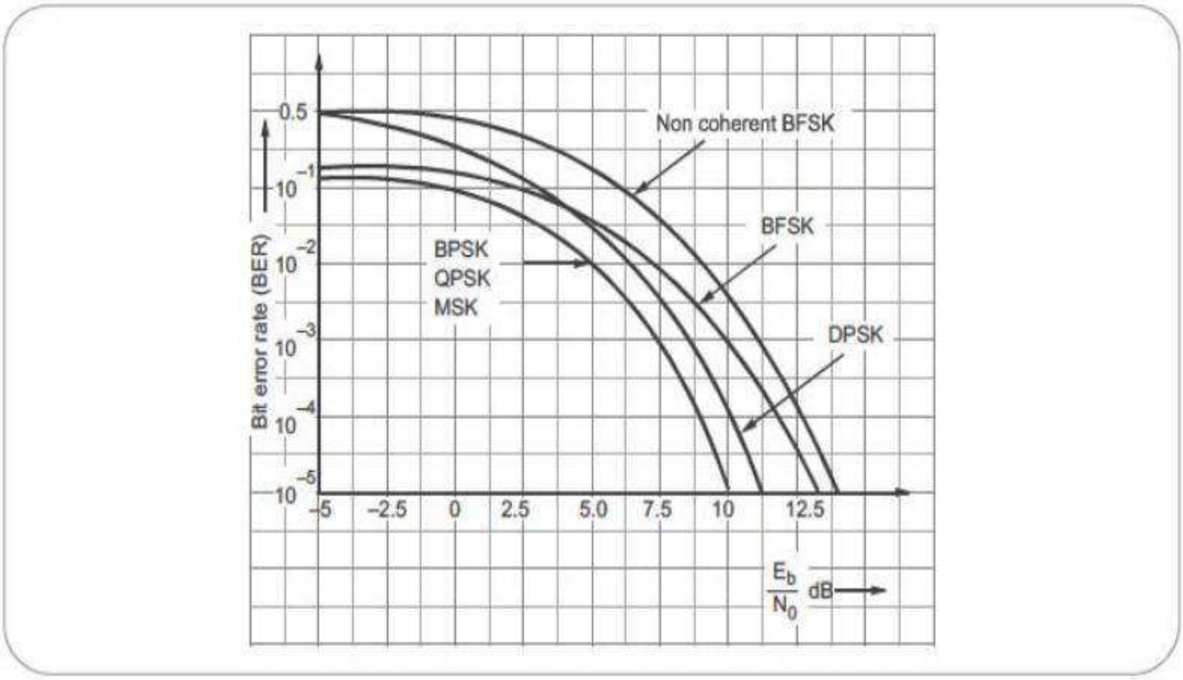
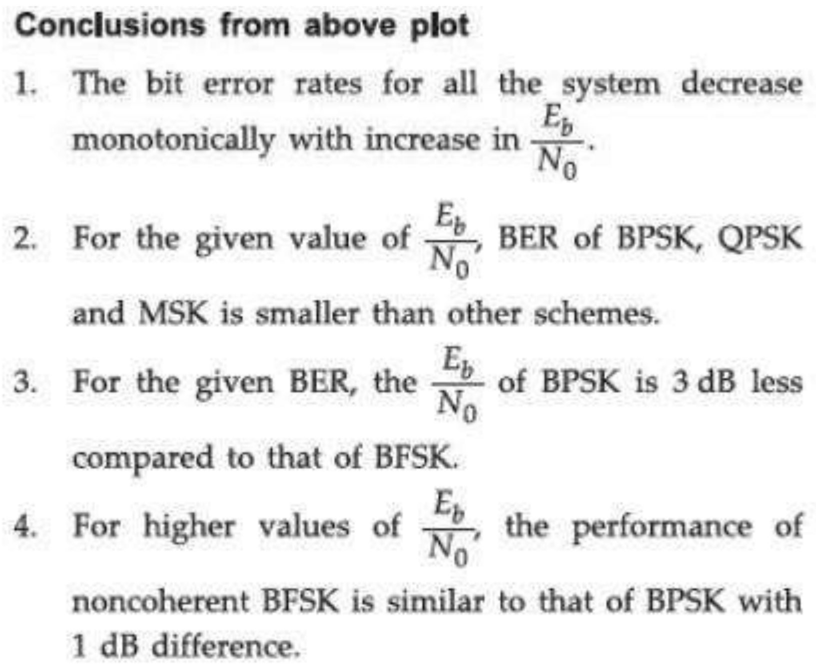


Fig. Comparison of *BER* Vs $\frac{E_b}{N_0}$



Conclusions from above plot

1. The bit error rates for all the system decrease monotonically with increase in $\frac{E_b}{N_0}$.
2. For the given value of $\frac{E_b}{N_0}$, BER of BPSK, QPSK and MSK is smaller than other schemes.
3. For the given BER, the $\frac{E_b}{N_0}$ of BPSK is 3 dB less compared to that of BFSK.
4. For higher values of $\frac{E_b}{N_0}$, the performance of noncoherent BFSK is similar to that of BPSK with 1 dB difference.

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Module-4

COMMUNICATION THROUGH BAND LIMITED CHANNELS

4.1 Digital Transmission through Band limited channels: Digital PAM Transmission through Band limited Channels.
4.2 Signal design for Band limited Channels: Design of band limited signals for zero ISI-The Nyquist Criterion (statement only).
4.3 Design of band limited signals with controlled ISI-Partial Response signals.
4.4 Probability of error for detection of Digital PAM:
  i. Probability of error for detection of Digital PAM with Zero ISI.
  ii. Symbol–by–Symbol detection of data with controlled ISI

4.5 Channel Equalization: Linear Equalizers (ZFE, MMSE), Adaptive Equalizers (Text 2: 9.1, 9.2, 9.3.1, 9.3.2) and (Text 2: 9.4.2).

Text 2: John G Proakis and Masoud Salehi, ―Fundamentals of Communication Systems‖, 2014 Edition, Pearson Education, ISBN 978-8-131-70573-5.

4.1 Digital Transmission through Band limited channels: Digital PAM Transmission through Band limited Channels.

1. With a neat block diagram of digital PAM system obtain the expression for inter symbol interference (ISI). **(06 Marks) June-July 2018.**

2. With a neat block diagram, explain the digital PAM transmission through band limited baseband channels. Also obtain the expression for inter symbol interference. **(06 Marks) Dec 2018-Jan 2019.**

3. What is ISI? Obtain the expression of output of a filter with inter symbol interference. **(8 Marks) Dec 2019-Jan2020.**

4.2 Signal design for Band limited Channels: Design of band limited signals for zero ISI-The Nyquist Criterion (statement only).

1. State and prove Nyquist condition for zero ISI. **(06 Marks) June-July 2018.**

2. With neat sketches and expressions, explain raised cosine spectrum solution to reduced ISI. **(06 Marks) Dec 2018-Jan 2019.**

3. Explain the Nyquist criterion for distortionless baseband binary transmission and obtain the ideal solution for zero ISI. **(08 Marks) Dec 2019-Jan 2020.**
4. Explain the following terms with related equations and diagram with respect to base band transmission.
   i. ISI and Nyquist condition for zero ISI
   ii. Duobinary signal pulse
   iii. Modified duobinary signal pulse
   iv. Partial response signals
   v. Raised cosine spectrum. **(10 Marks) June-July 2019.**

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4.3 Design of band limited signals with controlled ISI-Partial Response signals.

1. Explain the design of band limited signals with controlled ISI. **(10 Marks) June-July 2018.**

2. What is the advantage of controlled ISI partial response signalling? with block diagram, explain the duo-binary encoder with pre-coder. Mention the frequency response, impulse response and its features. **(06 Marks) Dec 2018-Jan 2019.**

4.4 Probability of error for detection of Digital PAM:

a. Probability of error for detection of Digital PAM with Zero ISI.

b. Symbol–by–Symbol detection of data with controlled ISI

1. For the binary data sequence $\{d_n\}$ given by 11101001. Determine the precoded sequence, transmitted sequence, received sequence and the decoded sequence. **(04 Marks) June-July 2018.**

2. Explain the modified duo-binary signalling scheme, with pre-coding. Illustrate the encoding for the binary sequence "011100101". Assume previous pre-coder outputs as 1. **(07 Marks) Dec 2018-Jan 2019.**

3. Explain the need for precoder in a duobiary signalling. The binary sequence 111010010001101 is the input to the precoder whose output is used to modulate a duobinary transmitting filter. Obtain the precoded sequence, transmitted amplitude levels, the received signal levels and the decoded sequence. **(6 Marks) June-July 2019.**

4. Draw and explain the time-domain and frequency domain of duobinary and modified duobinary signal. **(08 Marks) Dec 2019- Jan 2020.**

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5. With neat diagram, explain the timing features pertaining to eye diagram and it interpretation for baseband binary data transmission system. **(03 Marks) Dec 2018-Jan 2019.**

6. Write a note on Eye diagram. **(04 Marks) June-July 2019.**

4.5 Channel Equalization: Linear Equalizers (ZFE, MMSE), Adaptive Equalizers

1. What is a zero forcing equalizer? with a neat block diagram explain the operation of linear transversal filter. **(06 Marks) June-July 2018.**

2. With neat diagram and relevant expression, explain the concept of adaptive equalization. **(04 Marks) Dec 2018-Jan 2019.**

3. With a neat diagram, explain the concept of linear transversal filter. **(06 Marks) June-July 2019.**

4. Consider a channel distorted pulse x(t), at the input to the equalizer, given by $x(t) = \frac{1}{1+(\frac{2t}{T})^2}$ where 1/T is the symbol rate. The pulse is sampled at the rate 2/T and equalized by a zero-forcing equalizer. Determine the coefficients of a five-tap zero-forcing equalizer. **(06 Marks) June-July 2019.**

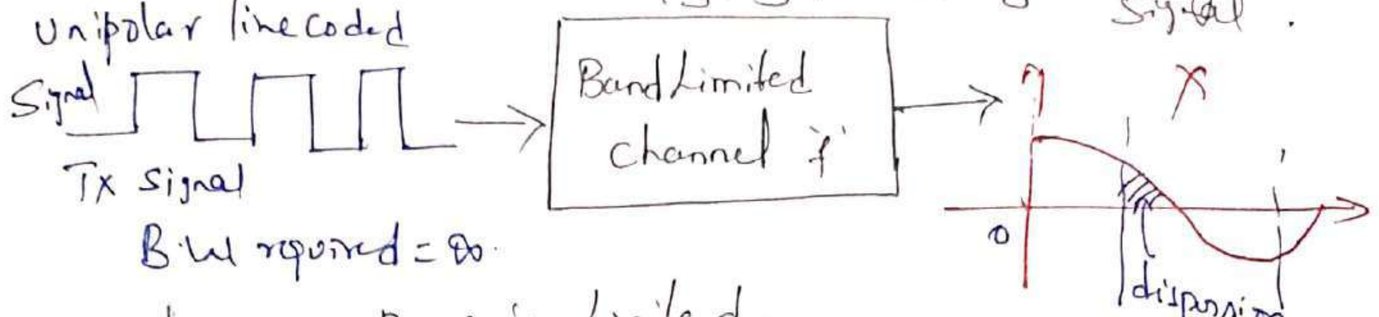
5. What is channel equalization? With a neat diagram, explain the concept of equalization using a linear transversal filter. **(08 Marks) Dec 2019-Jan 2020.**

Module-4

"Communication through Bandlimited channels."

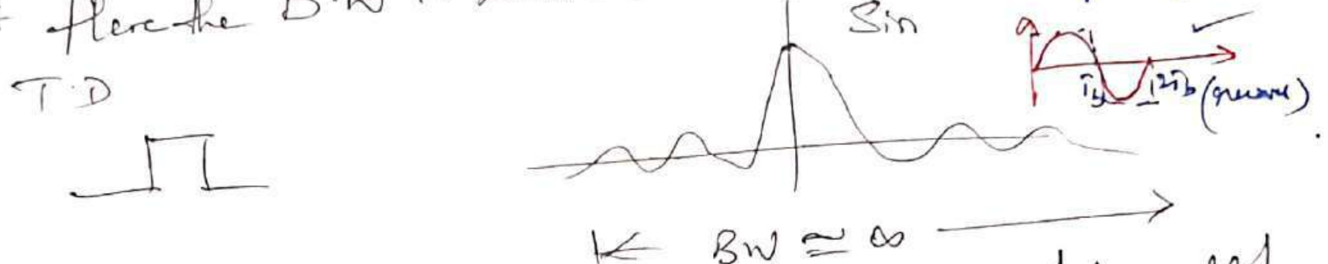
BaseBand Transmission and Reception technique.

→ Dealing with Low frequency signal. [ie we are not going to use high frequency carrier signal.



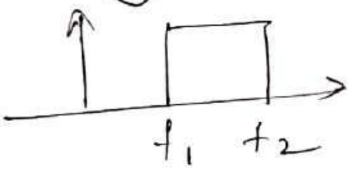
B.W required = ∞.

* Here the B.W is Limited.



* But in order to transmit the signals, we cannot be allot infinite B.W.

* channel is nothing but filter.



* passes frequencies b/w f1 + f2 all other frequencies are Eliminated.

Note:-

* Energy corresponding to one symbol, Expecting to fall from 0 to T_b, but that is not happening. Energy corresponding to one symbol is falls into the other consecutive symbols.

* One symbol interfere in the neighbouring symbol this called ISI. this is due to Band limited channel.

* Band Limited nature of the channel introduces a time dispersion Leads to Energy of One symbols fall's into the consective symbols.
So it may lead to wrong decision ∴ it is called
Inter Symbol interference.

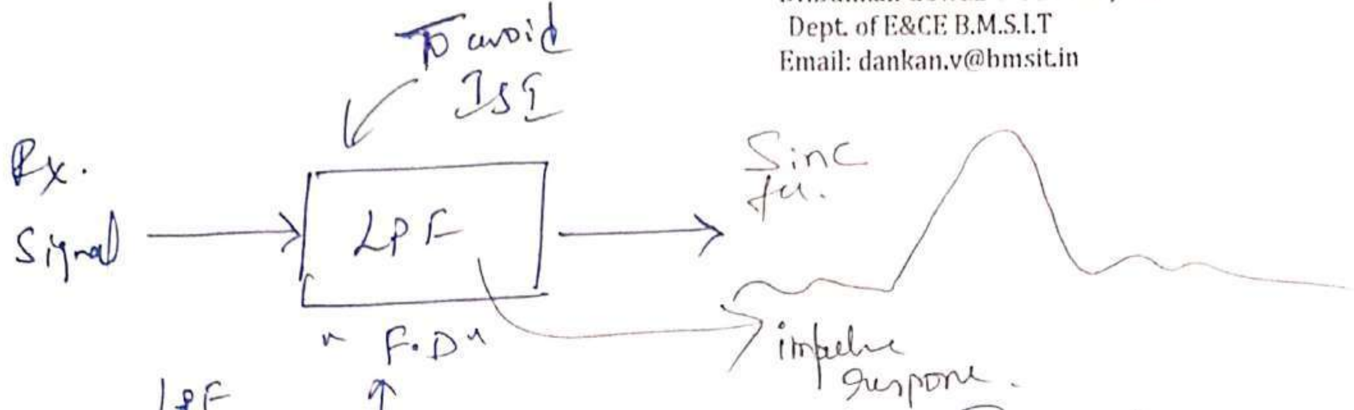
Inference/Remedy:

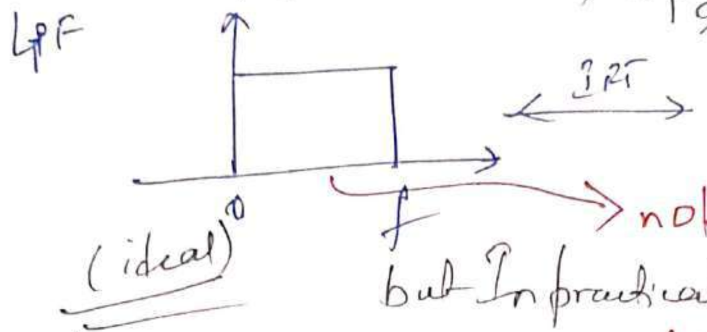
* BW is Limited for every application, we can't do anything with B.W.

* if the received signal having the frequency 'f' i.e fundamental frequency. (LPF)

Soln: Can i have a filter after the channel with a cutoff frequency 'f' Leads to be no ISI.
So use a LPF with impulse response is like a Sinc function. But to Realize a any filter without Transition Band? ⟹ not possible.

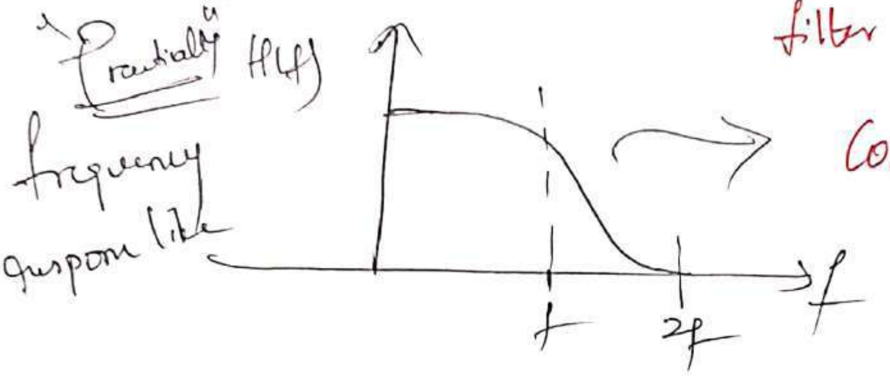
②





T.D must be "Sinc" fn

> not realizable ∴ we but In practical, we are going for another filter called Raised Cosine pulse".



Q. To avoid ISI we use LPF after the channel, with impulse response is ~~equal~~ ~~to~~ Raised Cosine pulse (Practically) but ideally ~~we~~ use it is "Sinc f^n".

This LPF is called "Receiver filter".

Here channel is "wired" ⇒ 'ISI due to only Band limited channel".

in wireless comm^n ISI leads to Multipath effect that in different.

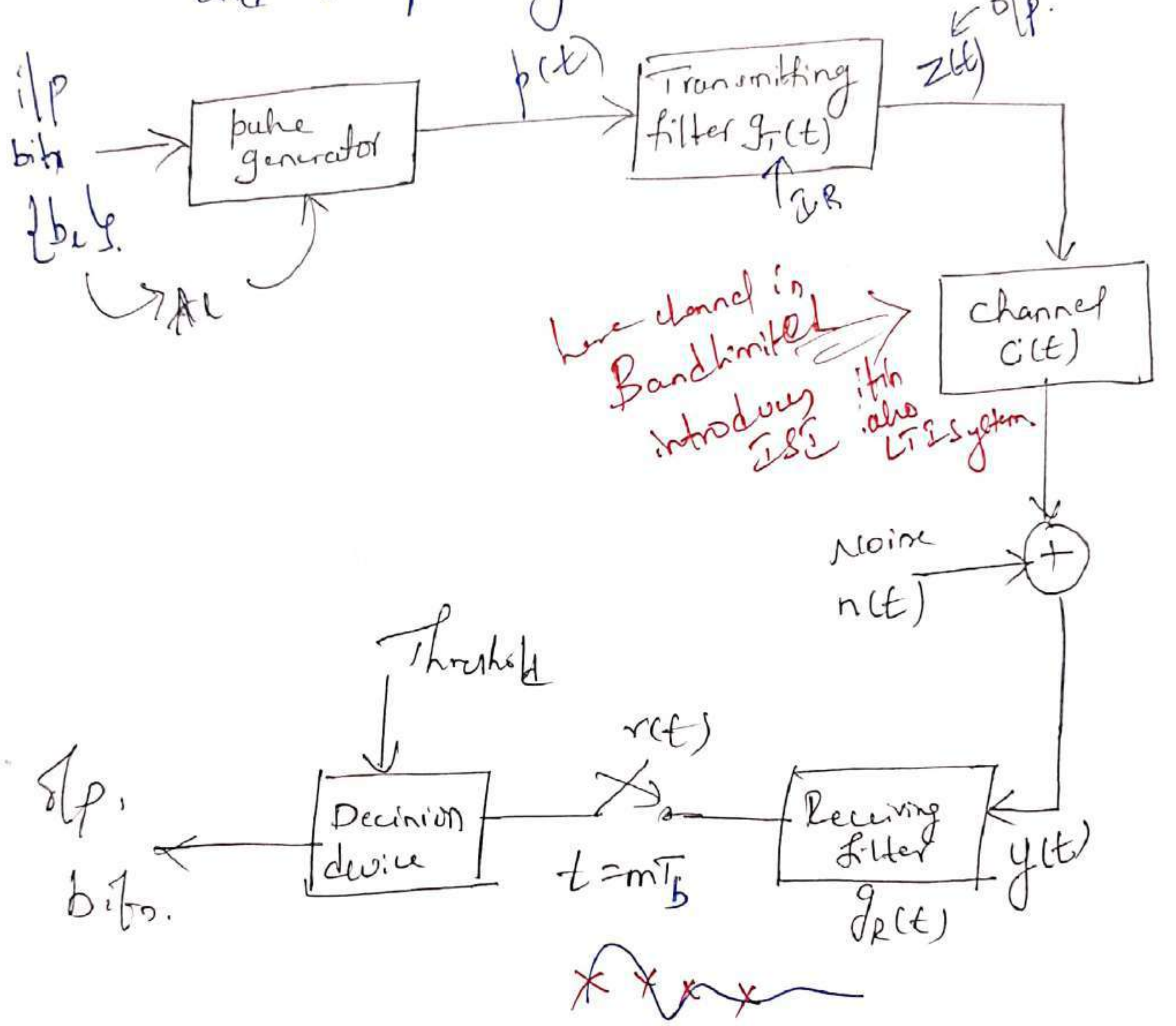
Here for simplicity we are not considering ISI due to multipath effect.

Band.

^ Time dispersion Introduces a ISI.

* Bandlimited nature of the channel introduces a Time dispersion in the received signal. Time dispersion introduces Inter Symbol Interference.

* Consider the Basic Block diagram of Base Band Transmission and Reception system.



(4)

* i/p bits mapped to levels. Based on the level's pulse generator generates Line coded Signal.

* Line coded signal in general represented by p(t).

* then we introducing 2 two filters, one Transmitter side called Transmitting filter $g_T(t)$ ←ir and Receiver side Receiving filter with impulse response $g_R(t)$.

Both the filters are Linear time invariant filters (LTI).

* with $g_T(t)$, we are going to pre-compensate the effect of ISI. Illy with $g_R(t)$, we are going to post compensate the effect of ISI.

* channel is Bandlimited and introduces ISI. its impulse response is represented by c(t).

→ Noise is added. this noise is AWGN represented by n(t).

→ Receiver filter output is continuous. im going to take Samples and this Sample values are compared with threshold.

$$\boxed{r \overset{1}{\underset{0}{\gtrless}} T}$$

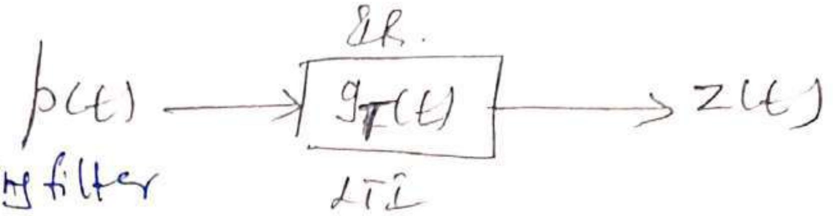
Mathematical Expression of I.S.I

* pulse generator output is Basically a Line coded Signal

$$p(t) = \sum_{l=-\infty}^{\infty} A_l \underbrace{x(t-lT_b)}_{\text{can be NRZ, RZ, or Manchester pulse.}}$$

Here $x(t)$ is NRZ, RZ etc.

Now



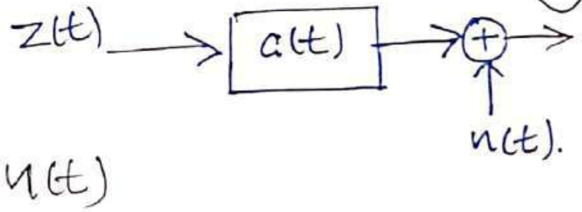
o/p of Transmitting filter

$$z(t) = p(t) * g_T(t).$$

$$= \sum_{l=-\infty}^{\infty} \overset{\leftarrow \text{constant}}{A_l}\, \underline{x(t-lT_b) * g_T(t)}.$$

$$z(t) = \sum_{l=-\infty}^{\infty} A_l P_1(t-lT_b)$$

$z(t)$ is transmitted through the channel, the receiving filter input



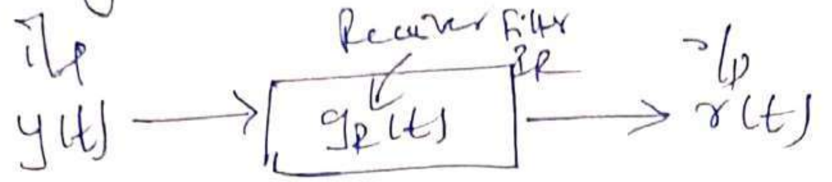
$$y(t) = z(t) * c(t) + n(t)$$

$$= \sum_{l=-\infty}^{\infty} A_l \underbrace{P_1(t-lT_b) * c(t)}_{P_2(t-lT_b)} + n(t)$$

$$y(t) = \sum_{l=-\infty}^{\infty} A_l P_2(t-lT_b) + n(t).$$

⑥

y(t) → i/p to the receiver filter.



$$r(t) = y(t) * g_R(t)$$

$$= \left[\sum_{l=-\infty}^{\infty} A_l\, P_2(t - lT_b) + n(t)\right] * g_R(t)$$

using distributive Law

$$r(t) = \sum_{l=-\infty}^{\infty} \underbrace{A_l\, P_2(t - lT_b) * g_R(t)}_{P_3(t - lT_b)} + \underbrace{n(t) * g_R(t)}_{u(t)}$$

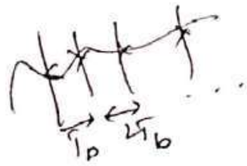
(another noise)

$$\boxed{r(t) = \sum_{l=-\infty}^{\infty} A_l\, P_3(t - lT_b) + u(t)}$$

The output of the receiver filter is sampled for every T_b sec.

$$r(t) = \sum_{l=-\infty}^{\infty} A_l\, P_3(t - lT_b) + u(t).$$

put $t = mT_b$ (general case).



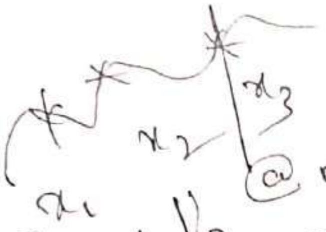
$$r(mT_b) = \sum_{l=-\infty}^{\infty} A_l\, P_3(mT_b - lT_b) + u(mT_b)$$

$l = m$.

$$r(mT_b) = \sum_{l=-\infty}^{\infty} A_l P_3\left[(m-l)T_b\right] + u(mT_b).$$

Threshold. Unipolar $\begin{cases} \nearrow a \\ \searrow 0 \end{cases}$ $\frac{a+0}{2} = a/2$

polar $\begin{cases} \nearrow a \\ \searrow -a \end{cases}$ $\Rightarrow \frac{a-a}{2} = 0$.



Symbol's Required Sample @ desired symbol is mth symbol

l = m.

$$\underbrace{r(mT_b)}_{\text{Received Signal @ } t = \text{mth Symbol}} = \underbrace{A_m P_3(0)}_{\text{Desired Symbol}} + \underbrace{\sum_{\substack{l=-\infty \\ l \neq m}}^{\infty} P_3\left[(m-l)T_b\right]}_{\text{ISI, Effect of other Symbol's}} + \underbrace{u(mT_b)}_{\text{Noise}}$$

→ This received signal $r(t)$ is continuous.

$$\boxed{r(mT_b) \gtrless_{0}^{1} \leftarrow \text{Threshold.}}$$

at mth Symbol its m not only getting level corresponding to the mth Symbol in addition to that the effect all other symbols on the required symbol is ~~from~~ called ISI.

BandLimited channel $\xrightarrow{\text{lead}}$ time dispursion $\xrightarrow{\text{Leads}}$ ISI

$$r(mT_b) = A_m\, p_3(0) + \underbrace{\sum_{l=-\infty}^{\infty} A_l\, p_3[(m-l)\,T_b]}_{\text{ISI}} + u(mT_b)$$

[Effect of other symbols on the required mth symbol].

Imp.

Time domain Condition for Zero ISI:-

$$\underbrace{r(mT_b)}_{\text{mth Symbol}} = \underbrace{A_m}_{\text{desired Symbol}}\, \underbrace{p_3(0)}_{\text{not required}} + \overbrace{\underbrace{\sum_{l=-\infty}^{\infty} A_l\, p_3[(m-l)\,T_b]}_{\text{ISI}}}^{2^{nd}\text{ term}} + \overbrace{\underbrace{u(mT_b)}_{\text{term corresponding noise}}}^{\text{Noise}}$$

if we know Am we can demap & get the bit.

∴ Am is the desired bit.

Note: for time being, let us not consider the effect of noise.

if 2nd term is Zero, Can we say the ISI is Zero.

Am, Al . . all are the levels corresponding to the Transmitted bit.

if b_l is the transmitted bits say

1 0 1 1 0 1 1 1 . . .

A_l a 0 a a 0 . . . ← which corresponding to the transmitted bits. So in A_l cannot be make zero.

So To make ISI = 0, Should concentrate on

⇒ term i.e if $p_3[(m-l)T_b] = 0$, for $m \neq l$ then ISI = 0

i.e

$$p_3[(m-l)T_b] = \begin{cases} 1; & m=l \\ 0; & m \neq l. \end{cases}$$

@ $m=l$ $p_3[(m-l)T_b] = p_3(0) = 1$ So that

then desired term $A_m p_3(0) = A_m \cdot (1)$

↳ demap to get the corresponding bit.

desired

$b_m \leftarrow A_m$ ← level.

$m, l \in Z$ (integer)

let say $(m-l) \in Z$ another integer

$m-l = n$ (integer).

$$P_3(nT_b) = \begin{cases} 1 ; & n=0 \\ 0 ; & \text{othe. } n \neq 0 . \end{cases}$$

Summary

$\Rightarrow$

$$x(t) * g_T(t) * c(t) * g_R(t) = P_3(t).$$

at $t = nT_b$

$$P_3(nT_b) = \begin{cases} 1 ; & n=0 \\ 0 ; & \text{othe} \end{cases}$$ $\Leftarrow$ This is the Time domain condition for Zero I.S.I.

Time and Frequency Domain Condition for Zero I.S.I.

Time domain condition.

$$x(t) * g_T(t) * C(t) * g_R(t) = P_3(t).$$

and $P_3(t)$ at $t = nT_b$ (or) $P_3(nT_b)$ $= \begin{cases} 1 ; & n=0 \\ 0 ; & n \neq 0 . \end{cases}$

Frequency domain cond^n :-

$$x(t) * g_T(t) * C(t) * g_R(t) = P_3(t)$$

$$\Big\updownarrow \text{F.T}$$

$$X(f) \cdot G_T(f) \cdot C(f) \cdot G_R(f) = P_3(f).$$

$G_T(f)$: Response of Transmitting filter

$C(f)$: channel

$G_R(f)$: Receiver filter response.

(11)

$x(t) \overset{in}{\to}$ i/p pulse (NRZ, RZ, Manchester pulse)

Frequency Domain Condition for Zero ISI :-

Cond^n:
$$\sum_{n=-\infty}^{\infty} P_3\left(f+\frac{n}{T_b}\right) = T_b$$

proof:- $P_3(f) \to$ frequency response of resultant pulse.

$$P_3(t) \xleftrightarrow{F.T} P_3(f)$$

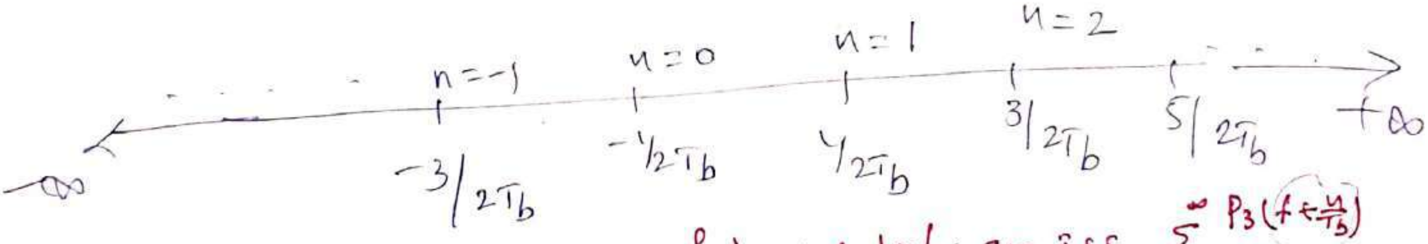
Inverse fourier transform

$$P_3(t) = \int_{-\infty}^{\infty} P_3(f)\, e^{j2\pi f t}\, df$$

$t = mT_b$

$$P_3(mT_b) = \underbrace{\int_{-\infty}^{\infty}}_{\to \text{replaced by one } \Sigma + \text{one integral.}} P_3(f)\, e^{j2\pi f m T_b}\, df$$

$$P_3(mT_b) = \sum_{n=-\infty}^{\infty} \int_{\frac{2n-1}{2T_b}}^{\frac{2n+1}{2T_b}} P_3(f)\, e^{j2\pi f m T_b}\, df$$



f domain cond^n for zero ISI $\sum_{n=-\infty}^{\infty} P_3\left(f+\frac{n}{T_b}\right) = T_b$

↑ Introduce

let $f = f' + \frac{n}{T_b} \Rightarrow f' = f - n/T_b$

$df = df'$.

L.L

$$f = \frac{2n-1}{2T_b} = \frac{\cancel{2}n}{\cancel{2}T_b} - \frac{1}{2T_b}.$$

$$f = \frac{n}{T_b} - \frac{1}{2T_b}$$

But $f' = f - n/T_b$

$$\Rightarrow f' = \cancel{\frac{n}{T_b}} - \frac{1}{2T_b} - \cancel{n/T_b}$$

$$\boxed{f' = -1/2T_b} \Rightarrow \text{new Lower Limit.}$$

Upper Limit (UL)

$$f = \frac{2n+1}{2T_b} = \frac{\cancel{2}n}{\cancel{2}T_b} + \frac{1}{2T_b}$$

$$f = \frac{n}{T_b} + \frac{1}{2T_b}$$

$$\Rightarrow f' = \cancel{\frac{n}{T_b}} + \frac{1}{2T_b} - \cancel{\frac{n}{T_b}} = + 1/2T_b$$

$$\boxed{f' = \frac{1}{2T_b}} \rightarrow \text{new upper Limit}$$

and $\boxed{df = df'}$.

$$f = f' + n/T_b.$$

$$P_3(mT_b) = \int_{-1/2T_b}^{1/2T_b} \boxed{\sum_{n=-\infty}^{\infty} P_3\left(f' + n/T_b\right)} \underline{e^{j2\pi\left(f' + n/T_b\right)\cdot mT_b}}\, df'$$

$\rightarrow = T_b$ cond^n of ISI

take exponential term

$$e^{j2\pi f' \cdot mT_b} \qquad e^{j2\pi n/\cancel{T_b} \cdot m\cancel{T_b}}$$

$$e^{j2\pi mn} = 1 \text{ (always).}$$

⑬

$f' \rightarrow f$ (change variable of integral).

$$\therefore P_3(mT_b) = \int_{-1/2T_b}^{1/2T_b} T_b \, e^{j2\pi f m T_b} \, df.$$

$$P_3(mT_b) = \int_{-1/2T_b}^{1/2T_b} \underbrace{T_b}_{\text{constant.}} e^{j2\pi f m T_b} \, df.$$

$$= T_b \int_{-1/2T_b}^{1/2T_b} e^{j2\pi f m T_b} \, df.$$

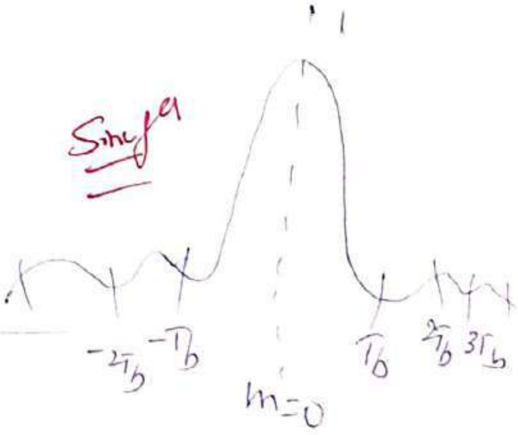
$$= \cancel{T_b} \cdot \left[\frac{e^{j2\pi f m T_b}}{j2\pi m \cancel{T_b}} \right]_{f=-1/2T_b}^{f=1/2T_b}$$

$$P_3(mT_b) = \frac{1}{(2j)\pi \cdot m} \left[e^{j2\pi \frac{1}{\cancel{2T_b}} \cdot m\cancel{T_b}} - e^{-j\cancel{2}\pi \cdot \frac{1}{\cancel{2T_b}} \cdot m\cancel{T_b}} \right]$$

$$= \frac{1}{\pi m} \left[\frac{e^{j\pi m} - e^{-j\pi m}}{2j} \right]$$

$$= \frac{\sin \pi m}{(\pi m)}$$

$$\boxed{P_3(mT_b) = \text{Sinc}(m)}$$



$$= \begin{cases} 1 \; ; & m = 0 \\ 0. \; ; & m \neq 0 \end{cases}$$ (n)

(14) Time domain condn for Zero ISI. ⇐

Frequency domain cond^n for Zero ISI.

$$\sum_{n=-\infty}^{\infty} P_3(f + n/T_b) = T_b.$$

To understand this eq^n

$\underline{n=0}$ $\quad n=1$ ✓ $\quad P_3(f)$ $\qquad\qquad n=1 \quad P_3(f + 1/T_b)$

$n=-1$ ✓

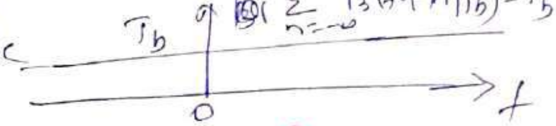
$P_3(f - 1/T_b)$

i.e.

$$\Rightarrow -\infty \ldots + P_3(f - 1/T_b) + P_3(f) + P_3(f + 1/T_b) - \ldots \infty$$

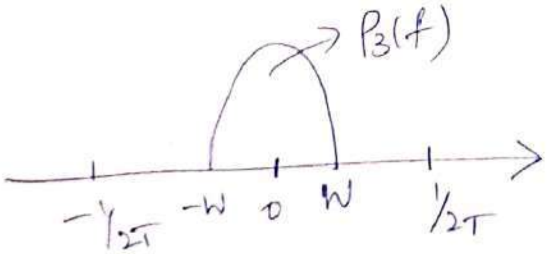
$$= T_b$$

$-\infty$ to $+\infty$ if we add all the replica of P_3 to get Constant i.e T_b. i.e



What should be $P_3(t)$ so that ISI is Zero?

Case ① $W < 1/2T$

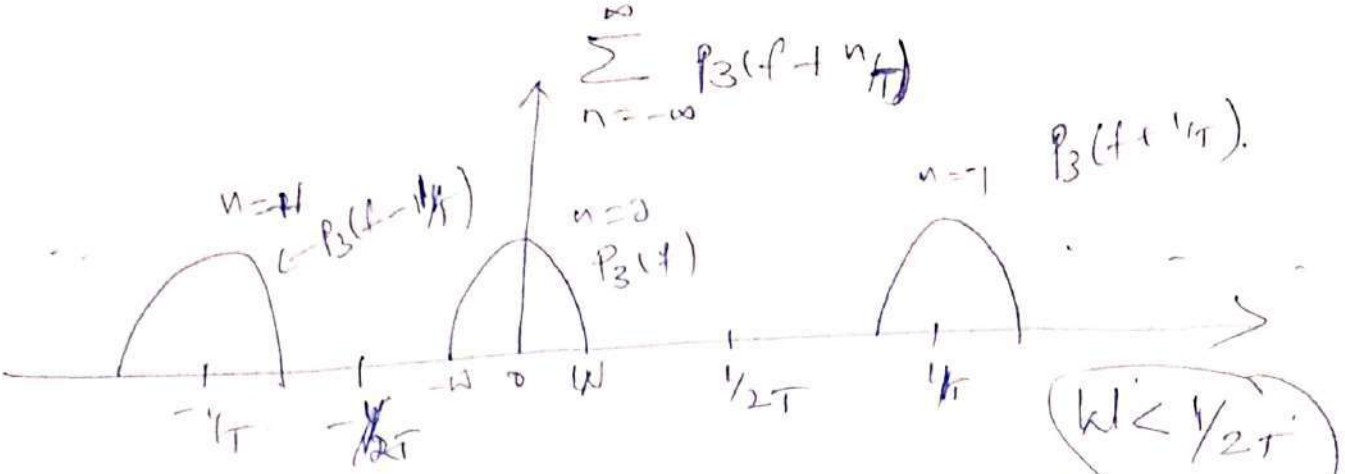


$f_s = 2W'$ → Max. frequency

$\frac{1}{T} = 2W'$

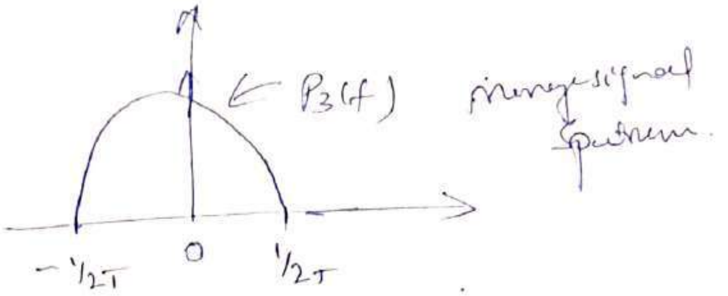
$\frac{1}{2T} = W.$

Zero ISI :- The addition of all Spectrum replicas should be constant.

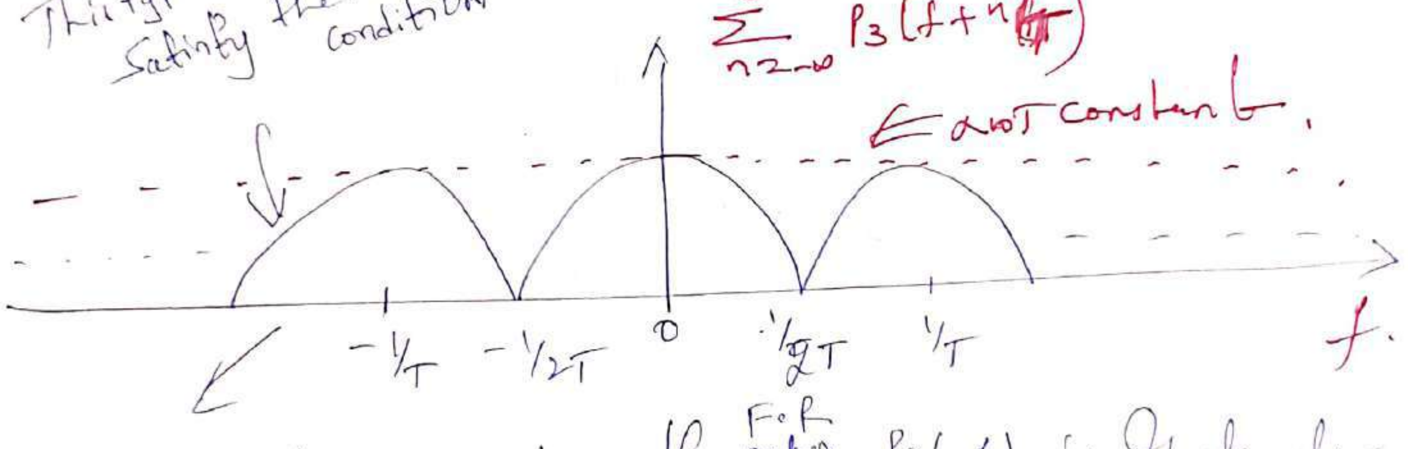


With any possible pulse, the constant Magnitude Spectrum Condition cannot be achieved.

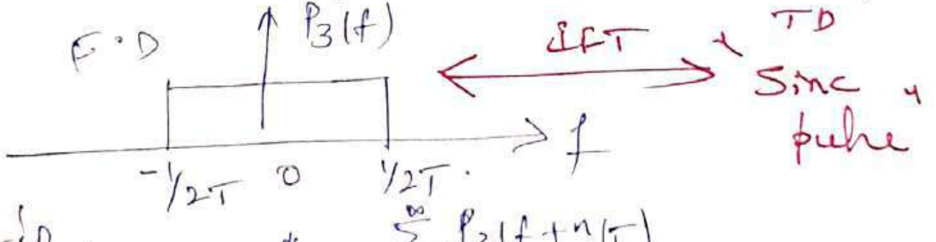
Case ii $W = \frac{1}{2T}$

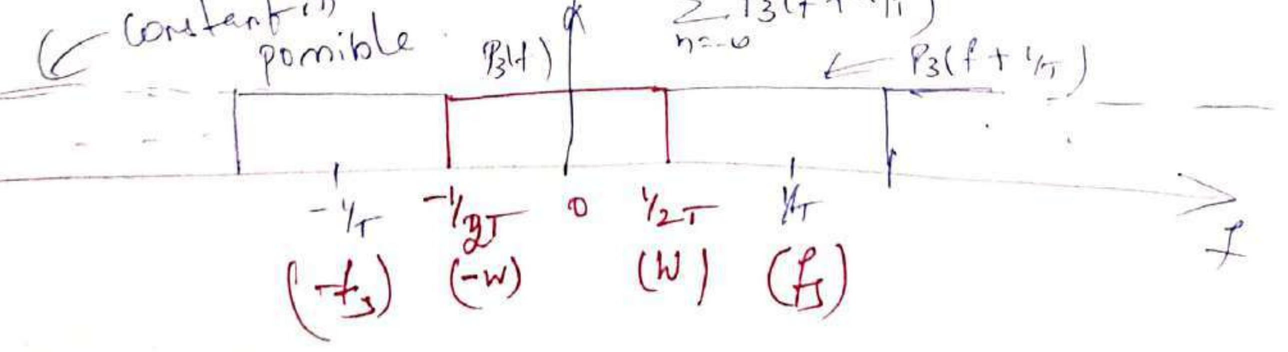


This type of pulse may not satisfy the required condition



Suppose if we consider the For pulse P3(f) is Rectangular





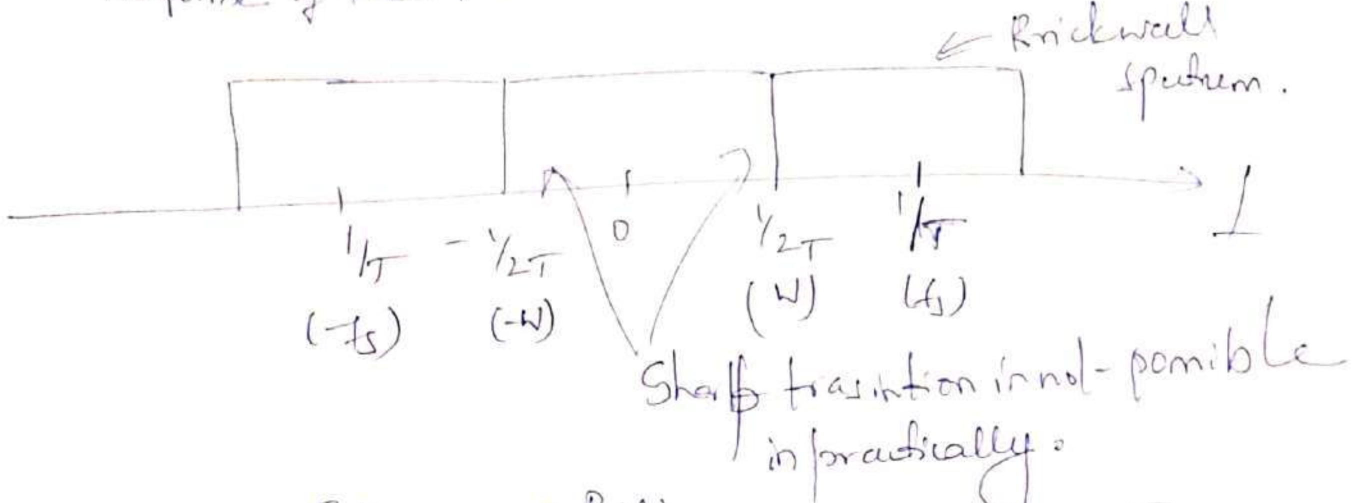
if we use $P_3(f)$ F.R as rectangular shape

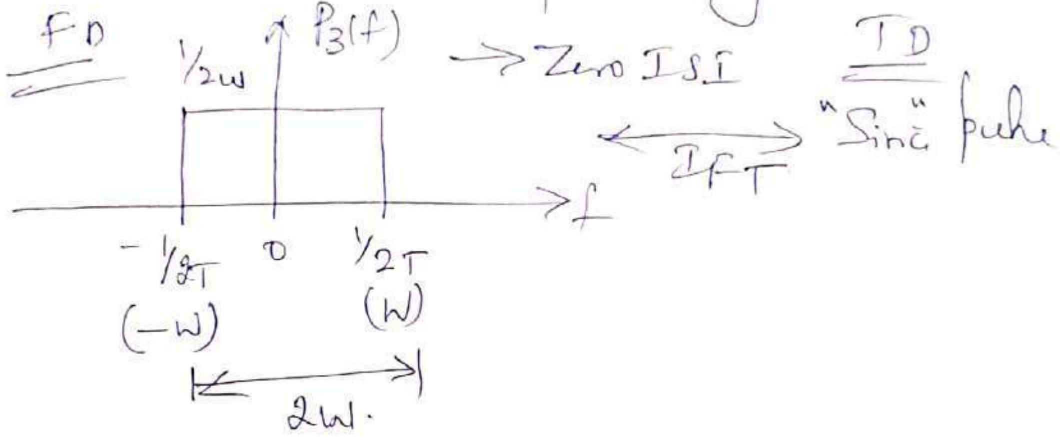
$\sum_{n=-\infty}^{\infty} P_3(f+n/T) = T_b$ can be achieved . flat results in

Zero ISI.

(i) Satisfy the required condition.

(ii) This is the Ideal solution for Zero ISI [Magnitude response of the pulse should be a brick wall].





$$p_3(t) = \int_{-W}^{W} P_3(f)\, e^{j2\pi f t}\, df .$$

$$= \int_{-1/2T}^{1/2T} \frac{1}{2W}\, e^{j2\pi f t}\, df$$

$$= \frac{1}{2W}\left[\frac{e^{j2\pi f t}}{j2\pi t}\right]_{-W}^{W}$$

⑰

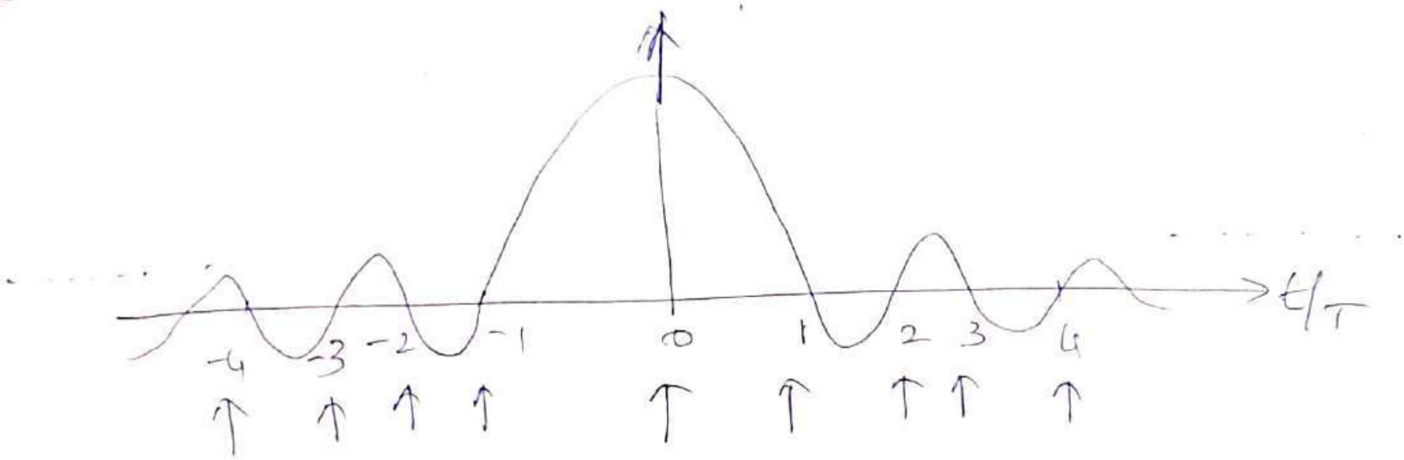
$$= \frac{1}{2W(2j)(\pi t)} \left[e^{j2\pi Wt} - e^{-j2\pi Wt} \right]$$

$$= \frac{\sin(2\pi Wt)}{2\pi Wt}$$

$$\boxed{P_3(t) = \text{Sinc}(2Wt)}$$ (or) Sinc(2πW

→ pulse which produces Zero ISI is a Sinc pulse.

Carefi. $W > 1/2T$



Reason for Not Realizable — Sampling instant

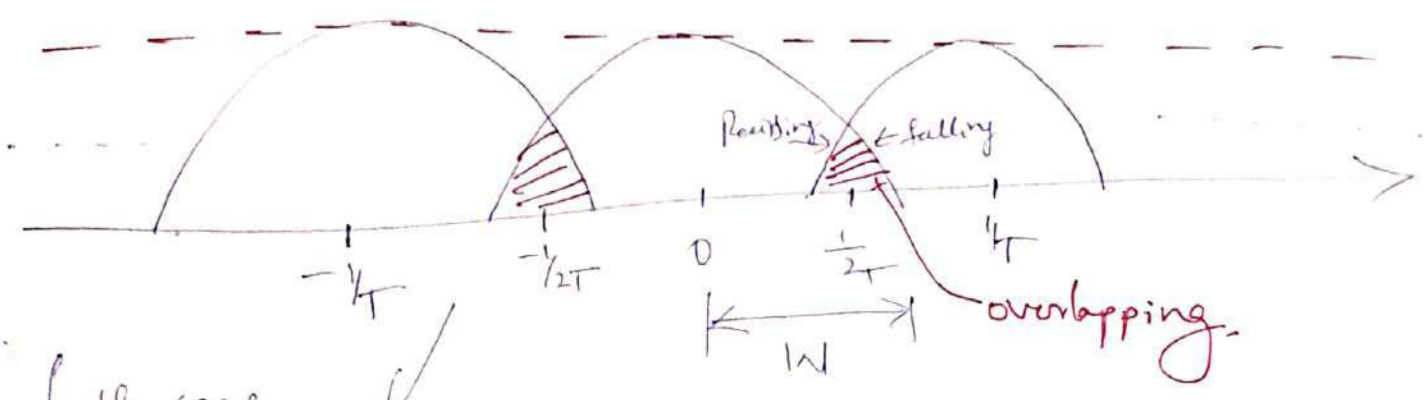
In T-D

⇒ infinite support (time domain) (duration) ⇒ Small drift will introduce interference.

In F-D

⇒ Brick wall Spectrum in practically unrealizable.

Case 3 $W > 1/2T$



for this case

Many possible pulses to achieve Zero ISI.

- Abrupt transitions at ~~also~~ ±W (in frequency domain) makes Sinc pulse not suited to achieve Zero ISI.

- To make smoother transitions at ±W, the sinc pulse is Multiplied by a Sinc Correlation factor → "Raised Cosine pulse".

$$P_3(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)} \cdot \left\{ \frac{\cos\left(\frac{\pi \alpha t}{T}\right)}{1 - \frac{4\alpha^2 t^2}{T^2}} \right\}$$

→ Sinc Correlation term.

mathematical eqn for raised cosine fn.

Sinc fn

Control the tail of Sinc fn.

'α' - correction factor known as roll off factor.

Note:- Raised cosine pulse is the modified Sinc fn.

(19)

$\alpha \rightarrow$ roll off factor

when $\alpha = 0$.

$$B(t) = \left[\frac{\sin(\pi t/T)}{(\pi t/T)}\right] = \text{Sinc}(t/T).$$

Note:-

i. Usually 'α' is not set to Zero (0). This may again require brick wall spectrum.

ii. 'α' is selected between 0 and 1.

iii. 'α' decides the bandwidth requirement.

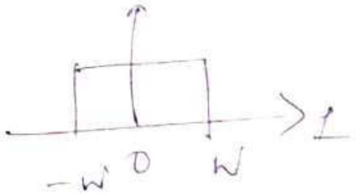
$\alpha \rightarrow$ gets smoother response (no abtrupt transition). but disadvantage is $\alpha \uparrow \Rightarrow$ BW requirement $\uparrow$.

Excess BW :- Bandwidth occupied beyond the nyquist frequency W (or) $1/2T$.

α - roll off factor.

$$\boxed{W_0 = W(1+\alpha)}$$

$\alpha = 0, \Rightarrow W_0 = W$.



$\alpha = 0.5 \Rightarrow W_0 = 1.5W$. (50% Excess BW)

$\alpha = 1 \Rightarrow W_0 = 2W$ (100% Excessive BW).

$\Rightarrow \alpha \uparrow \Rightarrow BW \uparrow$ but results Smoother response.

Question: Consider a T_1 Carrier System which is used to Multiplex (Based on pcm) 24 independent voice input Signals based on an 8-bit PCM codeword. The bit duration of the resultant TDM signal is 0.647 μsec. Find the BW requirement of the System when RC (Raised cosine) is used with different rolloff factor $\alpha = 0,\ 0.3,\ 0.5$ and 1.

Soln:

$T_b = 0.647\,\mu sec$

Nyquist Criteria

$$f_s = 2W \Rightarrow T_b = \frac{1}{2W}$$

Nyquist Bandwidth $$\Rightarrow W = \frac{1}{2T_b} = \frac{1}{2 \times 0.647 \times 10^{-6}}$$

$$\boxed{W = 772.797\ kHz}$$

Exam Bandwidth requirement for R.C.

$\alpha = 0,$ $$W_0 = W(1+\alpha)$$

$$\Rightarrow W_0 = W = \underline{772.797\ kHz}$$

$\alpha = 0.3$ $$W_0 = 1.3W = \underline{1004.637\ kHz}$$

$\alpha = 0.5$ $$W_0 = 1.5W = \underline{1159.196\ kHz}$$

$\alpha = 1$ $$W_0 = 2W$$

$$= \underline{1543.594\ kHz}$$

Raised Cosine pulses -

In T·D we represented the Raised Cosine pulse with the equation.

$$p_3(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)} \cdot \left\{ \frac{\cos\left[\frac{\pi\alpha t}{T}\right]}{1 - \frac{4\alpha^2 t^2}{T^2}} \right\} = \text{Sinc term} \times \text{Sinc correction term.}$$

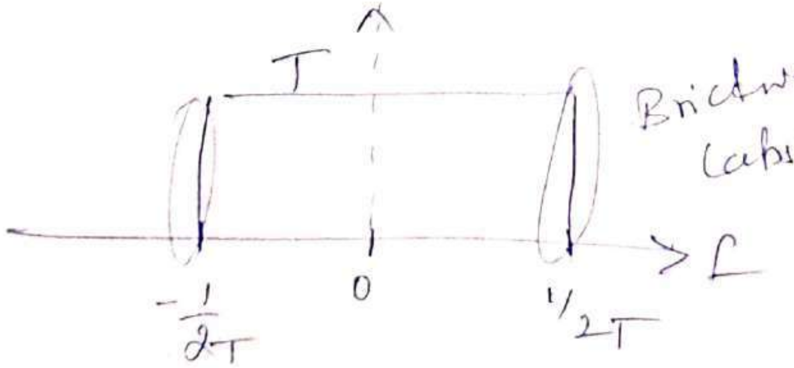
$$\updownarrow \text{F.T}$$

F.R of R.C pulse

$$P_3(f) = \begin{cases} T\ ; & |f| \le \frac{1-\alpha}{2T} \\ \frac{T}{2}\left\{1 + \cos\left[\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right]\right\}; & \frac{1-\alpha}{2T} < |f| \le \frac{1+\alpha}{2T} \\ 0\ ; & |f| > \frac{1+\alpha}{2T} \end{cases}$$

Case i: $\alpha = 0.$

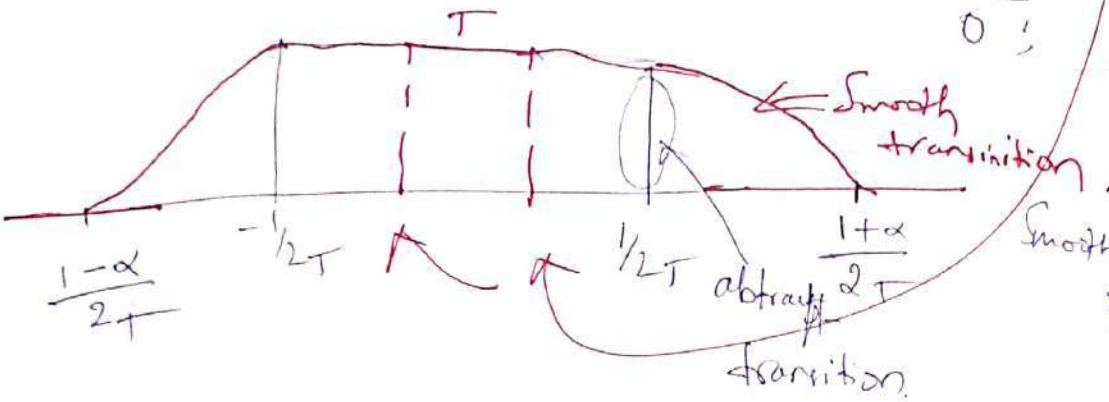
$$P_3(f) = \begin{cases} T\ ; & |f| \le 1/2T \\ 0\ ; & |f| \ge 1/2T. \end{cases}$$



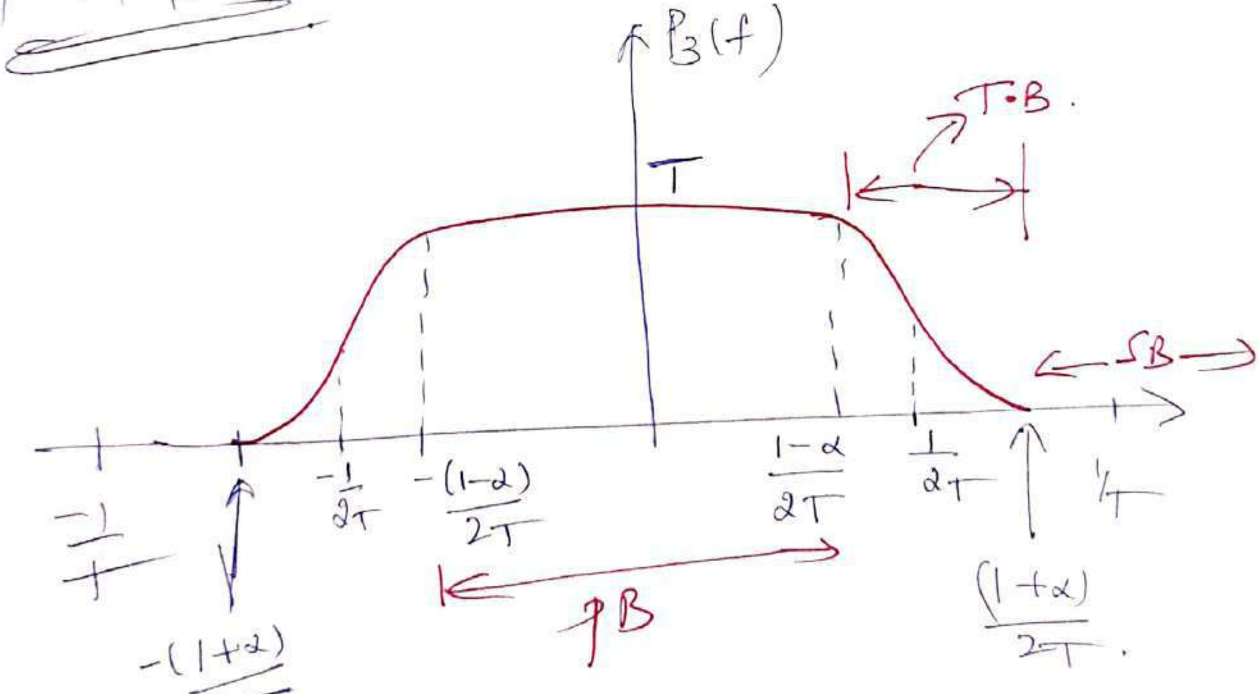
Brickwall F.R
[abrupt transition]

Case iii $0 < \alpha < 1$

$$P_3(f) = \begin{cases} T ; & |f| \le \frac{1}{2T} - \frac{\alpha}{2T} \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right]\right\} ; & \frac{1-\alpha}{2T} < |f| < \frac{1+\alpha}{2T} \\ 0 ; & |f| > \left(\frac{1+\alpha}{2T}\right). \end{cases}$$



plot of $P_3(f)$

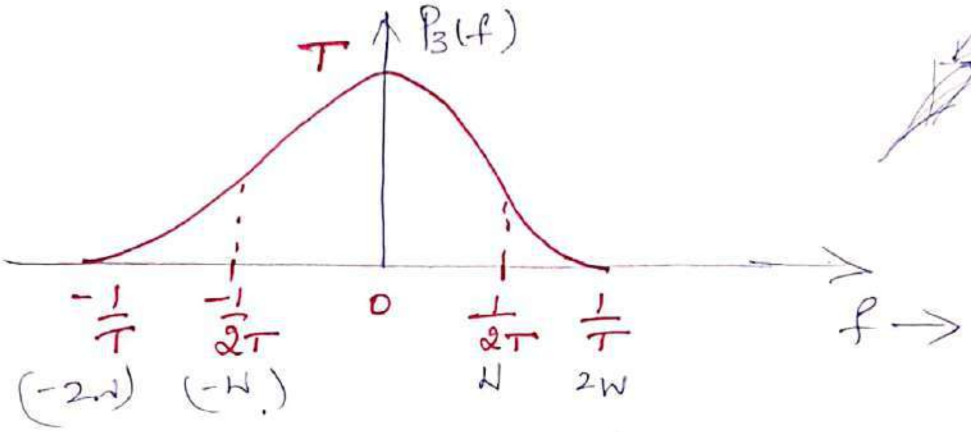


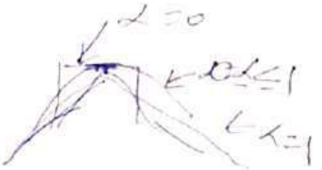
$$P_3(f) = \begin{cases} T ; & |f| \le \frac{1}{2T} - \frac{\alpha}{2T} \\ T/2\left\{1+\cos\left[\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right]\right\} ; & \frac{1}{2T} - \frac{\alpha}{2T} \le |f| \le \frac{1}{2T} + \frac{\alpha}{2T} \\ 0 ; & |f| > \frac{1}{2T} + \frac{\alpha}{2T}. \end{cases}$$

(23)

Case iii $\alpha = 1$

$$P_3(f) = \begin{cases} 0 ; & |f| \leq 0 \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right]\right\} ; & 0 \leq |f| \leq 1/T \\ 0 ; & |f| > 1/T \end{cases}$$





Significance of α :-

i. When α increases, the filter becomes more practically realizable.

ii. α ↑ ⇒ reduces ISI.

iii. α ↑ ⇒ BW requirement will also increases.

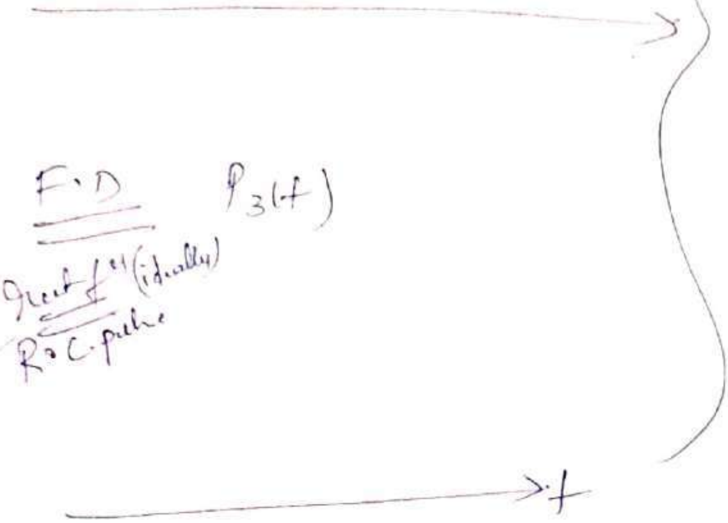
Note:- 'α' is chosen to be 0.3 for Many wireless applications.

T D $P_{13}(t)$

Sinc.

F.D $P_3(f)$

Great fall (ideally)
R.C. pulse



Comparision :

$P_3(t) \propto 1/t^3$, $\alpha > 0$

Tail of $P_3(t)$ decays as $1/t^3$.

Sinc → Smaller sampling offset will introduce more ISI.

RC ⇒ tail have very small power which leads to Smaller ISI.

Summary

i. $W < \frac{1}{2T}$ ⇒ Any type of pulse cannot achieve Zero ISI.

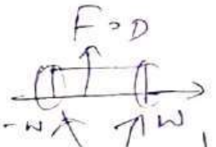
ii) $W = \frac{1}{2T}$ ⇒

Zero ISI possible with one type of signal i.e "Sinc".

T.D

Sinc
↓
$-\infty$ to $+\infty$

F.D



abrupt transition

ideal solu.

A filter without Transition Band practically not realizable.

iii) $W > \frac{1}{2T}$

Many pulse possible → "R.C"
Practically realizable pulses
One such pulse is Raised cosine pulse.

= Sinc × Sinc
Sinc term. correction term

$0 < \alpha < 1$

α → roll-off factor

$W_0 = W(1+\alpha)$

$\alpha = 0, W_0 = W$
$\alpha = 1, W_0 = 2W.$

$\alpha \uparrow \Rightarrow$ filter can be realizable practically.

obsP

Trade off b|w BW. BW & ISI

In comm^n system BW is costly resource.

Is their Any solu so that we can achieve trade off B|w BW and ISI.

Trade-off between ISI and BW i.e Correlative Coding

Based on:

1. Concept of controlled ISI.
2. Condition for zero ISI is relaxed here.

one such correlative coding is Duo-binary coding.

Duobinary Coding:

Time domain conditions for Zero ISI.

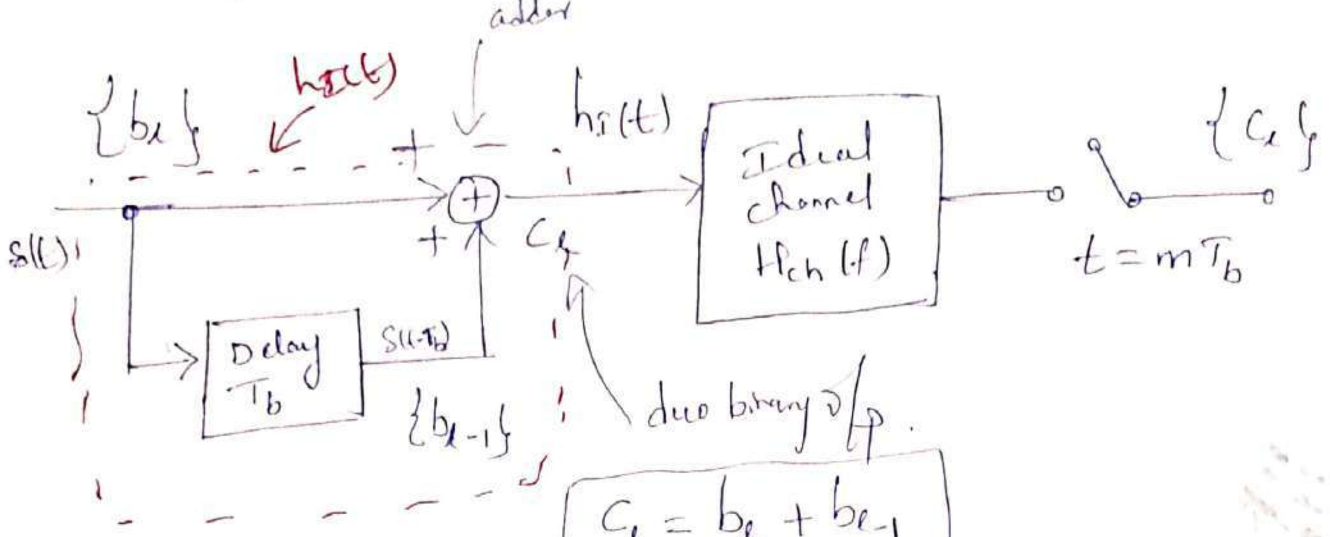
$$P_3(nT) = \begin{cases} 1 ; & n = 0 \\ 0 ; & n \neq 0 \end{cases}$$

Here this cond^n is now relaxed.

$$P_3(nT) = \begin{cases} 1 ; & n = 0, 1 \\ 0 ; & else. \end{cases}$$

allowing ISI b/w successive symbols not all the symbols.

Block diagram :-



$$\boxed{C_l = b_l + b_{l-1}}$$

* This o/p is passed through a filter, it will allow only the frequency component bel..

$$x(t) \longrightarrow \boxed{\begin{matrix}LTI\\ h(t)\end{matrix}} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

$$x(t) = \delta(t)$$

$$y(t) = x(t) * h(t) = \delta(t) * h(t)$$
$$= \underline{h(t)}$$

If we excite the system by impulse we will get o/p as a system response.

o/p of the system is

$$h_I(t) = \delta(t) + \delta(t - T_b)$$

The transfer function

$$H_I(f) = 1 + e^{-j2\pi f T_b}\cdot 1$$

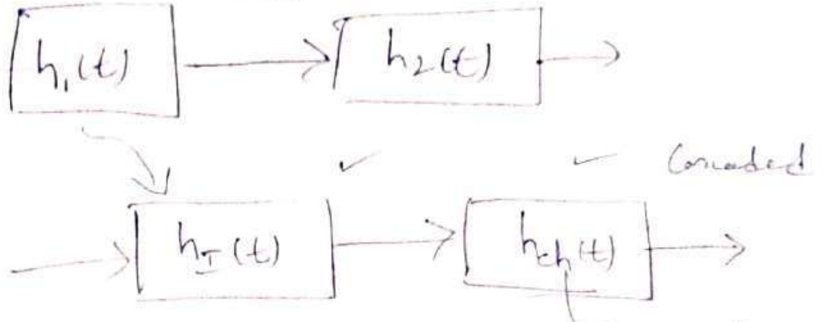
$$= e^{j\pi f T_b}\cdot e^{-j\pi f T_b} + e^{-j2\pi f T_b}$$

$$= e^{-j\pi f T_b}\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] \times \frac{2}{2}$$

(27)

$$\boxed{H_I(f) = 2e^{-j\pi f T_b}\cos(\pi f T_b).}$$

Two [LTI] systems are cascaded.



> used. restrict the other frequencies

Overall o/p in F.D

$$H_I(f)\cdot H_{ch}(f)$$

The ideal response of channel filter is

$$H_{ch}(f) = \begin{cases} 1; & |f| \le W \\ 0; & \text{else.} \end{cases}$$

Overall response in — The T.F of duobinary conversion filter.

$$H(f) = H_I(f)\cdot H_{ch}(f)$$

$$= \begin{cases} 2e^{-j\pi f T_b}\cos(\pi f T_b) & ;\ |f| \le W \\ 0 & ;\ \text{else.} \end{cases}$$

Magnitude response.

$$|H(f)| = 2\,|\cos(\pi f T_b)|$$

but $f = 0$.

$$|H(0)| = 2\,[1] = 2.$$

$f = R_b/2$, ie Nyquist BW $W = 1/2T_b = \underline{R_b/2}$.

(28)

$$\left|H\left(f_b/2\right)\right| = 2\cos\left[\pi \cdot f_b/2 \cdot T_b\right]$$

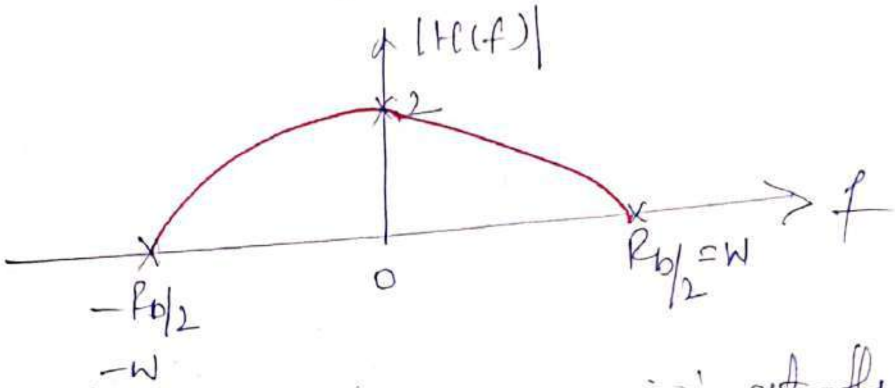
$$= 2\cancelto{0}{\cos(\pi/2)}$$

$$= 0.$$

note:-
$T_b f_b = 1$.
W ranges from
$-f_b/2$ to $+f_b/2$

$f = -f_b/2$

$$= 2\cos\left(-\pi \frac{f_b \cdot T_b}{2}\right) = 0.$$



Filter is self realizable. BW is 'W' at the cost of ISI, but ISI is in a controlled manner.

One such filter is duo-binary filter.

Advantages of Duo-binary

i. Controlled ISI
ii. Conserve the BW.
iii. Realizable Spectrum.

Drawback:

i. it has DC component.
i.e @ $f=0$, $H(0)=2$.

To make dc component to be zero we go for Modified duo-binary coding:-

㉙

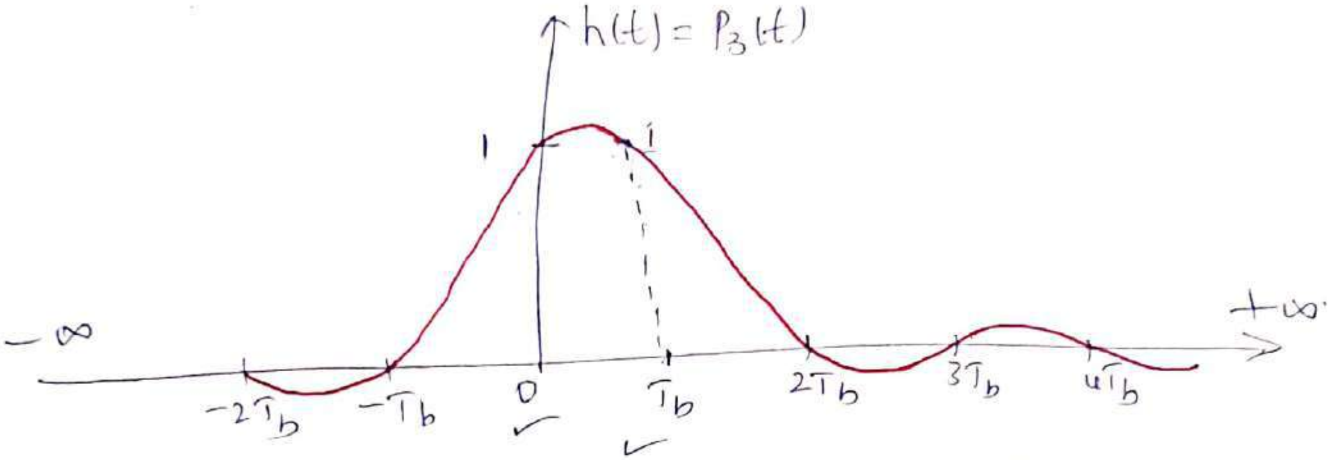
The impulse response of duo-binary filter

$$h(t) = \frac{\sin(\pi t/T_b)}{\underset{\uparrow \text{ sinc}}{(\pi t/T_b)}} + \frac{\sin \pi(t-T_b)/T_b}{\underset{\nwarrow \text{ delayed version of sinc}}{\pi(t-T_b)/T_b}}$$

$$P_2(t) = h(t) = \text{Sinc}\left(\frac{t}{T_b}\right) + \text{Sinc}\left(\frac{t-T_b}{T_b}\right).$$

i.e

$$P_3(nT) = \begin{cases} 1, & n = \overset{\nearrow 0}{0}, \; 1 \rightarrow T_b . \\ 0; & \text{ow} \end{cases}$$



ie @ $t=0,\ t=T_b \Rightarrow P_3(t)=1$

$=0$; elsewhere.

Problems related to duo-binary signaling :-

Example. if b_l ← i/p sequence

duo-binary output $\boxed{C_l = b_l + b_{l-1}}$.

Step 1 Convert $b_l \longrightarrow$ Polar $\begin{cases} 0 \to -1 \\ 1 \to +1 \end{cases}$ b_l^p.

Step 2 $\boxed{C_l = b_l^p + b_{l-1}^p}$ $\xrightarrow[\text{through channel}]{\text{transmit}}$ Received.

Use this two formulae.

Estimate of $\boxed{\hat{b}_{l_p}^p = C_l - \hat{b}_{l-1}^p}$

Example 1

No precoding (No Error).

if we go for duo-binary we are allowing ISI but the ISI pattern are known either 0, +2, −2 happen

Binary i/p $\{b_l\}$	0	0	1	1	0	1
polar o/p $\{b_l^p\}$	-1 b_l^{p-1}	-1 b_l^p	1	1	-1	1
Duo binary o/p $C_l = b_l^p + b_{l-1}^p$		-2	0	2	0	0
C_l		-2	0	2	0	0
Received sequence $\{C_l\}$ assume (without Error) $\hat{b}_l^p = C_l - \hat{b}_{l-1}^p$	-1	-1	$+1$	1	-1	1
polar to binary $\{\hat{b}_l\}$	0	0	1	1	0	1

$-2-(-1)$
$= -1$

b_l^{p-1} This bit is communicated to the receiver.

Same Transmitted Signal. even though ISI occurring we can able to get back b_l.

(31)

Ex2:- Without precoding (with Error).

(Same transmitted Sequence Ex1).

$\{c_l\}$		-2	0	2	0	0
$\{r_l\}$		-2	0	$\boxed{0}$	0	0
$\hat{b}_l^p$	⊖1	-1	1	-1	1	-1
$\hat{b}_l$	0	0	1	0 ✓	1 ✓	0 ✓

→ Error [due while transmitting channel introduces some error]

this bit is communicated to the Receiver

→ Error propogation.

Estimate of polar is

$$\boxed{\hat{b}_l^p = r_l - b_{l-1}^p}$$

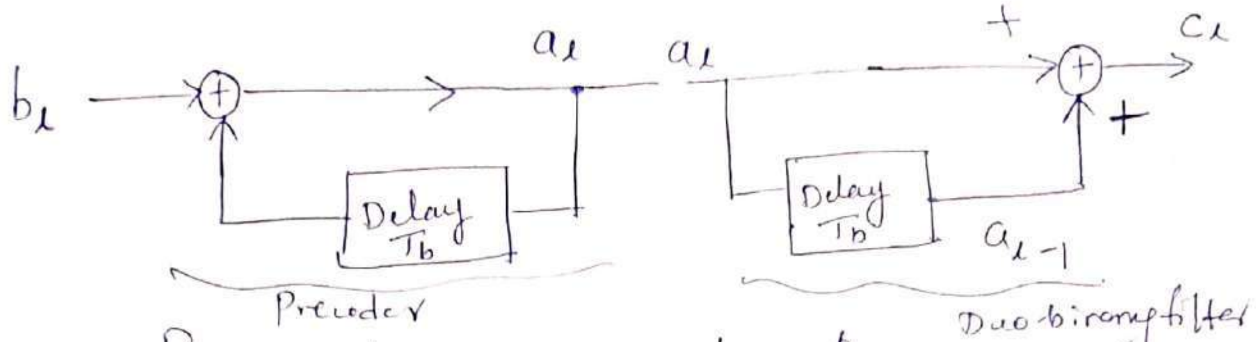
[Compare the Txted & detected bits. obs:- Error @ one bit leads to Error propogation. [ie Error occured that get propogated].

* In order to avoid Error propogation, Employ a precoder at the Transmitter side.

The job the precoder, it will convert the propogation Error to a localized Error.

Ex-3 With precoding [no Error].

Modified diagram, after including precoder.



Precoder output $a_l = b_l \oplus a_{l-1}$

Duo-binary filter o/p

$$C_l = a_l + a_{l-1}$$

Example.

XOR

$\{b_l\}$		0	0	1	1	0	1
precoder o/p $\{a_l\}$ $a_l = b_l \oplus a_{l-1}$	assume $\boxed{1}$	1	1	0	1	1	0
polar (a_l^p)	1	1	1	−1	1	1	−1
Duobinary o/p $\{c_l\} = a^p_l + a^p_{l-1}$		2	2	0	0	2	0

Detection Rule :-

$$b_l = \begin{cases} 0 ; & c_l = \pm 2 \\ 1 ; & c_l = 0 . \end{cases}$$

Duobinary o/p $\{c_l\}$	2	2	0	0	2	0
Received Sequence $\{c_l\}$	2	2	0	0	2	0
$\hat{b}_l$	0	0	1	1	0	1.

assume that NO Error

Ex:-4 With precoding [With Error].
(Consider the same duo binary o/p given in Ex:3).

Duobinary o/p $\{c_l\}$	2	2	0	$\boxed{0}$	2	0
Received Sequence $\{r_L\}$	2	2	0	$\boxed{2}$	2	0
Detected bit $\{\hat{b}_l\}$	0	0	1	$\boxed{0}$	0	1.

→ 1 bit Error.

Note:- precoding makes propogation Error to Localized Error. and it does Eliminates Error but it avoids propogation of Error.

34

Summary:-

Duo-binary:-

* it is Controlled ISI.
* without precoding → Error propogation occurs.
* With precoding → Error propogation does not occurs. [i.e it becomes Localized error].

drawback:

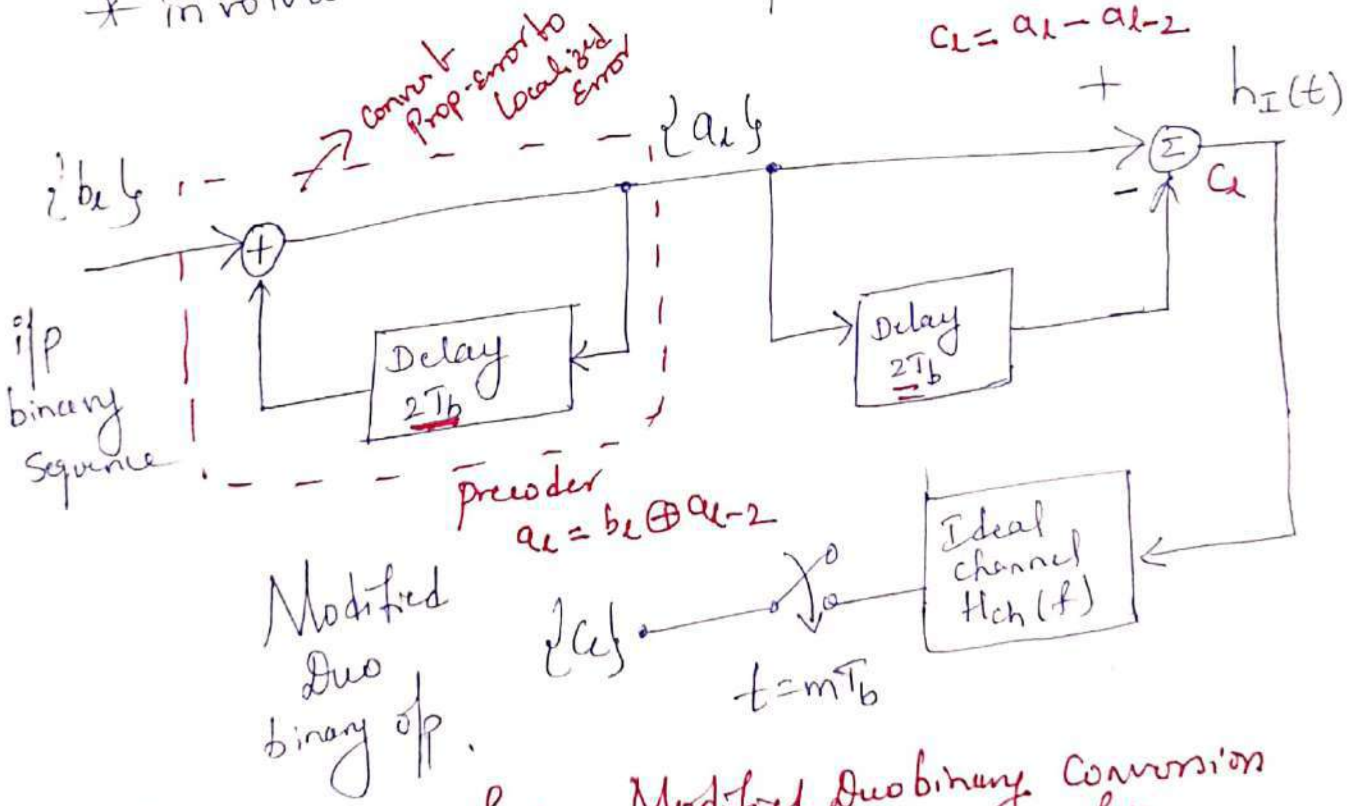
To Avoid dc component in duo-binary coding we go for modified duo-binary coding.

Modified duobinary Signalling

* Duobinary Suffers by DC component. [Cannot use for multistage, Transformer etc].

here also

* issue Controlled ISI.
* involves a correlation span of two binary digits.



fig!- Modified Duobinary Conversion filter.

Precoder output $a_k = b_k \oplus a_{k-2}$.

Precoder o/p given as input to modified duo binary filter. and its o/p is

$$c_k = a_k - a_{k-2}$$

The condition for Controlled ISI :-

$$P_3(nT_b) = h(nT_b) = \begin{cases} 1 ; & n=0 \\ -1 ; & n=2 \\ 0 ; & \text{else} \end{cases}$$

Impulse response is :-

$$P_3(t) = h(t) = \frac{\sin(\pi t/T_b)}{(\pi t/T_b)} \cdot \frac{\sin[\pi[t-2T_b]/T_b}{\pi(t-2T_b)/T_b}$$

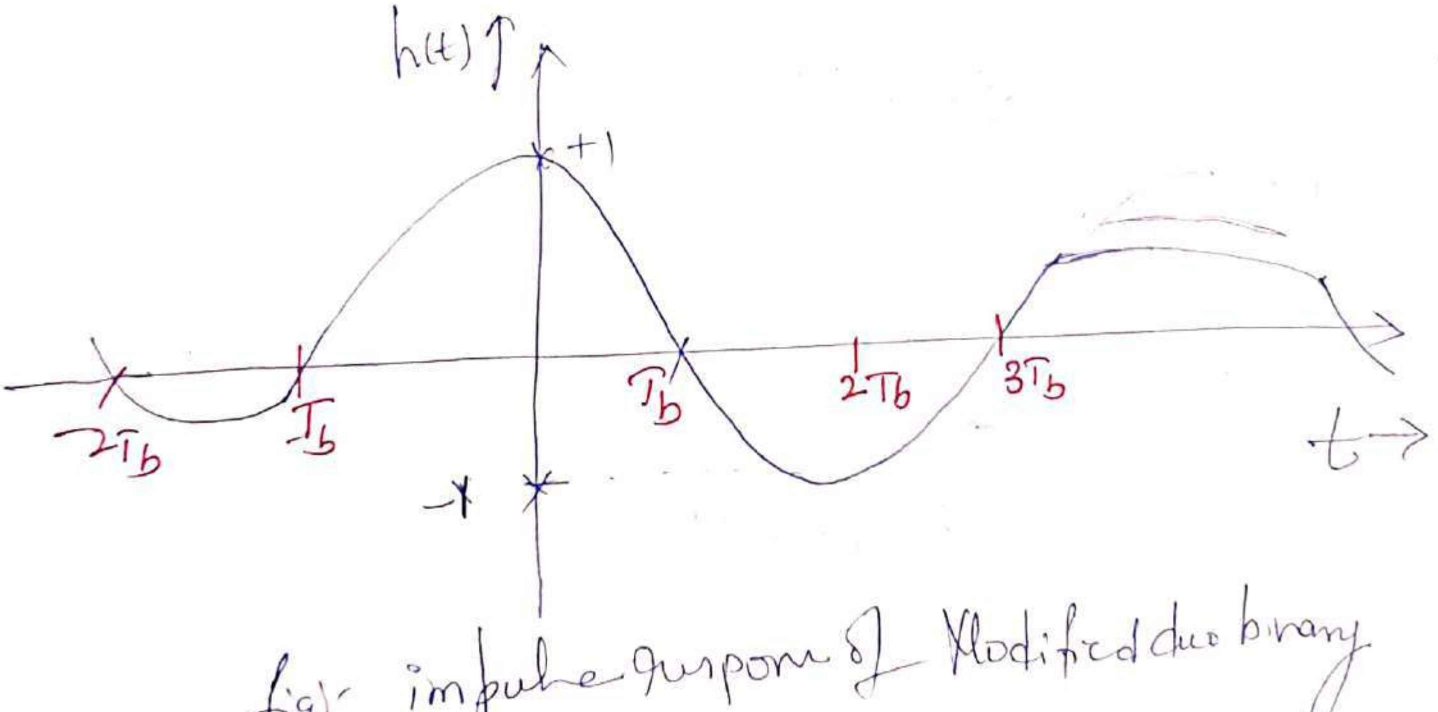
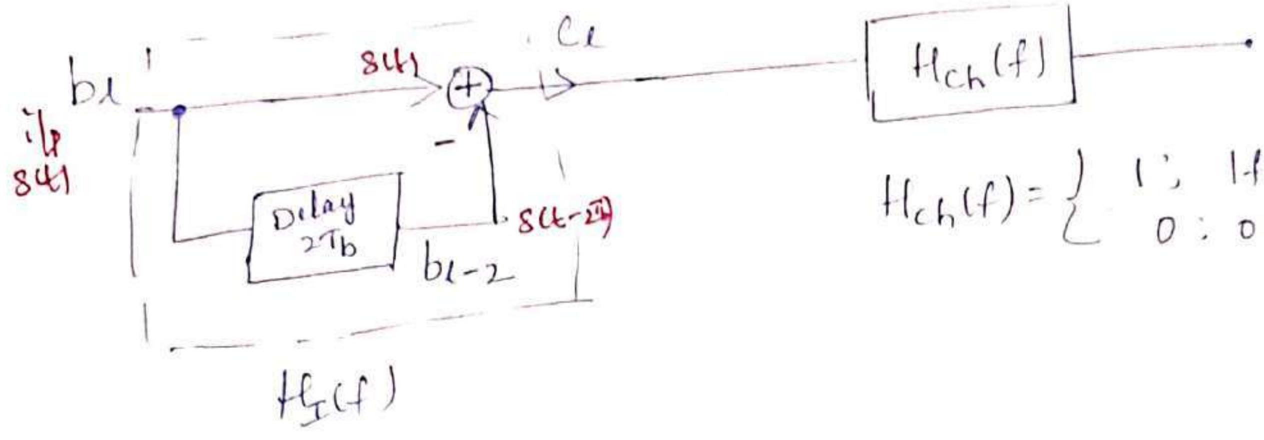


fig:- impulse response of Modified duo binary filter.

ⓓ 36

Frequency response :-

$c_l = a_l +$



$$H_{ch}(f) = \begin{cases} 1 ; & |f| \leq W \\ 0 : & o.w. \end{cases}$$

$$H(f) = H_I(f) \cdot H_{ch}(f)$$

$$h_I(t) = \delta(t) - \delta(t-2T_b)$$

$$\Updownarrow \text{F.T}$$

$$H_I(f) = 1 - e^{-j2\pi f(2T_b)}$$

$$= 1 - e^{-j4\pi f T_b}$$

$$= e^{j2\pi f T_b} \cdot e^{-j2\pi f T_b} - e^{-j4\pi f T_b}$$

$$= e^{-j2\pi f T_b}\left[e^{j2\pi f T_b} - e^{-j2\pi f T_b}\right] \times \frac{2j}{2j}$$

$$\boxed{H_I(f) = 2j\, e^{-j2\pi f T_b} \cdot \sin(2\pi f T_b)}$$

$$\boxed{H(f) = H_I(f) \cdot H_{ch}(f)}$$

$$\rightarrow H_{ch}(f) = \begin{cases} 1 ; & |f| \leq W \\ 0 ; & o.w \end{cases}$$

$$H(f) = \begin{cases} 2j\, e^{-j2\pi f T_b} \sin(2\pi f T_b) ; & |f| \leq R_b/2 \\ 0 ; & \text{else} \end{cases}$$

ⓔ 37

The ideal channel response is

$$H_{ch}(f) = \begin{cases} 1; & |f| \le R_b/2 . \\ 0; & else . \end{cases}$$

To check dc component present (or) not ?.

The Magnitude response.

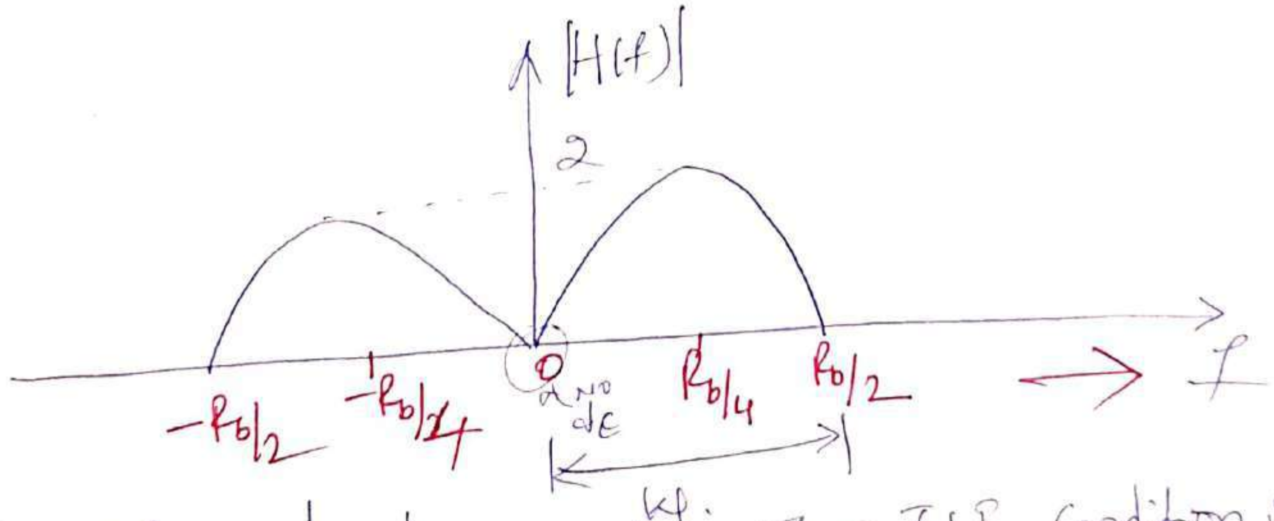
$$|H(f)| = \begin{cases} 2|\sin(2\pi f T_b)| & ; |f| \le R_b/2 \\ 0 & ; else . \end{cases}$$

$f=0$. $\sin(0)=0$ ∴ $\boxed{|H(f)|=0}$ → No dc component.

$R_b T_b = 0$

$f = R_b/2$ $$|H(R_b/2)| = \begin{cases} 2 \cancel{\sin}(2\pi \cdot R_b/2 \cdot T_b) \xrightarrow{} 0 ; = 0 . \end{cases}$$

$f = -R_b/2$ $|H(-R_b/2)| = 0$



For modified duo binary, the Zero ISI condition is slightly violated to successfully eliminate the dc component.

ⓐ f 38

Summary

Correlative coding

both comes under

Duo binary

Modified Duobinary {

* Zero ISI $\rightarrow W = \frac{1}{2T} \rightarrow$ ideal case. $\rightarrow$ Sinc but abrupt transition @ $\pm W$ & infinite BW.

$\downarrow$ practically go for R.C i.e

$RC = \text{Sinc} \times \text{Sinc correction term.}$

α — rolloff factor

$0 < \alpha < 1$

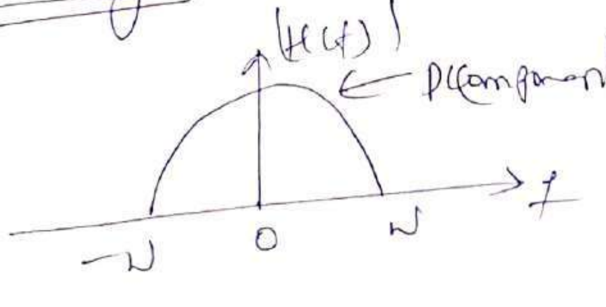
$\alpha \uparrow \rightarrow$ more & more realizable filter But BW$\uparrow$ also (increases)

$\alpha = 1 \Rightarrow \boxed{BW_0 = 2W}$ BW double

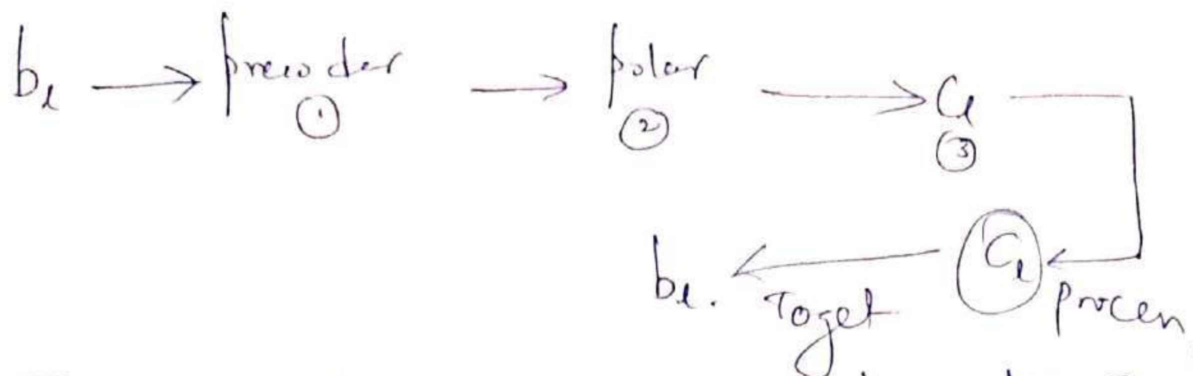
∴ Extent for correlative coding.

Concept of Controlled ISI introduced.

In Duobinary :— controlled ISI



To avoid dc component Modified duobinary filter used.



In Modified duo-binary, involves a correlation span of two binary digits.

The precoder o/p $a_l = b_l \oplus a_{l-2}$

Modified duo binary o/p $C_l = a_l - a_{l-2}$

Here no. of initial bits is two.

Detection Rule at the Receiver

$$\hat{b}_l = \begin{cases} 1; & |C_l| > 1 \\ 0; & |C_l| < 1 \end{cases}$$

fr 40.

Example-5 Using Modified duobinary.

$\{b_l\}$			1	0	1	1	0
precoder o/p $\{a_l\}$	$\boxed{0\ 0}$ initial.		1	0	0	1	0
polar o/p $\{a_l^p\}$	-1	-1	1	-1	-1	1	-1
$\{d_l^p\}$	-1	-1	1	-1	-1	1	-1
$\{c_l\} = a_l^p - a_{l-2}^p$			2	0	-2	2	0

$a_l = b_l \oplus a_{l-2}$

$c_l = a_l^p - a_{l-2}^p$

a_{l-2} a_l

decision rule

b-4

1 0 1 1 0 ✓

↑

Dl

ul.

&

Application,

Generalized form of correlative coding

i.e. general block for both duobinary and Modified duo-binary coding.

bcz some applications we may not consider of d.c. components and some applications we may ~~not~~ worried about the dc components ∴ the Question in can we have the Generalized form of correlative coding.

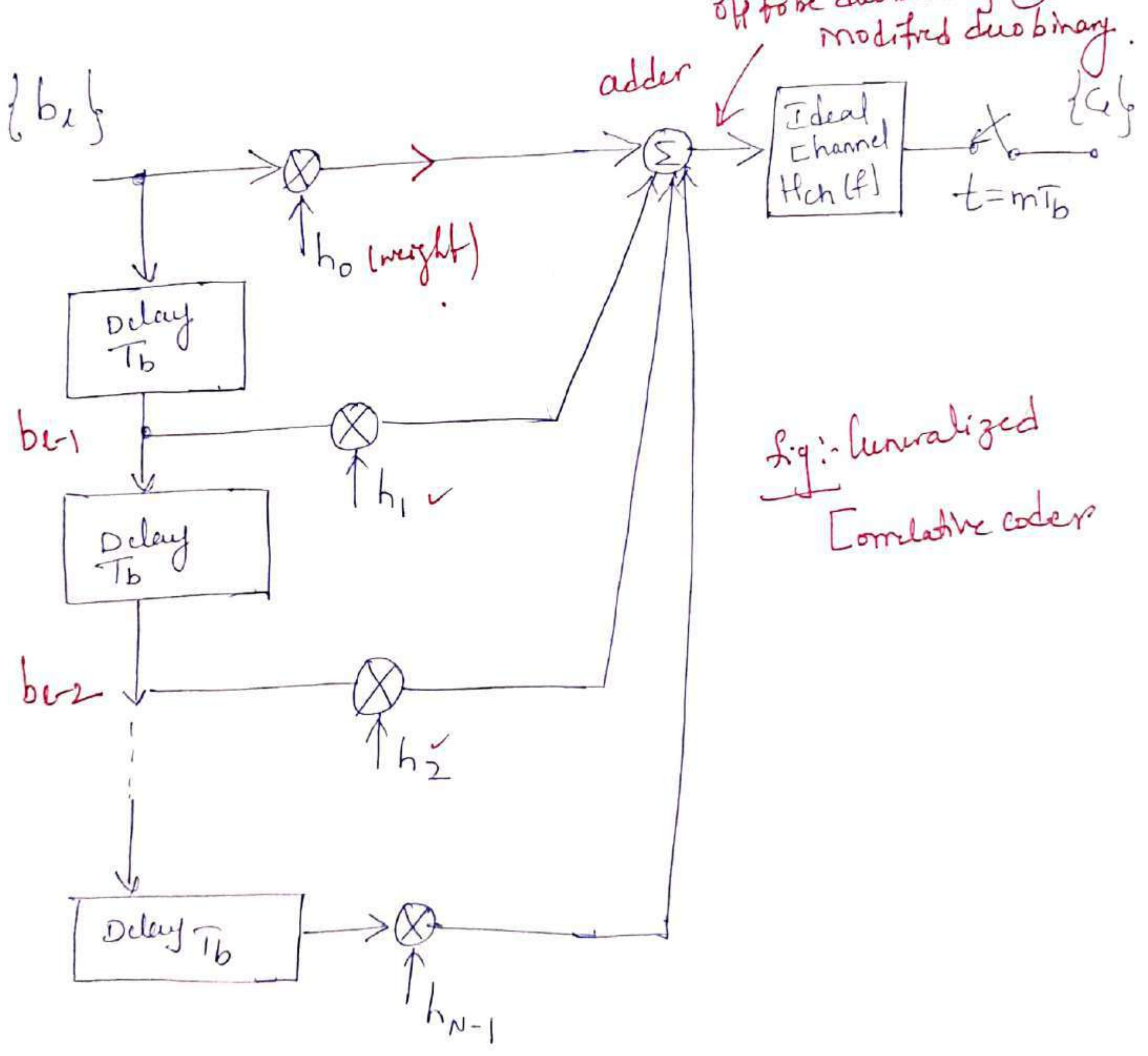


fig:- Generalized Correlative coder

o/p of duobinary o/p $C_e = b_e + b_{e-1}$;

makes $h_0 = h_1 = 1$ weight & all otherweights to zero.

it gives duobinary o/p.

and for Modified duobinary o/p:

$$C_l = b_l - b_{l-2}$$

$h_1 = 0, \quad h_0 = 1, \quad h_2 = -1$ and all other weights are zero.

that gives the o/p modified duo-binary.

for duobinary $\xrightarrow[\text{Correlation Span}]{}$ 1 bit.

modified duo binary $\xrightarrow[\text{Correlation Span}]{}$ 2 bits.

Tapped delay line filter structure with tap weights $h_0, h_1, h_2 \ldots h_{N-1}$.

$$C_e = \sum_{n=0}^{N-1} h_n b_{e-n}$$

$$C_l = h_0 b_l + h_1 b_{l-1} + h_2 b_{l-2} + \ldots + h_{N-1} b_{l-(N-1)}.$$

43

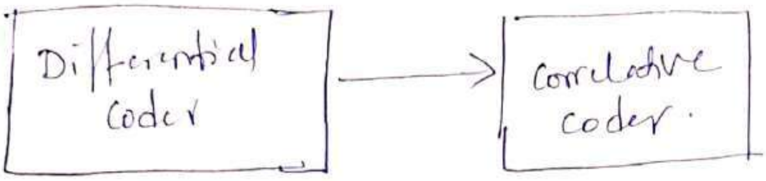
To obtain Duo-binary o/p

$$h_0 = 1 = h_1$$

$$C_l = b_l + b_{l-2} \quad \text{ie } h_n = \begin{cases} 1 ; n = 0,1 \\ 0 ; \text{else} \end{cases}$$

Application 2

Bipolar generation using correlative coding.



Binary sequence

1 0 1 1

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

1 0 -1 +1.

$0 \rightarrow 0$

alternate '1' mapped $\rightarrow \pm 1$

Biploar coding

Differential Encoder o/p :-

$$y_l = x_l \oplus y_{l-1}$$

x_l → i/p binary sequence

$\oplus$ → XOR

To generate Bipolar wave-form, requires Differential coder & correlative coder.

Block diagram of Bipolar waveform generator

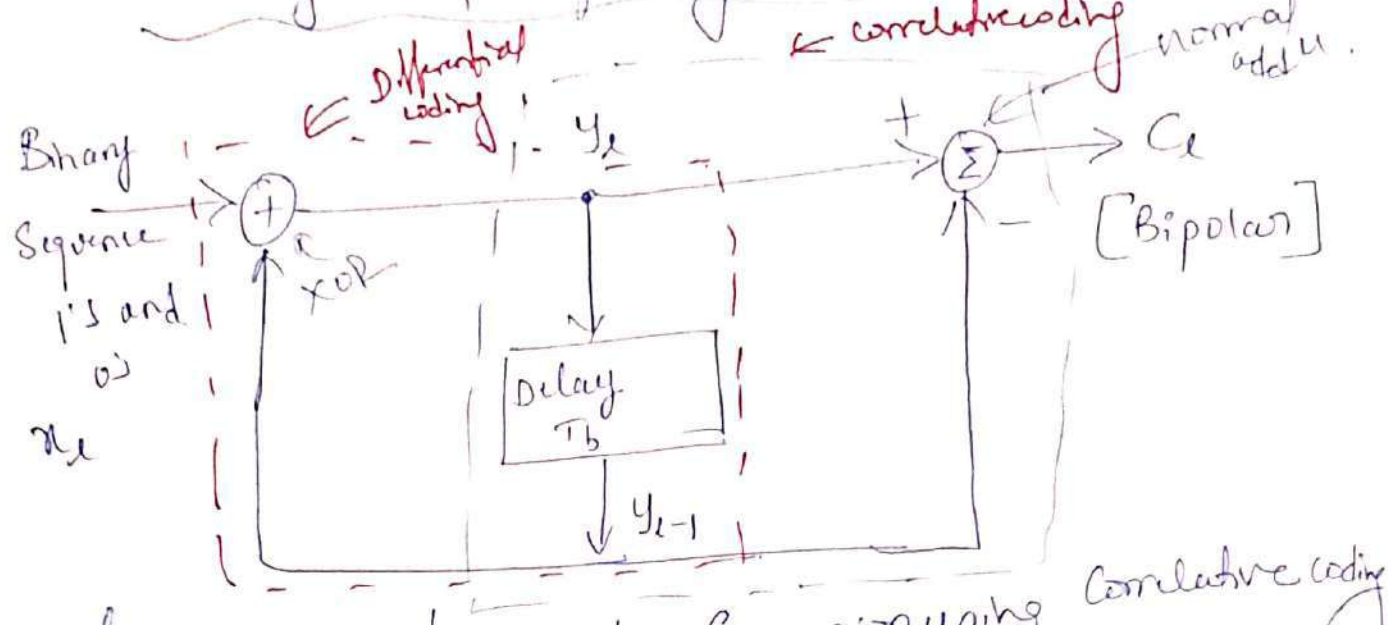


fig:- Binary to Bipolar Conversion using Correlative coding

i/p Binary sequence 0's & 1's $\longrightarrow$ Mapped to o/p. Bipolar waveform

$$y_l = x_l \oplus y_{l-1}$$ ← initialization bit either (0 or 1)

Correlative coding $C_l = y_l - y_{l-1}$.

Eg:-

x_l		0	1	1	1	0	1	0	1
y_l	1	1	0	1	0	0	1	1	0
$C_l = y_l - y_{l-1}$		0	-1	1	-1	0	1	0	-1
i/p sequence b_l		0	1	1	1	0	1	0	1

← Binary sequence

initial bit

} compare

0 $\xrightarrow{\text{mapped.}}$ 0

1 ⟶ alternative ±1 but first 1 mapped to −1.

45

M-ary PAM

pulse Amplitude Modulation.

M-ary format of pulse shaping

bits   Symbols → group of bits. it may be 1 bit (or) group of bits.

No. of bits in a Symbol (b) = $\log_2 M$.

Eg: $M=2$ $b = \log_2 2 = 1$

Where M is No. of constellation levels (or) no. of possible signals

symbol duration (T) = $T_b \log_2 M$

$$\boxed{T_s = T_b \log_2 M}$$

T_s → Symbol duration.

T_b → bit duration × No of bits

Eg: $M=4$. $\boxed{T = T_b \log_2 4 = 2T_b}$

(46)

M-PAM

if M=2, → called Binary PAM.

no of possible signal.

In general the amplitude levels $\boxed{A_m = 2m-1-M}$

, where m=1, ... M.

for M=2,

$m = 1, 2.$

$$A_m = 2m-1-2 = 2m-3$$

$$\left.\begin{array}{ll} m=1, \rightarrow \boxed{A_m = -1} & \text{i.e } A_1 = -1 \\ m=2 \rightarrow \boxed{A_m = 1}, & A_2 = +1 \end{array}\right\} 2$$

Bit	level
1 →	1
0 →	-1.

Illy polar Mapping.

for Binary PAM   Constellation diagram

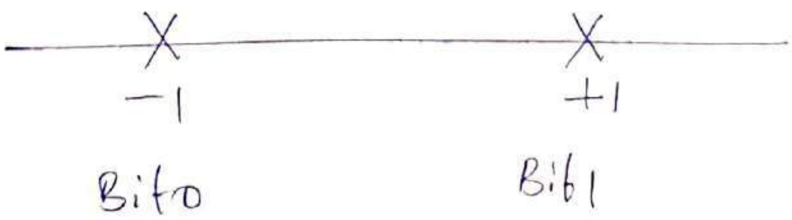


fig: Signal space diagram (or) Constellation diagram.

(47)

Eg:- 1 0 1 1

generate Binary PAM Signal.

Step 1:- map bits to level.

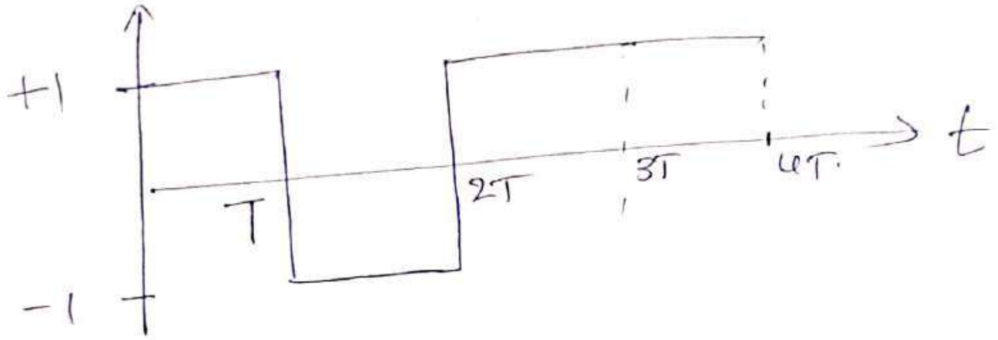
1 0 1 1

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

M=2 1 -1 1 1

$\therefore$ $T = T_b \log_2 2 \Rightarrow \boxed{T_b = T_b}$

bit duration = Symbol duration.



4-PAM

$M = 4$

$b = \log_2 4 = 2$ bits/symbol.

$\boxed{T = 2T_b}$.

$A_m = 2m - 1 - M$, $m = 1, \ldots M$

$M = 4$

$\therefore m = 1, 2, 3, 4$.

$\therefore \boxed{A_m = 2m - 1 - 4}$

48

$A_m = 2m - 5$

$m = 1 \Rightarrow A_1 = -3$ $\quad$ $m = 3 \Rightarrow A_3 = +1$

$m = 2 \Rightarrow A_2 = -1,$ $\quad$ $m = 4 \Rightarrow A_4 = +3$

Level	Natural Coding	Gray code.
-3	00	00
-1	01	01
+1	10	11
+3	11	10

differ by 1bit.

differ by 1bit

differ by 1bit.

Gray coding → Each level is coded in such a way that their will be a one bit difference between the Successive levels

Example!- 01 10 11 00 11 ⇒ genrat 4PAM Signal for the Given binary Sequence.

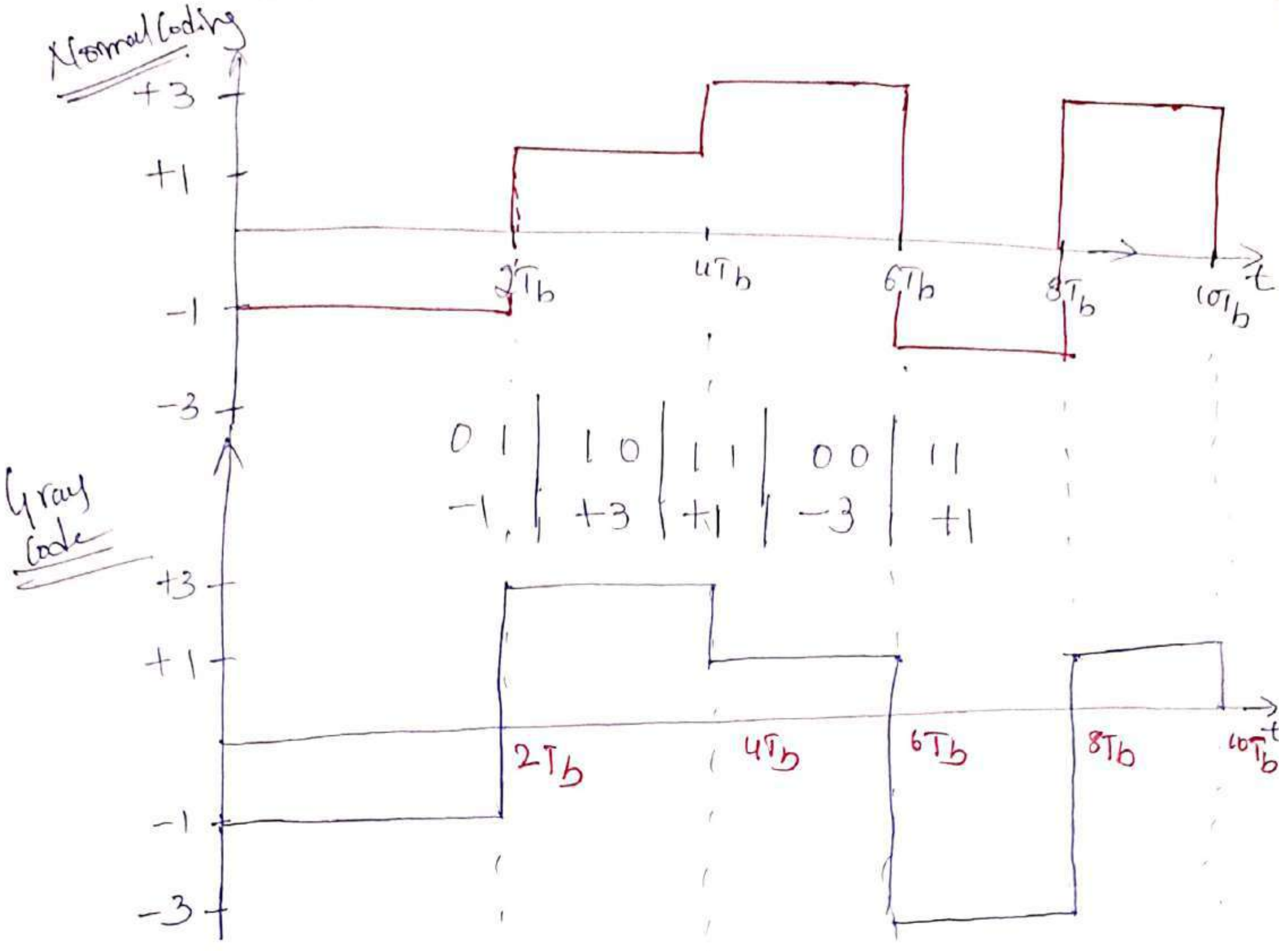
Soln:

i. Natural coding

01	10	11	00	11
-1	+1	+3	-3	+3

dibit to level Mapping.

(49)



0 1	1 0	1 1	0 0	1 1
-1	+3	+1	-3	+1

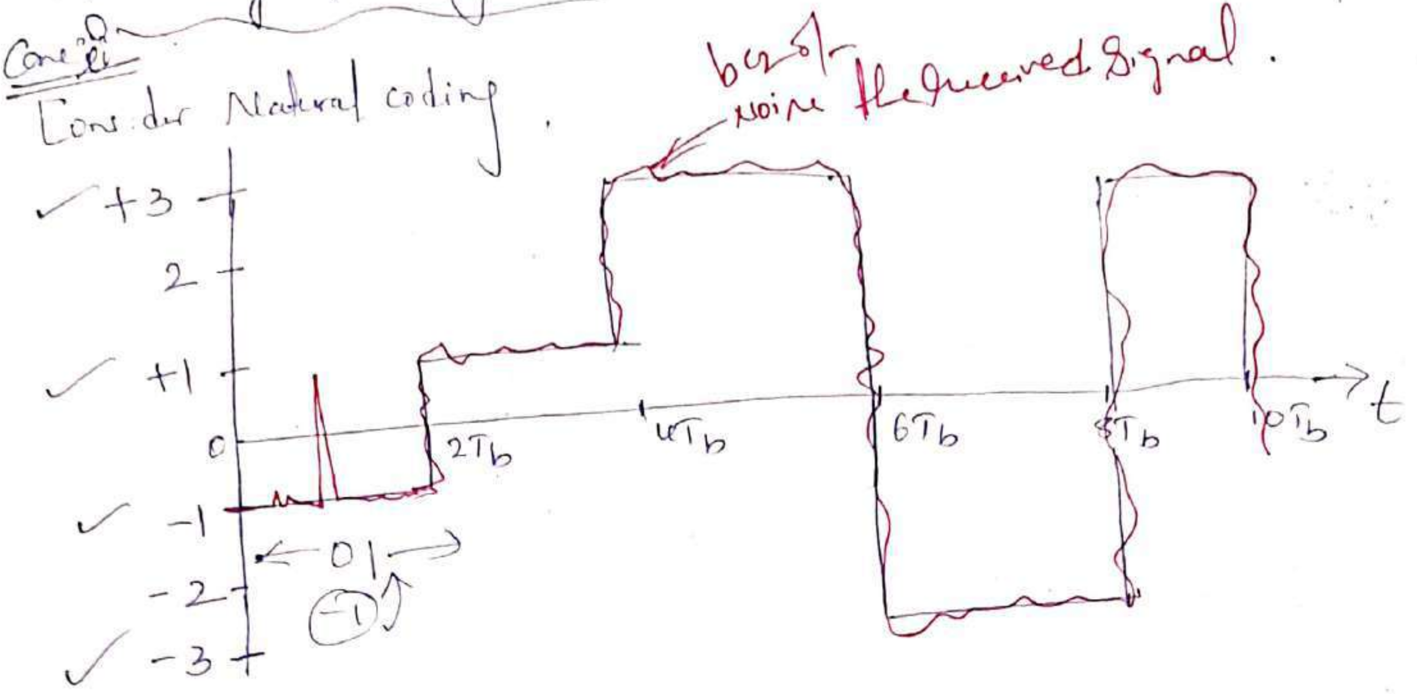
Advantage of going Graycode ?

(50)

Advantages of Gray code:-

Case i

Consider Natural coding.



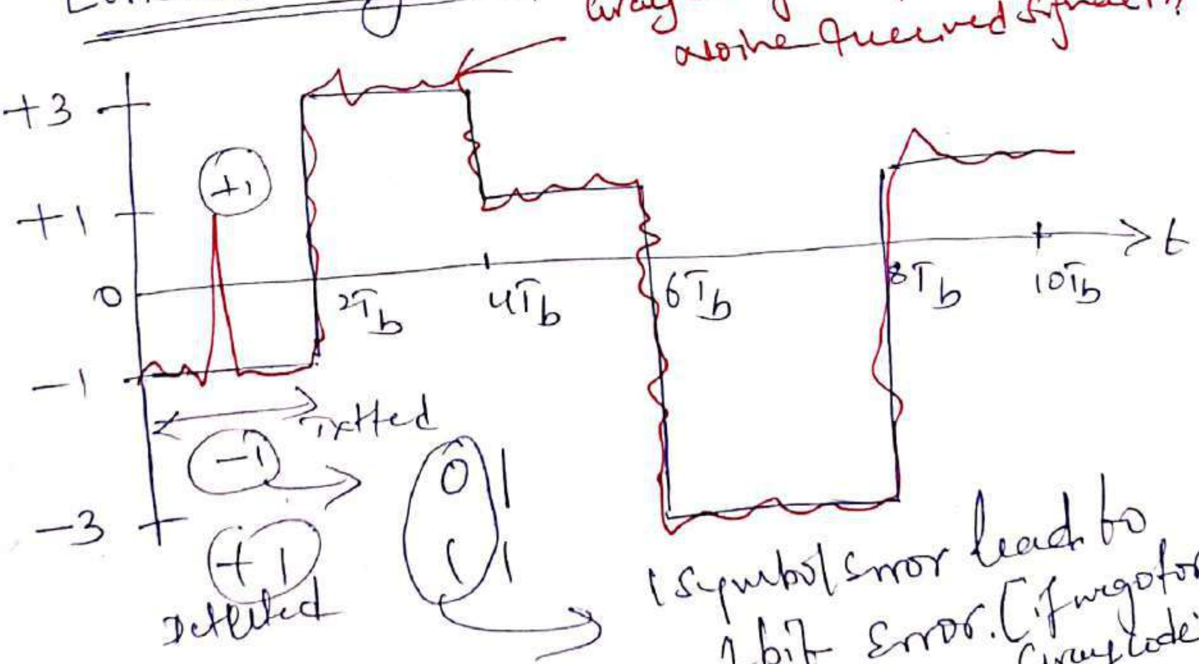
Txtted is -1 ie 01 Txtted

But detected +1 ie 10 Detected.

b'coz of noise bit

1 symbol $\xrightarrow{\text{leads}}$ 2 bit Error. [If we go for Natural coding].

Case ii Consider Gray code:-



1 symbol Error leads to 1 bit Error. [if we go for Gray coding].

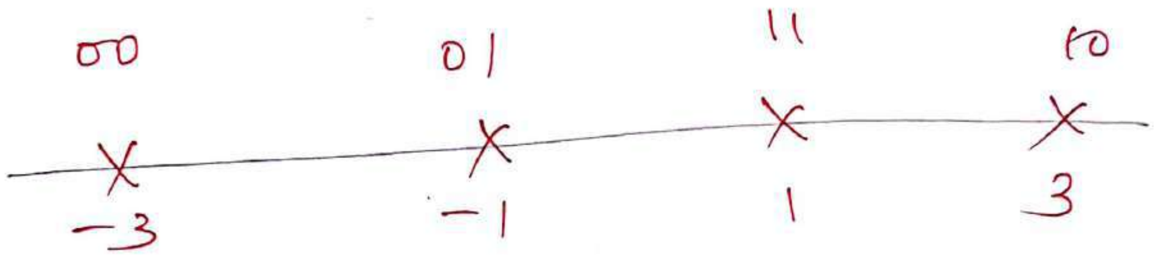
In Gray Code 1 symbol Error → leads to 1 bit Error.

Inference :- Gray coding Minimize the bit error rate. (BER).

Constellation diagram of 4-PAM

for M=4 $T_s = 2T_b$.

As per Gray code



for M=8 $T_s = 3T_b$

$b = 3$ bits / symbol.

→ Total 8 levels. ← Each 3 bit combination can be represented as a level.

Effect of increasing M↑ :- i.e By increasing Modl order data rate also increases.

−7	000
−5	001
−3	010
−1	011
+1	100
+3	101
+5	110
+7	111

↑ 8 level's ↑ Natural coding.

Note :- i.e. if M↑ ⟹ Data rate R_b ↑ by no. of bits / symbol increases.

Eg :- Say for R_b = 100 Symbols / sec

2-PAM ⟹ R_b = 100 b/sec (↳ only one bit)

4-PAM ⟹ R_b = 200 b/sec

8 PAM ⟺ R_b = 300 bits / sec.

(52)

~~X~~ ii. $M=2$ $-1, 1$

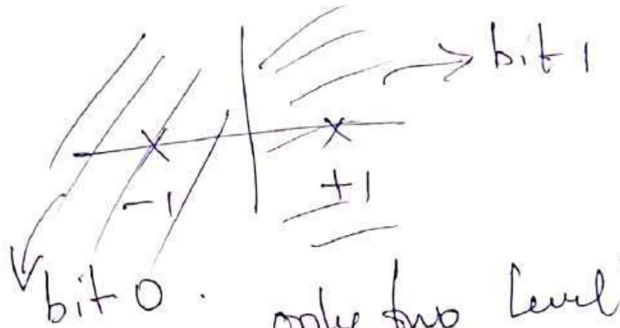
$M=4$ $-3, -1, 1, 3$

$M=8$ $-7, -5, -3, -1, 1, 3, 5, 7$

as M ↑ ⇒ voltage level's also ↑

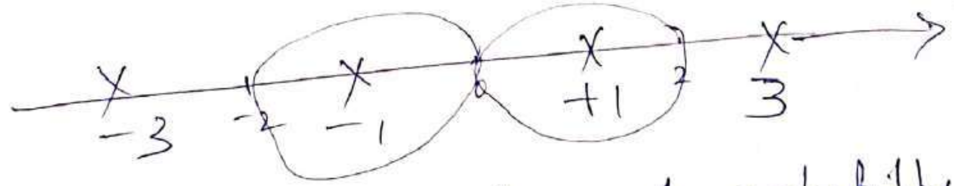
∴ as M↑ ⇒ power requirement will also increase.

iii. for $M=2$



only two level's chance of getting bit error is less.

When $M=4$. decision region shrinks.



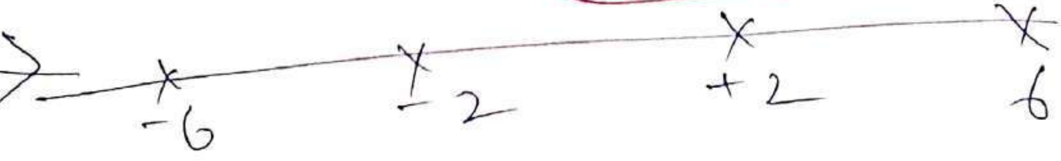
When M ↑ ⇒ more chances of probability of error.

Remedy:- increase the distance b/w the voltage level's and also Pe ↓

∴ Noise interference will be less

but this increases the Power requirement. Power↑

$\boxed{M\uparrow \to P_e\uparrow} \xrightarrow{\text{Remedy}} D\uparrow \to P_e\downarrow$



Power↑

M-level Signaling with Duobinary pulses

$\{d_l\} \rightarrow$ M level amplitude Sequence.

$$M_{d_l} \in \{0, 1, \ldots M-1\}$$

1. Precoder o/p

$$P_l = d_l - P_{l-1} \quad (\text{mod } M)$$

$$d_l \in \{0, \ldots M-1\}$$

Eg: 4 PAM
-3
-1
1
3

2. Map precoder o/p to different M-PAM levels.

$$I_m = 2m - 1 - M, \quad m = 1, 2, \ldots M.$$

3. O/p of the receiver filter is sampled, the following operation is performed with the obtained samples.

$$B_l = I_l + I_{l-1}$$

4. The received sequence is decoded using

$$\hat{d}_l = \frac{B_l}{2} + (m-1) \ (\text{mod } M).$$

(54)

Exp:- 4 level Signaling with Duobinary pulses.

M=4 → (0 to 3)

Data Sequence $\{d_k\}$ 0   0   1   3   1   2   0

precoder o/p $\{p_k\}$ $\boxed{0}$ 0   0   1   2   3   3   1

$\downarrow_3$ $\downarrow_3$ $\downarrow_{-1}$ mod 4.

$-1 \bmod 4 = +3$

$p_k = d_k - p_{k-4} \bmod 4$ -3 -3 -3 -1 1

Soln:-

Step1:- precoder o/p.

$$p_k = d_k - p_{k-1}$$

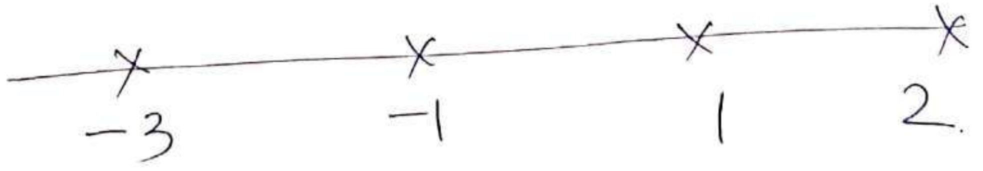
→ assume 0, 1, 2, 3 @ any value. Since M=4.

Note:- Modulus operation.

$$\boxed{-x \bmod M = -x + M}$$

Step 2 Maping. Precoder o/p in mapped to M-PAM levels.

4-PAM



0 → -3   2 → 1

1 → -1   3 → 3

Tx Sequence $\{I_k\}$ -3   -3   -3   -1   1   3   3   -1

i.e 4PAM o/p.

(55)

@Receiver Correlative coding.

$I_l \quad I_{l-1}$

TX $\{I_l\}$ $\quad -3 \quad -3 \quad -3 \quad -1 \quad 1 \quad 3 \quad 3 \quad -1$

B_l

$I_l + I_{l-1}$ $\quad -6 \quad -6 \quad -4 \quad 0 \quad 4 \quad 6 \quad 2$ } Correlative Coder O/P

$\quad /2 \quad /2 \quad /2 \quad /2 \quad /2 \quad /2 \quad /2$

Detected Sequence $\frac{B_l}{2}$ $\quad -3 \quad -3 \quad -2 \quad 0 \quad 2 \quad 3 \quad 1$

$\left(\frac{B_l}{2}+3\right)$ $\quad 0 \quad 0 \quad 1 \quad 3 \quad 5 \quad 6 \quad 4$

$0 \quad 0 \quad 1 \quad 3 \quad 1 \quad 2 \quad 0$ ⇒ Received Signal which is Same as Txted.

$$\hat{d}_l = \frac{B_l}{2} + (M-1) \text{ (mode } M)$$

$M=4.$

$$= \frac{B_l}{2} + 3 \text{ (mod. 4)}$$

$x \bmod M$

if $x < M$ $\qquad x \bmod M = x$.

if $\underline{\underline{x = -ve}}$ $\qquad x \bmod M = -x + M$.

if $x > M$ $\qquad x \bmod M =$ remainder value.

Exeeke **Equalization:-**

Summary:-

W.K.T Band Limited channel leads → Time dispersion

↓ due to (T.D) Energy from one Symbol falls into consecutive Symbols ie ISI

ISI

To Minimize ISI @ TX side

ie "Precompensate ISI @ Receiver"

- ✓ Sinc
- ✓ RC
- ✓ Duo-binary
- ✓ Modified Duo-binary

Controlled ISI { Duo-binary, Modified Duo-binary }

@ Receiver (Rx) side (Post compensate) of ISI we use "Equalizer" (ie "Inverse filter").

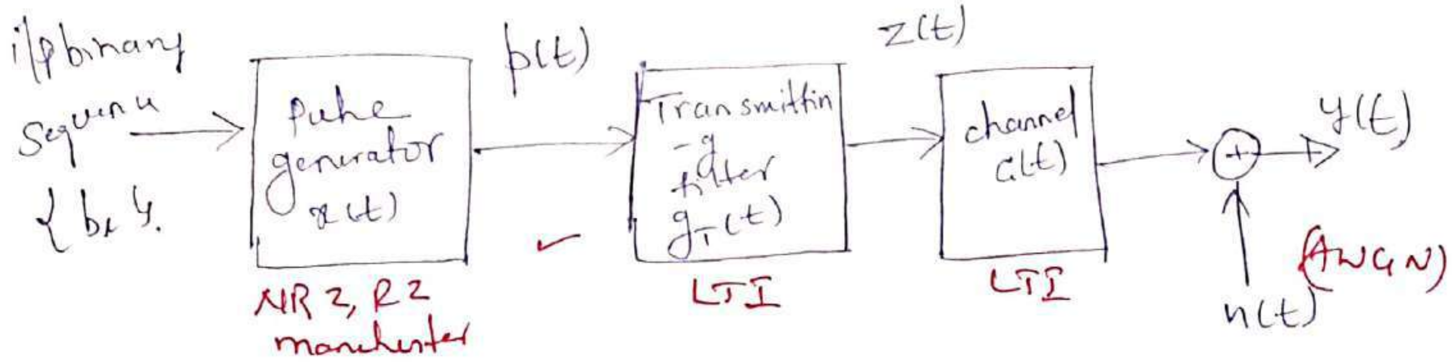
* Equalizer basically called Inverse channel. whatever the effects channel introduces, it will nullify using Equalizer.

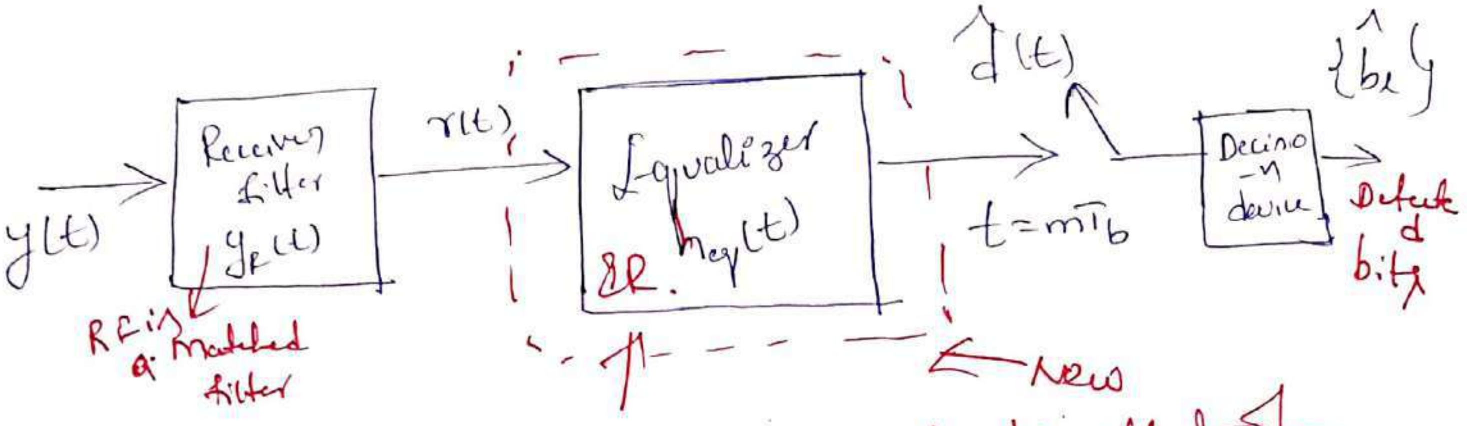
channel → filter.

Equalizer → Inverse filter.

* The effect of Pulseshaping, Tx filter, channel, & Rx filter can be nullified (Eliminated) by using Equalizer.

(57) (57)





* To post compensate the effect of Pulseshaping filter, $g_T(t)$, channel & Receiver filter that can be done by Equalizer.

$$y(t) = [z(t) * c(t)] + n(t).$$

The receiver filter input is

$$y(t) = \overset{z(t)}{\boxed{p(t) * g_T(t)}} * c(t) + n(t).$$

Receiver filter output

$$r(t) = y(t) * g_R(t)$$

$$r(t) = [p(t) * g_T(t) * c(t) + n(t)] * g_R(t).$$

Convolution is distributive.

$$r(t) = \underbrace{p(t) * g_T(t) * c(t) * g_R(t)}_{\text{Say } f(t).} + n(t) * g_R(t).$$

$$r(t) = p(t) * f(t) + n(t) * g_R(t).$$

Receiver filter output is passed through the Equalizer.

i.e.

$$\hat{d}(t) = r(t) * h_{eq}(t).$$

Equalizer output

$$\hat{d}(t) = \left[p(t) * f(t) + n(t) * g_R(t)\right] * h_{eq}(t).$$

using distributive property

$$\hat{d}(t) = \underbrace{p(t) * f(t) * h_{eq}(t)}_{\text{1st term.}} + \underbrace{n(t) * g_R(t) * h_{eq}(t)}_{\text{2nd term. } u(t).}$$

Receiver is usually the matched filter. and matched filter Maximizes the signal to noise ratio, when the channel is AWGN.

ie Matched filter takes care of noise . so we Concentrate only the first term .

$$\boxed{\begin{array}{c}\text{Equalizer}\\ h_{eq}(t).\end{array}} \longrightarrow \hat{d}(t)$$

-x. The o/p of equalizer is expected to be the actual signal. ie p(t).

$$\hat{d}(t) \simeq p(t) * \overbrace{\underline{\underline{[f(t) * h_{eqn}(t)].}}}^{\delta(t).}$$

ie when $f(t) * h_{eqn}(t) = \delta(t)$

$$\hat{d}(t) \simeq p(t) * \delta(t)$$

$$\underline{\underline{\simeq p(t)}}$$ ie the actual Txtted Signal.

<u>Note:-</u> i.e $f(t) * h_{eqn}(t) = \delta(t)$

$$\Big\Updownarrow \text{FT}$$

$$F(f) \cdot H_{eqn}(f) = 1$$

$$\Rightarrow \quad F(f) = \frac{1}{H_{eqn}(f)}.$$

so.

ⓐ $$\boxed{H_{eqn}(f) = \frac{1}{F(f)}}$$

Where ie. Equalizer is a "<u>inverse channel</u>"

⑥⓪

$$H_{eqn}(f) = \frac{1}{F(f)}.$$

where $f(t) = g_T(t) * c(t) * g_R(t)$

$\Big\downarrow$ F.T

$$\boxed{F(f) = G_T(f) \cdot C(f) \cdot G_R(f).}$$

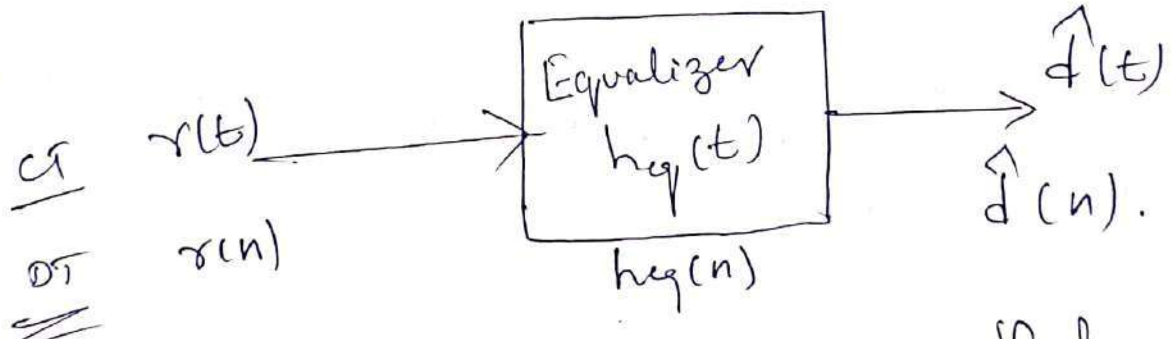
with the help of Equalizer we can Minimize the effect of Intersymbol Interference. (ie ISI) at the Receiver Side.

Example: Construct an Equalizer for the given Specifications.

Equalizer order $N = 3$.

Equalizer Co.efficient $h_{eq}(0), h_{eq}(1), h_{eq}(2)$.

Solu:



assume that r(t) itself Sampled and that samples are passed through Equalizer.

Z.T of

$$T.F = \frac{\hat{D}(z)}{R(z)} = \frac{o/p}{i/p}$$

i.e $H_{equ}(Z) = \frac{\hat{D}(Z)}{R(Z)}$

$$\frac{\hat{D}(Z)}{R(Z)} = h_{eq}(0) + h_{eq}(1) Z^{-1} + h_{eq}(2) Z^{-2}.$$

$$H_{equ}(Z) = \sum_{n=0}^{\infty} h_{equ}(n) z^{-n}$$; ... Right-sided Z.T.

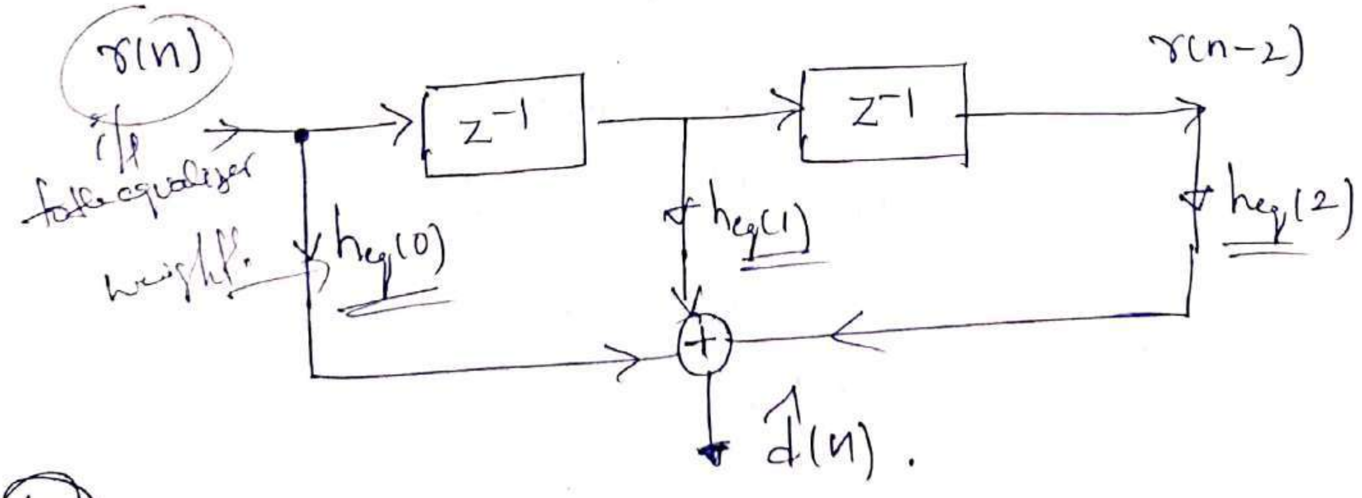
$$h_{eq}[n] = [h_{eq}(0), \; h_{eq}(1), \; h_{equ}(2)].$$
n=0 n=1, n=2.

$$\frac{\hat{D}(z)}{R(z)} \boxed{H_{eq}(z) = h_{equ}(0) + h_{eq}(1) z^{-1} + h_{eq}(2) z^{-2}}$$

$$\hat{D}(z) = h_{eq}(0) R(z) + h_{eq}(1) z^{-1} R(z) + h_{eq}(2) z^{-2} R(z)$$

$$\Big\updownarrow \text{ I.Z.T}$$

$$\hat{d}(n) = h_{eq}(0) r(n) + h_{eq}(1) r(n-1) + h_{eq}(2) r(n-2).$$



Note:-

- * If the channel is not changing then, the equalizer coefficients also not changing i.e just an one time estimation and use it forever.

- * if channel is changing then the equalizer coefficients also should change (vary) that type of equalizers are called "Adaptive Equalizers"

ie channel varying (Timevarying) ⟶ Equalizer coefficient varied

↓

called Adaptive equalizer.

Adaptive Equalizers :—

- * If the channel is varying channel, the weights used in the channel also should vary as per the varying nature of the channel.
- * ie weights are not fixed, they should be updated. Such type of Equalizers are called **Adaptive Equalizers.**

$h_{eq}(0)$ $h_{eq}(1)$ $h_{eq}(2)$ These weights should be updated as per the varying channel.

To update :

Adaptive Equalizers work in two Modes

i. Training Mode :—

ii. Decision Directed Mode.

i. Training Mode :—

in the Training Mode, they don't send the actual data, through channel, instead pilot (or) Reference (or) Training Signal is transmitted.

* This pilot signal is known for the Transmitter as well as the Receiver.

* i.e Receiver must be knowing what the information the transmitter is going to send. that signal is

⇒ represented by $d(n)$.

now the

* ⇒ Equalizer will make some decision and it obtains $\hat{d}(n)$.

* ⇒ find the difference $\underset{\substack{\uparrow \\ \text{actual} \\ \text{or Expected}}}{\underline{d(n)}} - \underset{\text{Obtain}}{\underline{\hat{d}(n)}}$

$$e(n) = d(n) - \hat{d}(n)$$

This Error Signal $e(n)$ is used to update the weight of the Equalizer.

* when $e(n) = 0$ i.e $d(n) - \hat{d}(n) = 0$ $\Rightarrow \boxed{d(n) = \hat{d}(n)}$, then

Stop updating the Equalizer Co-efficients.

* when $e(n)=0$, the equalizer Switches to the "decision Directed Mode"

Now we can transmitt the "actual data" through Transmitter. again after sometime. the equalizer Switches to Training Mode and this repeats. . . .

This type of Equalizer is called Adaptive Equalizer.

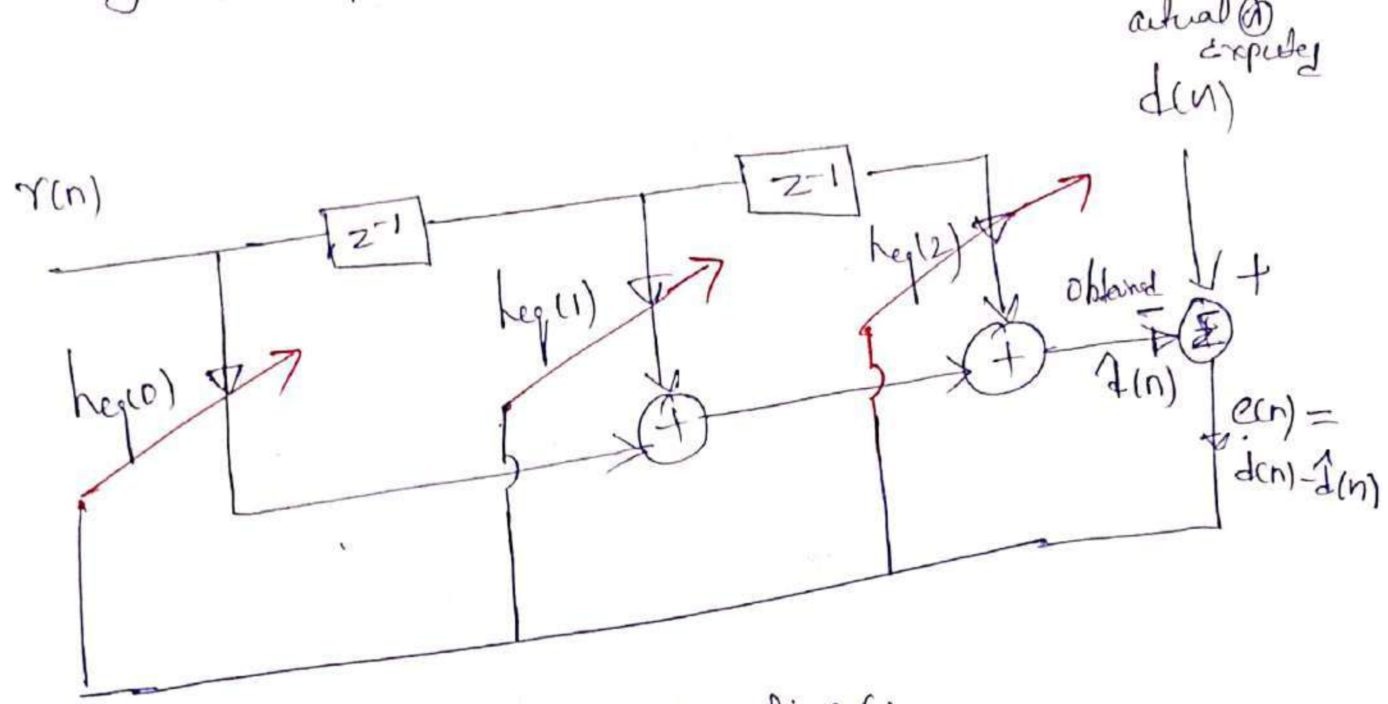


fig:- Adaptive Equalizer.

* There are Many algorithms are available in the Literature to update the equalizer Coefficients. such as i. Least Mean Square algorithm. (LMS)

Note:- Adaptive equalizer works under two modes
→ i. Training mode
→ ii. Decision Directed Mode.

i. In Training Mode.

* pilot sequence (Training sequence) is first transmitted.
* This sequence is known to both Transmitter and Receiver.
* The Equalizer Co-efficients are updated in this mode.

- The training mode continues, ~~[illegible]~~ update the weights until the error sequence becomes zero (or minimum).
- Then the equalizer is switched to decision directed mode where the actual data is transmitted.
- For time varying channel conditions there training sequence is transmitted periodically.

Least Mean Squere algorithm (LMS algorithm) *

i. Filter co-efficients are initialized to zero.

$h_{eq}(n)$ ⓐ both are same.

$$\hat{w}_k(0) = 0 , \quad k = 0, 1, 2 \ldots M .$$

M - equalizer order. if M=1, order is 2.

ii. Error Sequence computation.

$$e(n) = d(n) - \hat{d}(n)$$

$$= d(n) - \sum_{k=0}^{M} \hat{w}_k(n) r(n-k)$$

M=1

$$\hat{d}(n) = \sum_{k=0}^{1} \hat{w}_k(n) r(n-k)$$

$$= \hat{w}_0(n) r(n) + \hat{w}_1(n) r(n-1)$$

r(n) - matched filter o/p.

r(n) is X^2 with weights.

iii. Compute kth filter co-efficient

$$\hat{w}_k(n+1) = \hat{w}_k(n) + \mu e(n) r(n-k) , \quad k = 0, 1, 2, \ldots M.$$

μ - Step size, that decides the convergence. helps to update the weights.

(67)

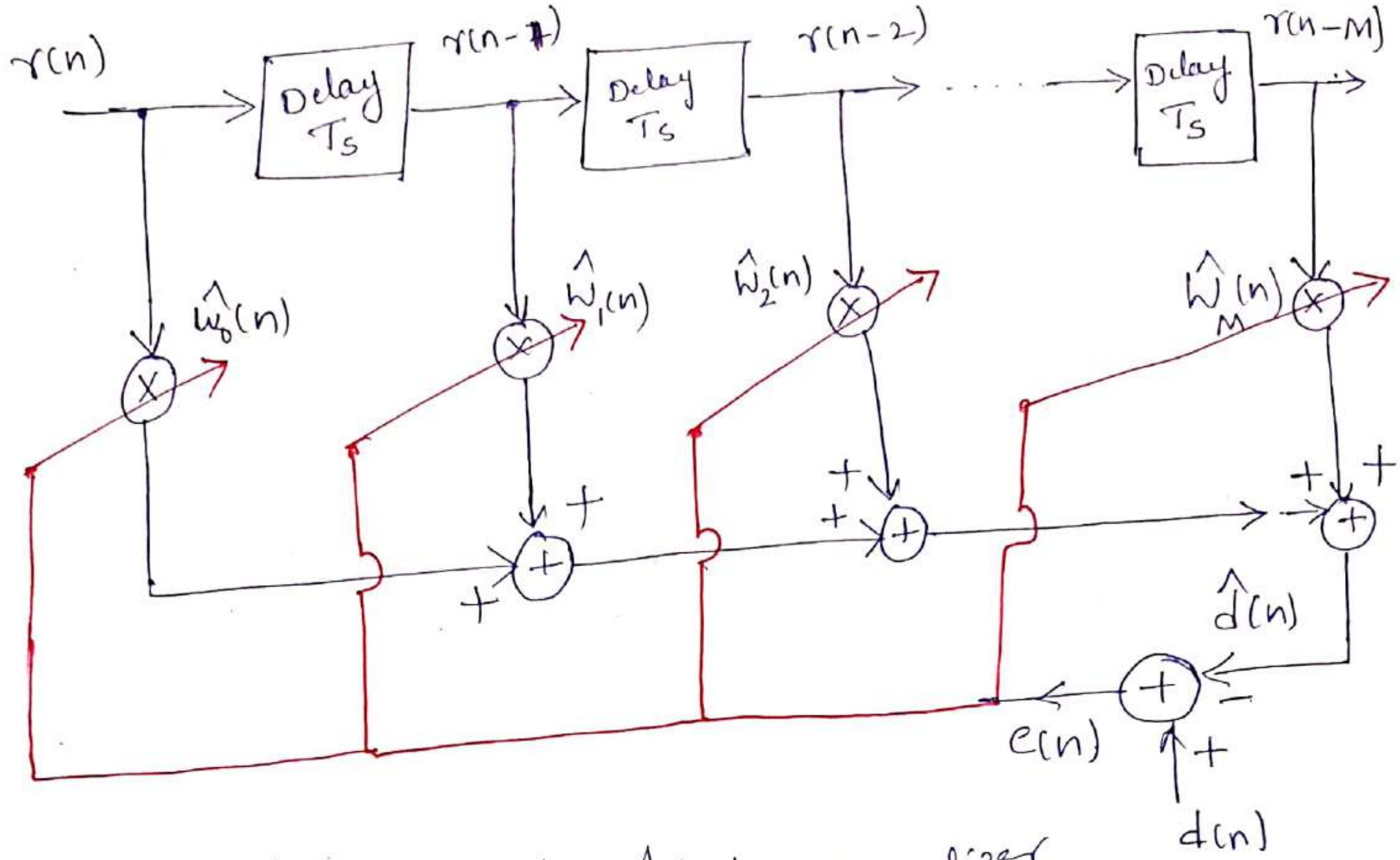


fig:- mth order Adaptive equalizer using LMS algorithm.

Problem. A first order equalizer is to be designed using LMS algorithm, where the desired sequence is $d(n) = \{1, 2, 3, 4\ 5, \ldots\}$ and the input sequence is $r(n) = \{1.2, 2.01, 3.17, 4.21, 5.2 \ldots\}$. Show how equalizer coefficients are updated. also assume the system to be causal. assume the Equalizer initial co-efficients to Zero and Convergence constant $\mu = 0.1$.

68

Soln: Since given it is a M=1, first order Equalizer.

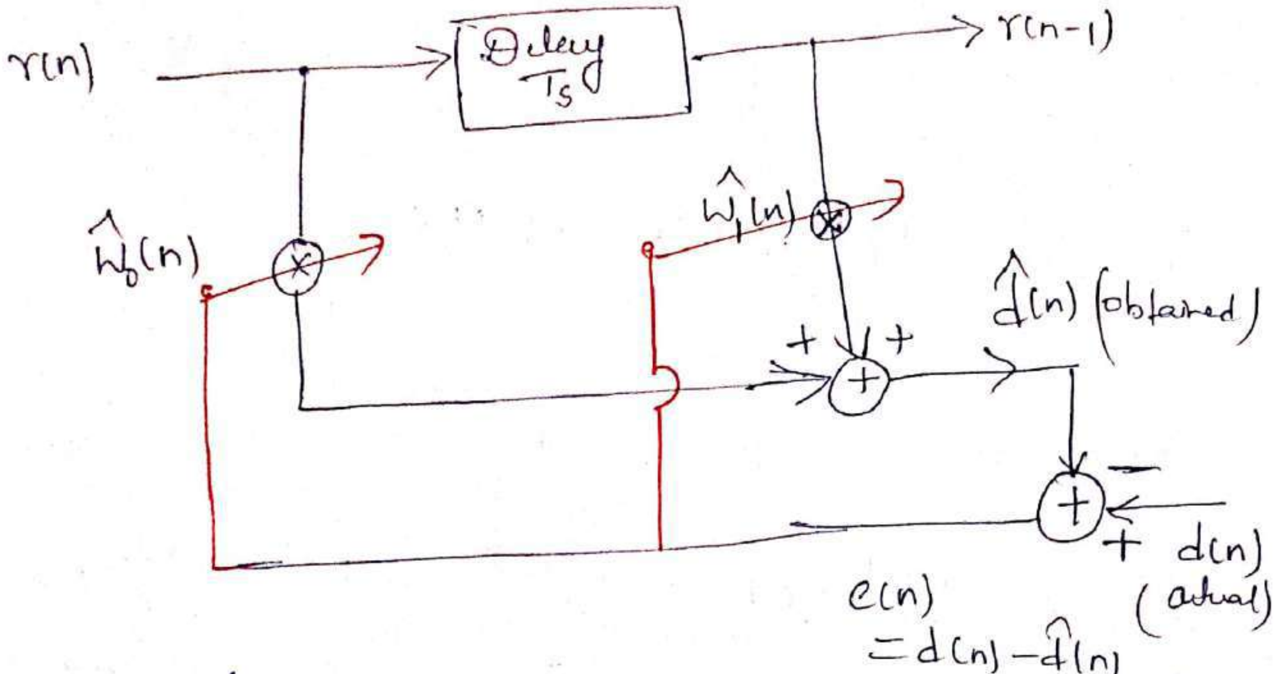


fig: First order Adaptive Equalizer (LMS algorithm).

initially $\hat{w}_0(0) = 0$, $\hat{w}_1(0) = 0$ .. fix initial weights to Zero

Error Signal $e(n) = d(n) - \hat{d}(n)$

$$= d(n) - \sum_{k=0}^{M} \hat{w}_k(n) r(n-k)$$

order

M = 1.

$$e(n) = d(n) - \sum_{k=0}^{1} \hat{w}_k(n) r(n-k)$$

$$e(n) = d(n) - \hat{w}_0(n) r(n) - \hat{w}_1(n) r(n-1)$$

$d[n] = \{1, 2, 3, 4, 5 \ldots\ldots\}$

$r[n] = \{1·2, 2·01, 3·17, 4·21, 5·2 \ldots\ldots\}$

n=0, $e(0) = \cancelto{1}{d(0)} - \cancelto{0}{\hat{w}_0(0)}\, r(0) - \cancelto{0}{\hat{w}_1(0)}\, r(-1)$

$\boxed{e(0) = 1}$: now update the Equalizer Co-efficients.

$$\hat{w}_k(n+1) = \hat{w}_k(n) + \mu\, e(n)\, r(n-k),$$

$k = 0, 1, 2, \ldots M.$

$M = 1,$ $k = 0, 1.$ $\mu = 0.1$ (given)

k=0 $\boxed{\hat{w}_0(n+1) = \hat{w}_0(n) + 0.1\, e(n)\, r(n).}$

k=1 $\boxed{\hat{w}_1(n+1) = \hat{w}_1(n) + 0.1\, e(n)\, r(n-1)}$

n=0 $\hat{w}_0(1) = \hat{w}_0(0) + 0.1\, e(0)\, r(0)$

$= \cancelto{0}{\hat{w}_0(0)} + 0.1(1) \times 1.2$

$\boxed{\hat{w}_0(1) = 0.12}$

n=0 $\hat{w}_1(1) = \cancelto{0}{\hat{w}_1(0)} + 0.1\, e(0)\, \cancelto{0}{r(-1)}$ bcz causal system

$= 0.1(1) = 0.1.$

$\boxed{\hat{w}_1(1) = 0.1}$

find $\underline{\underline{e(1)}}$ i.e $e(n)$ @ $n=1$.

$$e(n) = d(n) - \hat{w}_0(n)\, r(n) - \hat{w}_1(n)\, r(n-1)$$

$$e(1) = d(1) - \underset{0.12}{\underset{\downarrow}{\cancel{\hat{w}_0(1)}}}\, r(1) - \underset{0.1}{\underset{\downarrow}{\cancel{\hat{w}_1(1)}}}\, r(0)$$

$$e(1) = 2 - 0.12\,(2.01) - 0 \times 0.2$$

$$\boxed{e(1) = 1.7588}$$

$$\underline{\underline{n=1}} \quad \hat{w}_0(2) = \hat{w}_0(1) + \overset{\mu}{0.1} \times \overset{e(1)}{1.7588} \times \overset{r(1)}{2.01}$$

$$\boxed{\hat{w}_0(2) = 0.47352}$$

lly $\quad \hat{w}_1(2) = \hat{w}_1(1) + \mu\, e(1)\, r(0)$

$$\hat{w}_1(2) = 0 + 0.1 \times 1.7588 \times 1.2$$

$$\boxed{\hat{w}_1(2) = 0.211}$$

$$e(2) = 1.075.$$

$\underline{\underline{\text{lly}}}$ verify ~~the~~ for all the samples.

$\underline{\underline{\text{obs:}}}$ based on the Error Signals weights are updating automatically till the error becomes zero.

$$\left.\begin{aligned}\hat{w}_0(3) &= 0.8143\\ \hat{w}_1(3) &= 0.4272\end{aligned}\right\} \text{ and } e(3) = -0.7824.$$

$$\left.\begin{aligned}\hat{w}_0(4) &= 0.4849\\ \hat{w}_1(4) &= 0.1792\end{aligned}\right\} \& \; e(4) = 1.724.$$

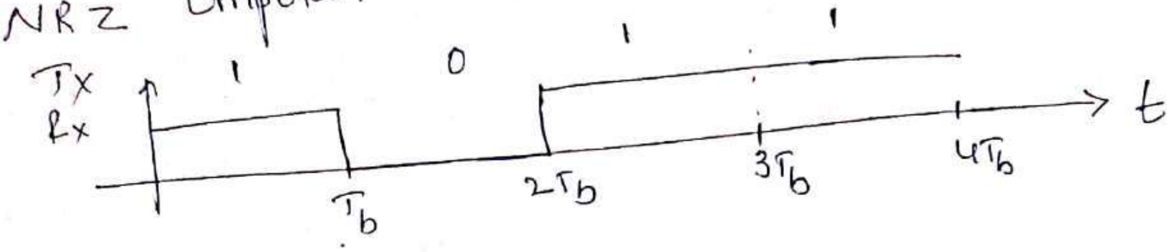
$$\hat{w}_0(5) = 1.3812, \quad \hat{w}_1(5) = 0.905$$

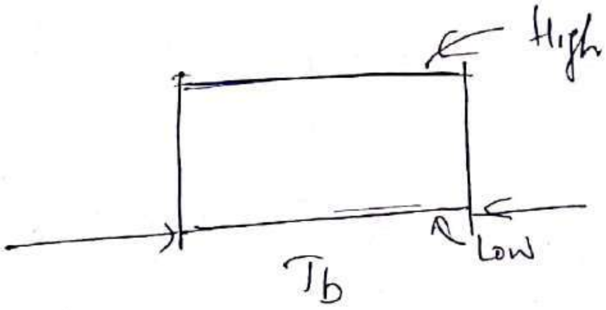
Eye diagrams! → used to understand the effect ISI.

* An eye diagram summarizes the effect of ISI by showing the responses to 0's and 1's respectively.

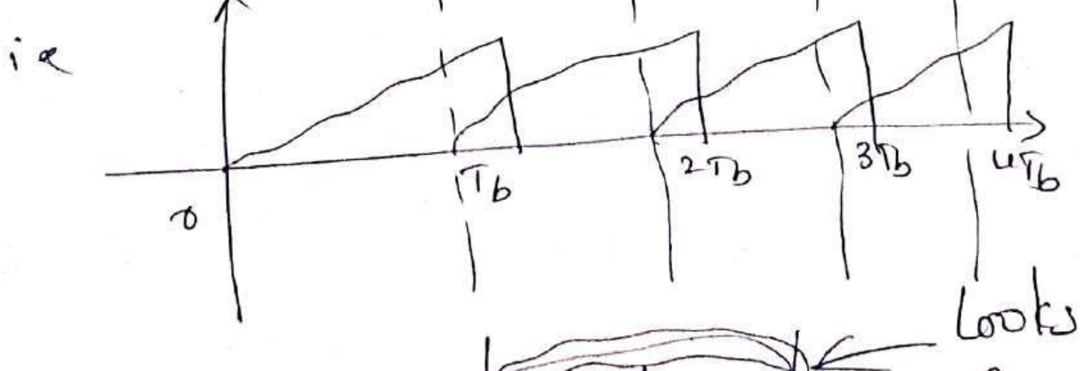
* ~~Eye~~ Consider the Txtted signal in NRZ format.

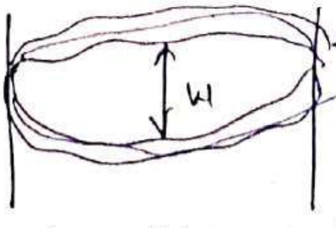
NRZ unipolar





bcz of ISI, one signal energy may fall into the other symbol.

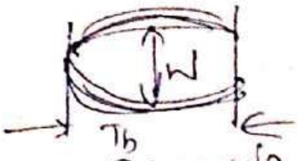




Looks like an Human eye

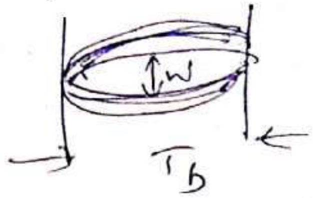
Hals the Name Eye diagram.

⑦②

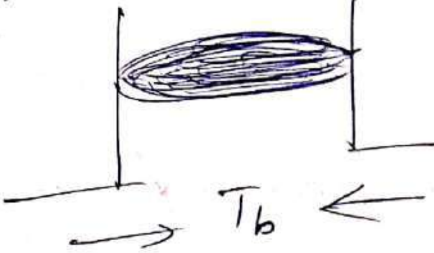


Note:

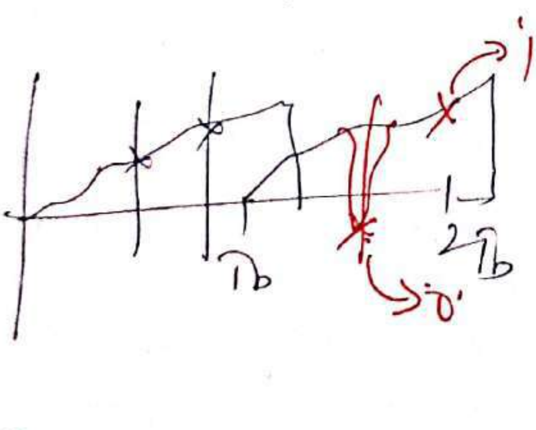
- * If the (W) width of the eye opener is very high, that indicates the amount of ISI is very less (or) No ISI.
- * If the width (W) of Eye opener is much smaller, then ISI is high.



- * if the Eye is completely closed, then signal is completely corrupted by ISI.



→ Note: instant in which the sample taken is also matter, otherwise it may leads to wrong error decision



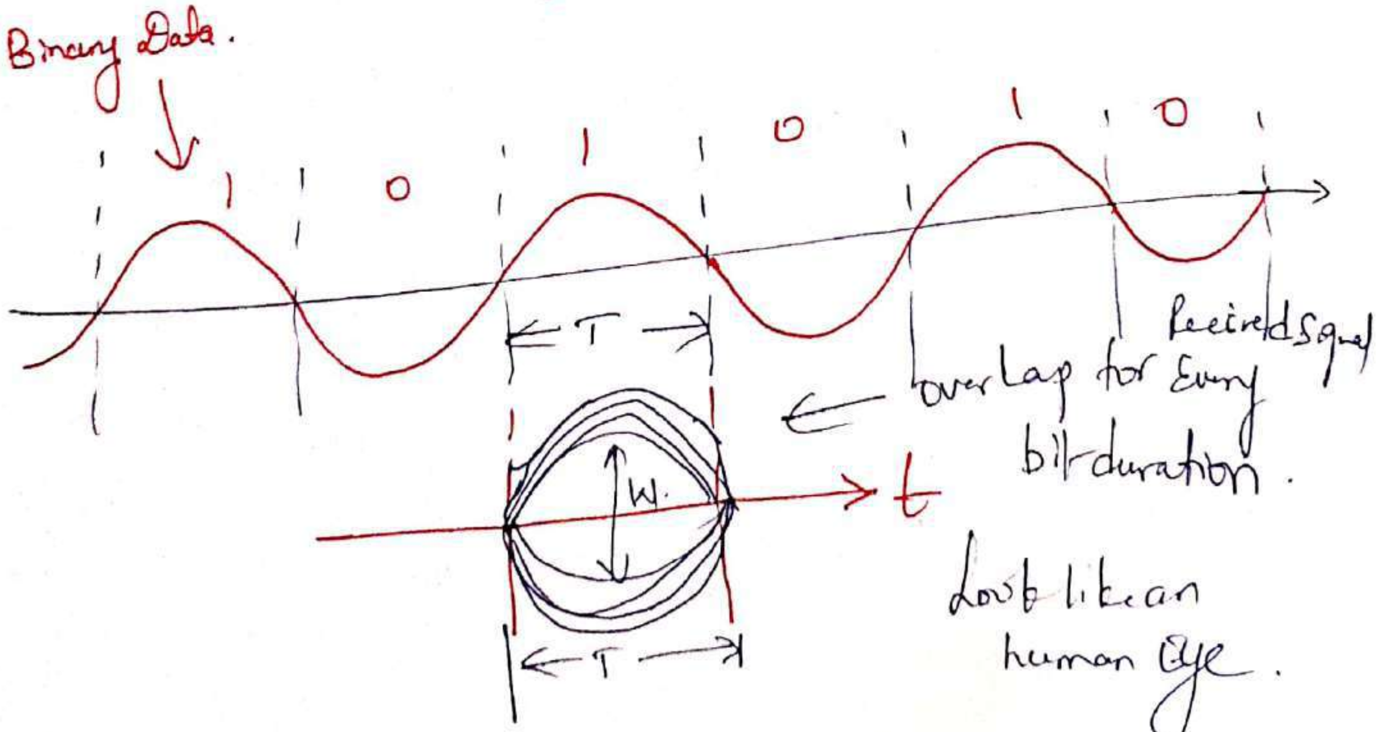
⇒ This Eye diagram gives an idea where we can take the sample value.

(73)

Generation of Eye diagram:-

- Eye diagram can be generated by overlapping plots of the received sequence (channel output) for every symbol time.
- The resulting display will look like human eye for binary waves.
- The interior region of the eye pattern is called ~~eye diagram~~ eye opening.
- Eye opening gives more useful information about the performance of the system.

Received distorted binary wave.



Received signal

overlap for every bit duration.

Look like an human eye.

Interpretation of Eye pattern :- / Three important information from Eye diagram.

1. Width of the Eye opening :- Time interval over which the received distorted Sequence can be Sampled with Minimum Error.

2. Sensitivity of the Syltem to Timing Error :- Rate of closure of eye pattern when the Sampling time is Varried.

3. Margin over Noise :- Height of the eye opening for a Specified Sampling time.

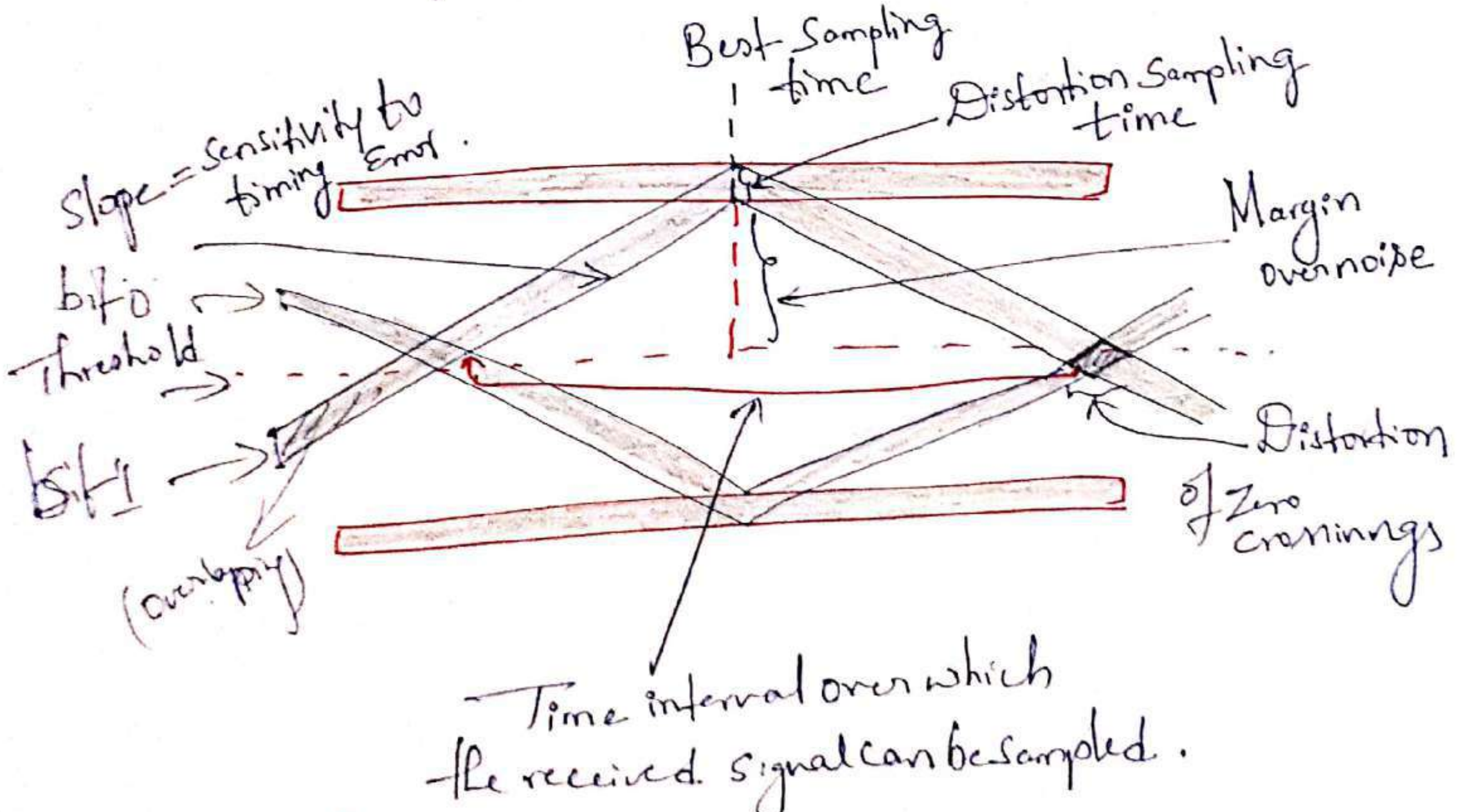
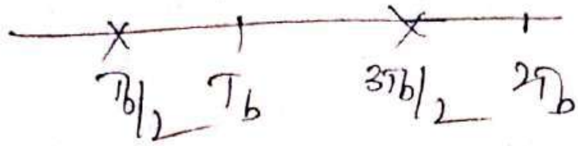
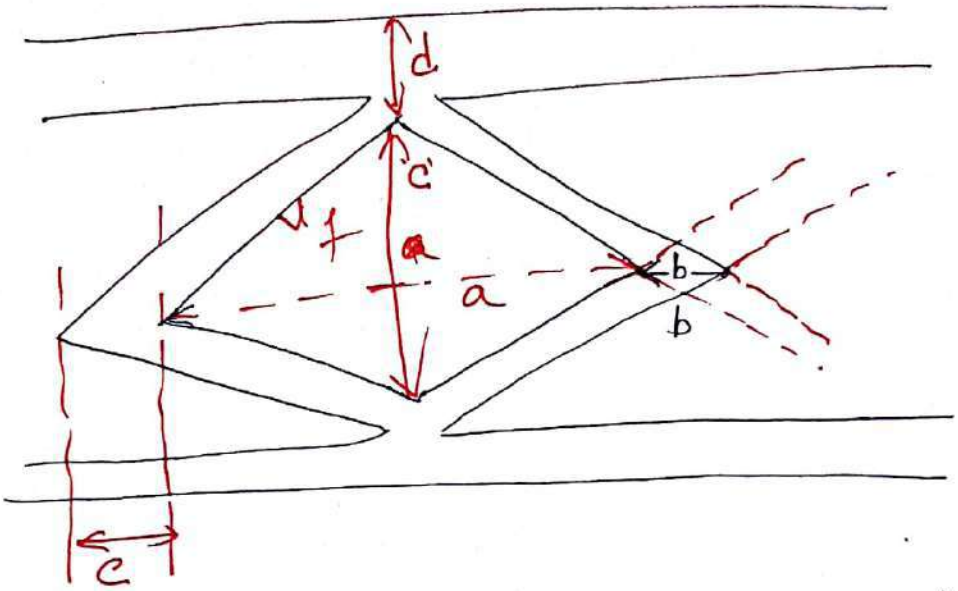


fig:- Eye diagram.



→ fix the Middle point as a Sampling point bcz of Low ISI. i.e called Best Sampling time.



a. Time interval for Sampling (Eye Width).

b. Distortion at Zero crossings.

c. Eye opening height → Best sampling time.

↑ ⇒ Noise margin

more ⇒ (more SNR) ↑

d. Distortion at the sampling instant.

e. Total Jitter (or) Timing Error:-

Jitter occurs when a rising (or) falling edges occur at times that differ from ideal time.

Jitter is Random by

i. Reflections,
ii. ISI
iii. Crosstalk.

Measure of Jitter (Time variation of Zero crossings)

Jitter ↑ ⟹ Available eye Width ↓

↓

Poor Signal quality

↓

Higher BER.

Ideal case:- all Zero crossings must be Some integer multiples of T.

practical case:- There will be Δ away from T.

f. Slope = Sensitivity of the system to timing Error, rate of closure of eye, when the sampling time is varied.

Module-5

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PRINCIPLES OF SPREAD SPECTRUM

5. Spread Spectrum Communication Systems
5.1 Model of a Spread Spectrum Digital Communication System
5.2 Direct Sequence Spread Spectrum Systems,
5.2.1 Effect of De-spreading on a narrowband Interference
5.2.2 Probability of error (statement only)
5.3 Some applications of DS Spread Spectrum Signals
5.4 Generation of PN Sequences
5.5 Frequency Hopped Spread Spectrum
5.6 CDMA based on IS-95

(Text 2: 11.3.1, 11.3.2, 11.3.3, 11.3.4, 11.3.5, 11.4.2).

Text 2: John G Proakis and Masoud Salehi, —Fundamentals of Communication Systems||, 2014 Edition, Pearson Education, ISBN 978-8-131-70573-5.

5.1 Model of a Spread Spectrum Digital Communication System

1. Explain the model of a spread spectrum digital communication system **(06 March) June-July 2018.**

5.2 Direct Sequence Spread Spectrum Systems

5.2.1 Effect of De-spreading on a narrowband Interference
5.2.2 Probability of error (statement only)

1. Explain the generation and demodulation of direct sequence spread spectrum signals with necessary equation and block diagram. **(07 Marks) June-July 2018.**

2. Explain the effect of dispreading on a Narrow band interference in direct sequence spread spectrum systems. A direct sequence spread spectrum signal is designed to have the power ration P_R/P_N at the intended receiver is 10^{-2}. If the desired $\frac{E_b}{N_0} = 10$ for acceptable performance, determine the minimum value of processing gain. **(06marks) June-July 2018.**

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3. Explain the working of Direct Sequence Spread Spectrum transmitter and receiver with neat diagram, waveform and expression. **(08 Marks) Dec 2018-Jan 2019.**

4. In a DS/BPSK system, the feedback shift register used to generate the PN sequence has length m=19. The system is required to have an average probability of symbol error due to externally generated interfering signals that does not exceed 10^{-5}. Calculate the following system parameters in decibels:
i. Processing gain ii. Antijam margin
(Assume $Q(4.25)=10^{-5}$ *or* $erfc(3) = 2x10^{-5}$) **(04 Marks) Dec 2018-Jan 2019 / (04 Marks) June-July 2019.**

5. Explain the generation and demodulation of DS spread spectrum signal. **(06 Marks) June-July 2019.**

6. Explain the generation of direct sequence spread spectrum with relevant waveforms and spectrums. **(07 Marks) Dec 2019- Jan 2020.**

5.3 Some applications of DS Spread Spectrum Signals

1. Write a note on low detectability signal transmission as an application of direct sequence spread spectrum. **(03 Marks) June-July 2018.**

2. Write a note on code division multiple access as an application of direct sequence spread spectrum. **(03 Marks) June-July 2018.**

3. List and briefly explain any 3appliations of direct sequence spread spectrum. **(05 Marks) Dec 2018-Jan 2019.**

4. Write a note on application of spread spectrum in wireless LAN's. **(04 Marks) June-July 2019 / (03 Marks) Dec 2019- Jan 2020.**

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5.4 Generation of PN Sequences

1. What is a PN sequence? Explain the generation of maximum-length sequences (ML- sequence). What are the properties of ML sequences? **(04 Marks) Dec 2018-Jan 2019.**

2. With a neat diagram explain the generation of PN sequences and state its properties. **(06 Marks) June-July 2019.**

3. Draw the 4-stage linear feedback shift register with 1st and 4th stage is connected to Modulo-2 adder. Output of Modulo-2 is connected to 1st stage input. Find the output PN sequence and write he autocorrelation function with initial state 1000. **(06 Marks) Dec 2019-Jan 2020.**

5.5 Frequency Hopped Spread Spectrum

1. With a neat block diagram, explain the frequency hopped spread spectrum. **(07 Marks) June-July 2018.**

2. A slow frequency Hopped/MFSK system has the following parameters,
   i. The number of bits/MFSK symbols=4.
   ii. The number of MFSK symbols per hop=5.
   iii. Calculate the processing gain of the system in decibles. **(03 Marks) Dec 2018-Jan 2019.**
3. With a neat block diagram, explain frequency Hopped spread spectrum technique. Explain the terms chip rate, Jamming Margin and Processing gain. **(08 Marks) Dec 2018-Jan 2019.**

4. Explain with neat block diagram FH spread- Spectrum system. **(06 Marks) June-July 2019**

5. With necessary block diagram, explain the transmitter and receiver of frequency ho spread spectrum. (08 Marks) Dec 2019- Jan 2020.

5.6 CDMA based on IS-95

1. With a neat block diagram, explain the IS-95 reverse link. **(06 Marks) June-July 2019.**
2. With a neat block diagram explain the CDMA system based on IS-95. **(08Marks) Dec 2019- Jan 2020.**

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Module-5

Spread Spectrum Communication

→ Mainly developed for Secured Communication.
it is a very old technique, in 1945's it was used
Eg:- in Military based Application's. for secured comm^n.
- presently in 3G cellular phones, CDMA.

Spread Spectrum ⇒ Spectrum "spreading"

Important Note:-
* In Spread spectrum communication, the transmission data occupies a Bandwidth in Excess of the Minimum B.W necessary to transmit the data."
[i.e required Largest B.W].
* Spectrum "spreading" is accomplished by codes that is independent of data. Same code is used at the receiver to "despread" the received signal so that the original data may be recovered.

Codes:-
Pseudo Noise Sequence (or) Pseudo Random Sequence
(or) Maximum Length (ML) Sequence
↑ PN Sequence.

How to generate PN Sequence

→ (no.of FF's)

Next
2. Linear Feedback shift registers (LFSR)
[No External i/p, Works based on the F/B mechanism]

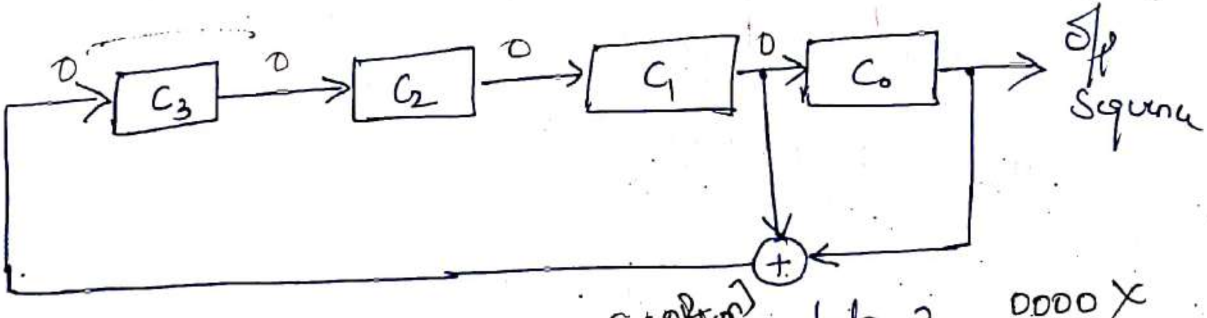
- → LFSR contains 'm' number of flipflops.
- → no External i/p for LFSR.
- → initial content of the all FF's shouldnot be Zero.
  [i.e all FF's should not be initialized by Zero].
- → 4 Length of the generated PN Sequence is $N = (2^m - 1)$.

Eg:- $m = 3$
$N = 2^3 - 1 = 8 - 1 = 7$.

* Generated ~~pn~~ Sequence is periodic in nature with fundamental period is N.

Example:-

(0000) X
* initialization of all FF's content should not be with Zero's.



[Ex-or operation] modulo-2 adder

fig:- Generation of P.N Sequence.

0000 X
000 1
:
1111
carry there

Assume initial content is 1000 (initial state).

$C_3 = C_1 \oplus C_0$

Clk	C_3	C_2	C_1	C_0
	1	0	0	0 ← Initial content
1	0	1	0	0
2	0	0	1	0
3	1	0	0	1
4	1	1	0	0
5	0	1	1	0
6	1	0	1	1
7	0	1	0	1
8	1	0	1	0
9	1	1	0	1
10	1	1	1	0
11	1	1	1	1
12	0	1	1	1
13	0	0	1	1
14	0	0	0	1
15	1	0	0	0 ← initial sequence repeated
16	0	1	0	0

$\boxed{\text{o/p sequence is } C_0}$

i.e pn sequence.

m = 4 ff

$N = 2^m - 1 = 2^4 - 1$

$\boxed{N = 15}$

↳ Length of pn Sequence is 15 it is periodic with fundamental period is 15.

PN Sequences:

0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

obs:-

* Sequence is random over a period $N = 15$.

* This pn sequence should satisfy certain properties i.e are

Properties of PN sequence

i. Balanced property.

$N = 15$

no. of 1's $= 8$

no. of Zero's (0's) $= 7$

$\Rightarrow$ no. of 1's = no. of 0's + 1

if PN Sequence length is very very large

ie $N \uparrow$

$$P(1) = \frac{\text{No. of 1's}}{N}$$

$$P(0) = \frac{\text{No. of 0's}}{N}.$$

Note:- ie for a Larger value of N

$$\boxed{p(1) \simeq p(0) = \frac{1}{2}}$$

Say eg!: $N = 1000$

$$p(1) = \frac{501}{1000}$$

$$p(0) = \frac{499}{1000}$$

$\rightarrow p(1) \simeq p(0)$.

i.e PN Sequence is Balanced

Note:- for a Better performance the value of N must be Larger as possible.

ii. Run property :-

* "Out of total number of runs $\frac{1}{2^n}$ of total runs are of length n".

* run's are nothing but Continuous 1's (or) Continuous 0's.

Eg:- P-N Sequence

$$\underbrace{0\ \ 0\ \ 0}_{1}\ \ \underbrace{1}_{2}\ \ \underbrace{0\ \ 0}_{3}\ \ \underbrace{1\ \ 1}_{4}\ \ \underbrace{0}_{5}\ \ \underbrace{1}_{6}\ \ \underbrace{0}_{7}\ \ \underbrace{1\ \ 1\ \ 1\ \ 1}_{8}$$

Total number of runs = 8

In general total no. of runs in any PN sequence of length 'N' $= \frac{N+1}{2}$ | N - Length of the PN sequence.

here $N = 15$

$\therefore$ no. of runs $= \frac{15+1}{2} = \frac{16}{2} = 8$.

By Defn

$n = 1 \Rightarrow \frac{1}{2^1} \times 8 = \frac{8}{2} = 4 \Rightarrow$ Length 1.

i.e 4 run's of Length 1 . i.e Runs 2, 5, 6, 7 }

$n = 2 \Rightarrow \frac{1}{2^2} \times 8 = \frac{8}{4} = 2 \Rightarrow$ Length is 2

i.e 2 run's of length 2 . i.e Runs {3, 4 }

$n = 3 \Rightarrow \frac{1}{2^3} \times 8 = \frac{8}{8} = 1 \Rightarrow$ Length 3

i.e 1 run's of Length 3 . i.e Runs { 1 }.

Finally

$$\underset{\text{total no of runs}}{8} - \underset{\text{(completed till)}}{7} = \overset{\text{run}}{\underline{1}} \text{ of length } 4.$$

i.e Run 8.

iii. Auto correlation property :-

$m=4, \; N=2^4-1=15.$

	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
Mapping	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
C(n):	1	1	1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1

Note:- $c^2(n) = 1$ for all n.

Auto correlation :-

$$R(d) = \frac{1}{N}\sum_{n=1}^{N} C(n)\,C(n-d)$$

d ← time lag.
N ← Length of pn sequence.

let $d=0$, & $N=15$

$$R(0) = \frac{1}{15}\sum_{n=1}^{15} C(n)C(n) = \frac{1}{15}\sum_{n=1}^{15} C^2(n)$$

C(n):	1	1	1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1
C(n):	1	1	1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1
$c^2(n) =$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$\therefore G^2(n) = 1$

$$\sum_{n=1}^{15} c^2(n) = \sum_{n=1}^{15} 1 = 15 \Longleftarrow$$

6

$$R(0) = \frac{1}{15}\sum_{n=1}^{15} c^2(n) = \frac{1}{15}(15) = 1.$$

$$\therefore \boxed{R(0) = 1}$$

d=1 $\Rightarrow R(1) = \frac{1}{15}\sum_{n=1}^{15} c(n)\,c(n-1)$

first bit

$c(n)$:	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1
$c(n-1)$ {Circular Shift}	−1	1	1	1	−1	1	1	−1	−1	1	−1	1	−1	−1	−1
$c(n)c(n-1)$	−1	1	1	−1	−1	1	−1	1	−1	−1	−1	−1	1	1	1

$$\sum_{n=1}^{15} c(n)\,c(n-1) = -1$$

$$R(1) = \frac{1}{15}\sum_{n=1}^{15} c(n)\,c(n-1) = \frac{-1}{15}$$

IIIy **d=2** $R(2) = \frac{1}{15}\sum_{n=1}^{15} c(n)\,c(n-2)$ ← Circular right shift by 2 unit

the value of $\sum_{n=1}^{15} c(n)\,c(n-2) = -1.$

$$\therefore \boxed{R(2) = \frac{-1}{15}}$$

IIIy $R(3) = R(4) = \ldots\ldots = R(14) = \frac{-1}{15}$

when $d = 15$ $R(15) = 1 = R(0)$ bcz $R(d)$ is periodic
i.e ACF is periodic.

ie With $N = 15$

$$R(0) = R(15) = R(30) = R(K \cdot N) = 1.$$

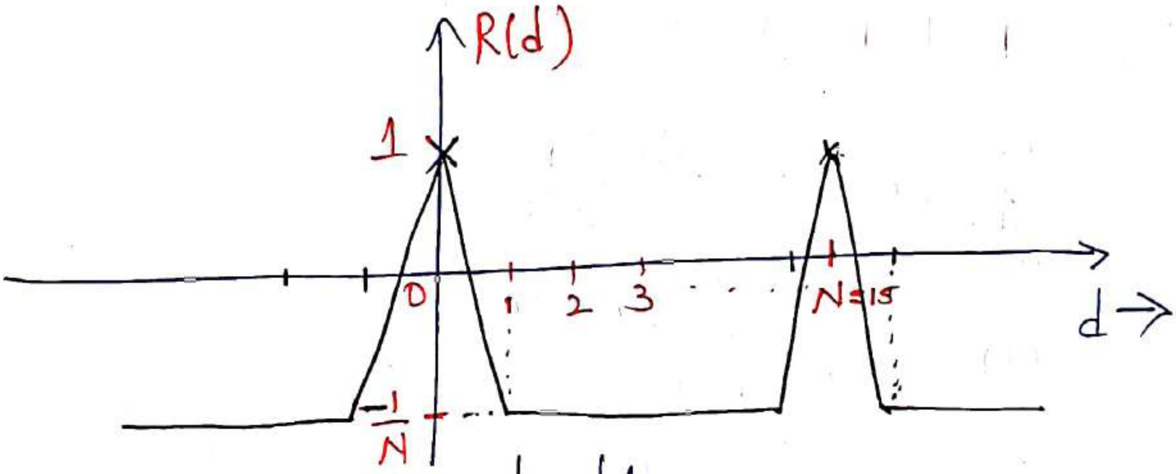
$$R(1) = R(2) = \ldots = R(14) = -1/15.$$

$K \in Z$ integer

In general

$$R(d) = \begin{cases} 1 & ; d = 0, \pm N, \pm 2N \ldots \\ -\frac{1}{N} & ; \text{others} \end{cases}$$

N - Length of the PN sequence.



Note:- PN Sequence inherently Satisfies the Auto correlation property.

Problem:-

A 3-stage shift register with a Linear feedback generates the sequence:

0 1 0 1 1 1 0 0 1 0 1 1 1 0

a. Determine the period of the given infinite sequence.

b. Verify the three properties of PN sequence for the given Sequence.

Soln: given no. of shift registers m = 3.

$N = 2^m - 1 = 2^3 - 1 = 8 - 1 = 7.$

$\boxed{0\ 1\ 0\ 1\ 1\ 1\ 0}\ 0\ 1\ 0\ 1\ 1\ 1\ 0$

Repeated.

∴ Fundamental period $\boxed{N = 7}$

$c(n) = \{0, 1, 0, 1, 1, 1, 0\}$

i. Balance property:

no. of 1's = 4

no. of 0's = 3

no. of 1's = no. of 0's + 1

i.e no. of 1's one more than the no of zero's i.e

∴ Balance property is satisfied.

9

ii. Run property :-

no of run's $= \frac{N+1}{2} = \frac{7+1}{2} = 4$.

shift .

$$\underbrace{0}_{1} \quad \underbrace{1}_{2} \quad \underbrace{0}_{3} \quad \underbrace{1\ 1\ 1}_{4} \quad \underbrace{0}_{5}$$

violating the rule.

Remedy :- Circular shift by one bit.

i.e

$$\underbrace{0\ 0}_{1} \quad \underbrace{1}_{2} \quad \underbrace{0}_{3} \quad \underbrace{1\ 1\ 1}_{4} .$$

$$\boxed{\frac{1}{2^n} \times \text{total no of run} = \text{runs of length } n}$$

n=1 : $\frac{1}{2^1} \times 4 = 2$ run's of length 1 .
ie Run [2, 3]

n=2 : $\frac{1}{2^2} \times 4 = 1$ run of length 2
i.e Run 1 .

∴ Remaining
$4-3 = 1$ run of length 3.
ie Run 4 .

iii. Autocorrelation property :-

$$\begin{array}{ccccccc} 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & -1 & 1 & -1 & -1 & -1 \end{array}$$

Map
$0 \rightarrow 1$
$1 \rightarrow -1$

$$R(d) = \frac{1}{N} \sum_{n=1}^{N} c(n)\, c(n-d)$$

$$R(0) = \frac{1}{7}\sum_{n=1}^{7} c^2(n) = \frac{1}{7}\sum_{n=1}^{7}(1) = \frac{1}{7}(7)$$

$$\boxed{R(0) = 1}$$

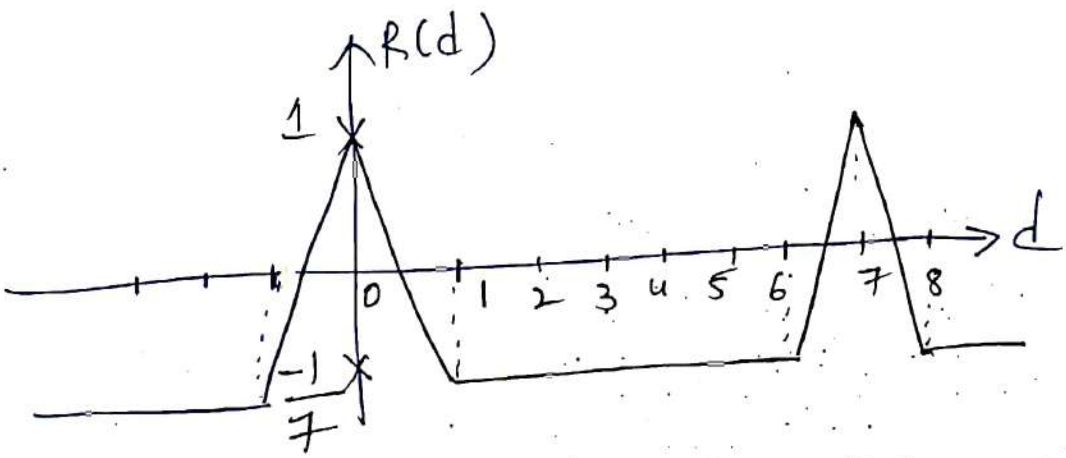
$$R(1) = \frac{1}{7}\sum_{n=1}^{7} c(n)\,c(n-1)$$

c(n)	1	1	-1	1	-1	-1	-1
c(n-1)	-1	1	1	-1	1	-1	-1
c(n)c(n-1)	-1	1	-1	-1	-1	1	1

$$\sum_{n=1}^{7} c(n)\,c(n-1) = -1$$

$$R(1) = \frac{1}{7}\sum_{n=1}^{7} c(n)\,c(n-1) = \frac{-1}{7} = R(2) = \cdots = R(6)$$

and $R(7) = 1$.



5.4 Generation of PN Sequences

- What is a PN sequence? Explain the generation of maximum-length sequences (ML- sequence). What are the properties of ML sequences? **(04 Marks) Dec 2018-Jan 2019.**

- With a neat diagram explain the generation of PN sequences and state its properties. **(06 Marks) June-July 2019.**

Soln:- Defn:- A pseudo-Noise (PN) Sequence is defined as a Coded sequence of 0's and 1's with Certain Auto-correlation properties.

* The PN Sequence used in Spread spectrum communication are periodic.

* The Length of the PN Sequence is given by $N = 2^m - 1$.
where m - Number of Flip-flops.

Generation of PN Sequence:-

* Using m-stage shift register's [LFSR] → Linear feedback shift register's, it is possible to generate a periodic Sequence of Length i.e $(2^m - 1)$ bits. Such a Sequence are also called Maximum Length (ML) Sequence.

* initial Content of all the shift Register's should not be Zero.

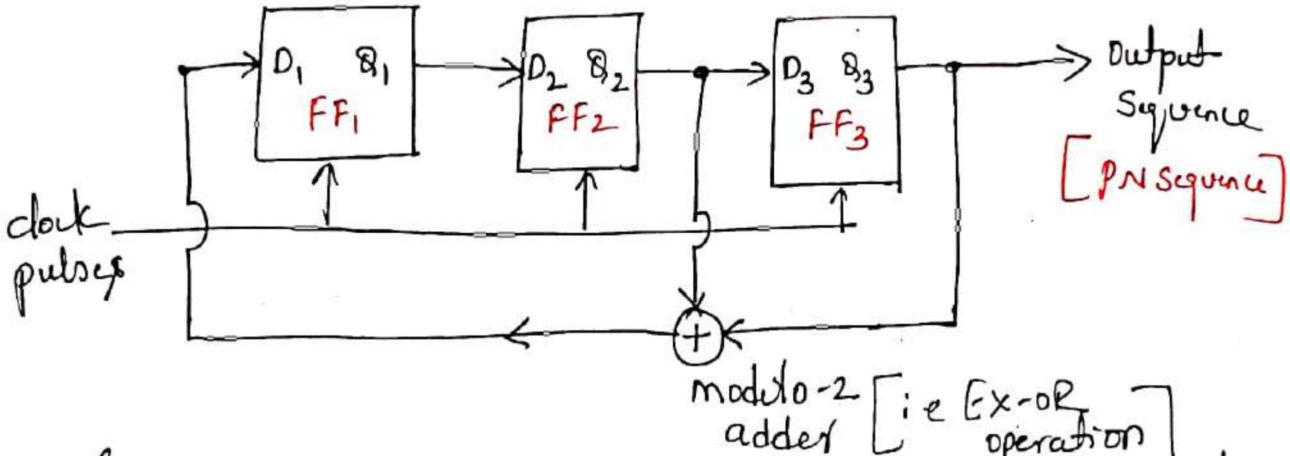


fig a:- 3 stage LFBSR used for generating Maximum Length Sequence.

- * The Shift register operation is controlled by a sequence of clock pulses. for every clock pulse the contents of each stage of the register is shifted by one position to the right.
- * For every clock pulse the contents of 2nd and 3rd stages are modulo-2 added and the result is fed back to the 1st stage.
- * The output of shift register is taken at the Last stage of the flip-flop i.e Q_3.
- * The Length of the PN sequence is given by $N = 2^m - 1$.

i.e $N = 2^3 - 1 = 8 - 1 = 7$.

$\boxed{N = 7 \text{ bits}}$

i.e the generate PN sequence is periodic with fundamental period is $\boxed{N = 7}$.

* Consider the initial state of the shift register is $\underset{Q_1}{1}\ \underset{Q_2}{1}\ \underset{Q_3}{0}$

$Q_1 = Q_2 \oplus Q_3$

clk	Q_1	Q_2	Q_3
—	1	1	0
1	1	1	1
2	0	1	1
3	0	0	1
4	1	0	0
5	0	1	0
6	1	0	1
7	1	1	0
8	1	1	1

← initial state/sequence

← PN sequence

$\boxed{N=7}$

← initial Sequence is repeated

The generate PN Sequence = O/p of shift register-3 ie Q_3

$$= \underbrace{0\ 1\ 1\ 1\ 0\ 0\ 1}_{N7} \mid \underbrace{0\ 1\ 1\ 1\ 0\ 0\ 1}_{N=7} \mid 0 \ldots$$

i.e the generate PN Sequence is periodic with a fundamental period $N = 7$ bits

Properties of PN sequence | ML sequence

There are three properties of PN sequence

i. Balanced property
ii. Run property
iii. Auto correlation property.

i. Balanced property :-

In Each period of a ML sequence, the number of 1's is always one more than the number of 0's.

i.e no of 1's = (no. of 0's) + 1.

Eg:- Consider the generated PN sequence.

0 1 1 1 0 0 1

$$\left.\begin{array}{l}\text{no of 1's} = 4 \\ \text{no of 0's} = 3\end{array}\right\} \Rightarrow \boxed{\text{no of 1's} = \text{no of 0's} + 1}$$

$$4 = 3 + 1 \checkmark$$

Note:- for a Very Large value of N, probability of occurrence of 0's & 1's i.e $p(0) \simeq p(1) = \frac{1}{2}$.

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ii: Run property :-

* A run is defined as a subsequence of identical symbols within ML sequence.

* The Length of the subsequence is known as the run-length.

* the total number of runs = $\frac{(N+1)}{2}$.

where N - length of PN sequence.

"Out of total no. of runs $\frac{1}{2n}$ of total runs are of length n."

Eg:- Consider the generated PN sequence

$$\underbrace{0}_{1}\quad \underbrace{1\ \ 1\ \ 1}_{2}\quad \underbrace{0\ \ 0}_{3}\quad \underbrace{1}_{4}$$

total no of Runs $= \frac{N+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$.

ie run's 1, 2, 3, 4.

∴ $\frac{1}{2^n} \times \left(\frac{N+1}{2}\right)$ of runs of length n.

n=1, $\frac{1}{2^1} \times 4 = \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}} = 2$ run's of length 1.
[ie run's 1, 4] ⇒ {0, 1}

n=2, $\frac{1}{2^2} \times 4 = \frac{1}{4} \times 4 = 1$ run's of length 2
[ie run 3] ⇒ {00}

Remaining $4 - 3 = 1$ run of length 3.
[ie run 2] ⇒ {111}

iii. Autocorrelation property :-

The autocorrelation function of a ML-sequence is periodic and binary valued.

$$\boxed{R(d) = \frac{1}{N}\sum_{n=1}^{N} C(n)\,C(n-d)}$$

Time lag of A.C sequence

Length of PN sequence

In general $$R(d) = \begin{cases} 1 \; ; & d = 0, \pm N, \pm 2N, \pm 3N \ldots \\ \frac{-1}{N} \; ; & \text{otherwise.} \end{cases}$$

ie Consider a generated p.n sequence

$$\boxed{N=7} \quad \begin{array}{cccccccc} & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ C[n] \Rightarrow & 1 & -1 & -1 & -1 & 1 & 1 & -1 \end{array}$$

Note:-
Map
0 → 1
1 → -1
(or) Vice versa.
$C^2(n) = 1$

C[n-d] ⇒ Circularly shifted sequence by 'd' units.

d=0
$$R(0) = \frac{1}{N}\sum_{n=1}^{N} c(n)\,c(n) = \frac{1}{N}\sum_{n=1}^{N} c^2(n) = \frac{1}{7}\sum_{n=1}^{7}(1)$$
$$= \frac{1}{7}(7) = 1.$$

d=1
$$R(1) = \frac{1}{N}\sum_{n=1}^{N} c(n)\,c(n-1) = \frac{1}{7}\sum_{n=1}^{7} c(n)\,c(n-1) = \frac{-1}{7}.$$

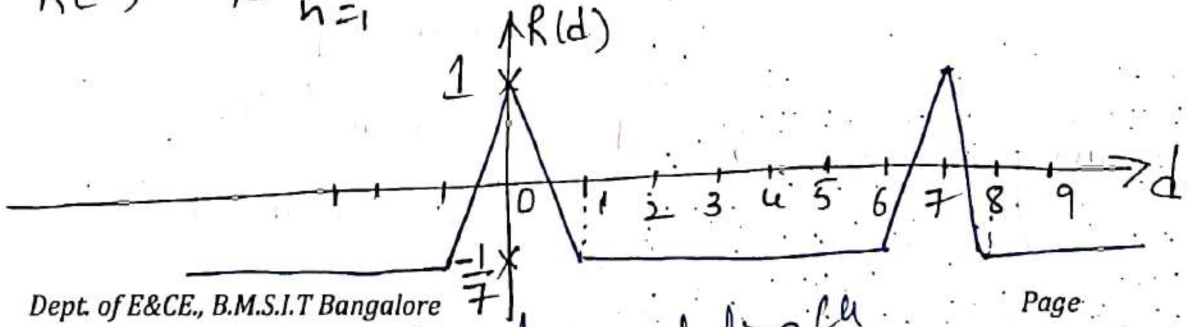
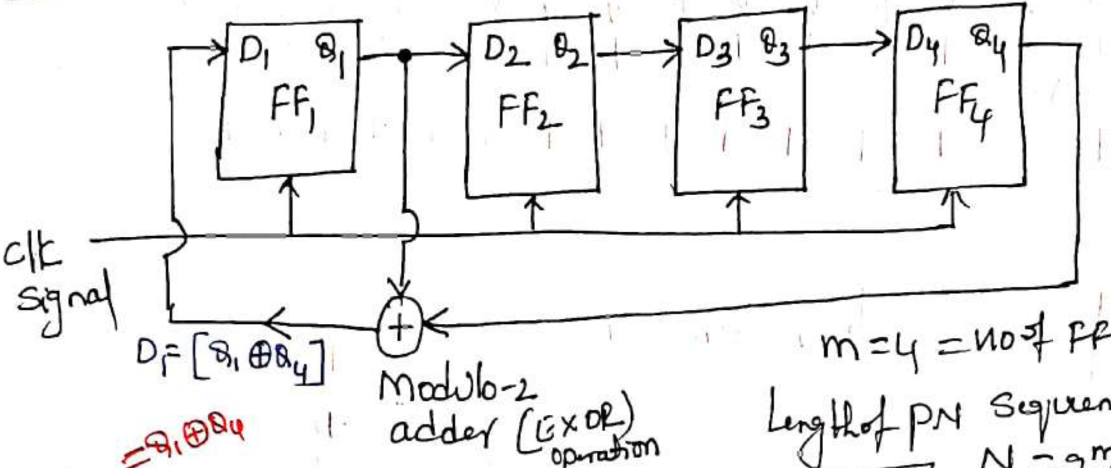


fig a:- Autocorrelation fn

Draw the 4-stage linear feedback shift register with 1st and 4th stage is connected to Modulo-2 adder. Output of Modulo-2 is connected to 1st stage input. Find the output PN sequence and write he autocorrelation function with initial state 1000. **(06 Marks) Dec 2019-Jan 2020.**

Soln:-



m = 4 = no of FF (4 stages)

Length of PN sequence

$N = 2^m - 1 = 2^4 - 1$
$= 16 - 1 = 15$

$\boxed{N = 15}$

clk	Q_1 $= Q_1 \oplus Q_4$	Q_2	Q_3	Q_4
–	1	0	0	0
1	1	1	0	0
2	1	1	1	0
3	1	1	1	1
4	0	1	1	1
5	1	0	1	1
6	0	1	0	1
7	1	0	1	0
8	1	1	0	1
9	0	1	1	0
10	0	0	1	1
11	1	0	0	1
12	0	1	0	0
13	0	0	1	0
14	0	0	0	1
15	1	0	0	0
16	1	1	0	0

← initial state / sequence (given)

← PN sequence of Length N=15

= {0 0 0 1 1 1 1 0 1 0 1 1 0 0 1}

PN-sequence in a periodic with fundamental period N=15 bits.

← PN-sequence repeated.

PN Sequence with $N=15$, Map $\left.\begin{matrix} 1 \rightarrow -1 \\ 0 \rightarrow 1 \end{matrix}\right\}$

$$= [\,0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\,]$$

$$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$$

$c(n)$ $1\ 1\ 1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ 1\ 1\ -1\ 1$

$c(n-1)$ $1\ 1\ 1\ 1\ -1\ -1\ -1\ -1\ 1\ -1\ 1\ -1\ -1\ 1\ 1\ -1$

Circular shift by unity

Autocorrelation function (ACF)

$$R(d) = \frac{1}{N} \sum_{n=1}^{N} c(n)\, c(n-d) = \begin{cases} 1 ; & d=0, \pm N, \pm 2N \ldots \\ \frac{-1}{N} : & \text{Otherwise} \end{cases}$$

$R(d)$ ↑ Time lag of ACF

N ↑ PN sequence Length

$c(n-d)$ ↑ Circularly shifted Sequence with 'd' unit

$R(d)$ is a periodic with ACF 'N'.

$N=15$,

$\underline{d=0}$, $$R(0) = \frac{1}{15}\left[\sum_{n=1}^{15} c^2(n)\right] = \frac{1}{15}[15] = 1$$

$$\Rightarrow \boxed{R(0)=1}$$

$\underline{d=1}$ $$R(1) = \frac{1}{15} \sum_{n=1}^{15} c(n)\, c(n-1)$$

and $$\sum_{n=1}^{15} c(n)\, c(n-1) = -1$$

$$R(1) = \frac{1}{15}(-1) = \frac{-1}{15}$$

$$R(1) = R(2) = \cdots = R(14) = -1/15$$

$\underline{\text{ACF}}$ $$R(d) = \begin{cases} 1 : & d = 0, \pm 15, \pm 30, \ldots \\ \frac{-1}{15} ; & \text{otherwise} \end{cases}$$

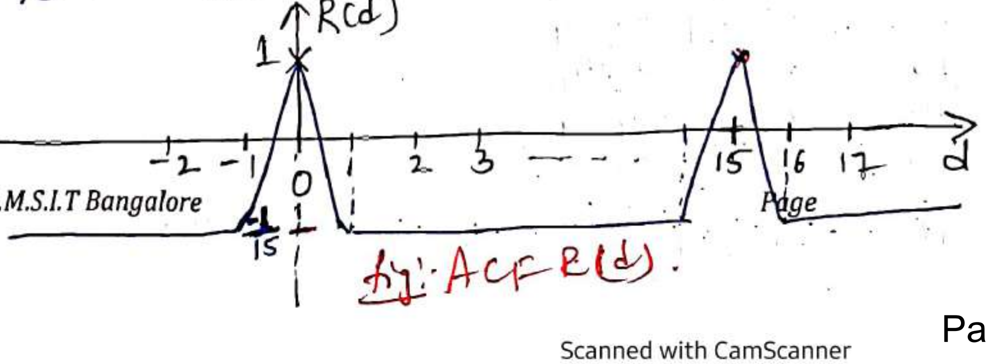


fig: ACF R(d).

"Tell me and I Forget, Show me and I remember, Let me do and I Understand"

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Module-5

5.1 Model of a Spread Spectrum Digital Communication System

Explain the model of a spread spectrum digital communication system **(06 March) June-July 2018.**

Soln:- * The basic Elements of a spread spectrum digital Communication System are illustrated in fig a.

* The channel Encoder and decoder and the modulator and demodulator are the basic Elements of a Conventional digital communication System.

* In addition to these Elements, a Spread-spectrum System Employs two identical pseudorandom Sequence generators, one which interfaces with the modulator at the transmitting End and the Second which interfaces with the demodulator at the receiving end.

Transmitter:-

* The channel Encoder Encodes the input binary Sequence according to some Error control coding technique.

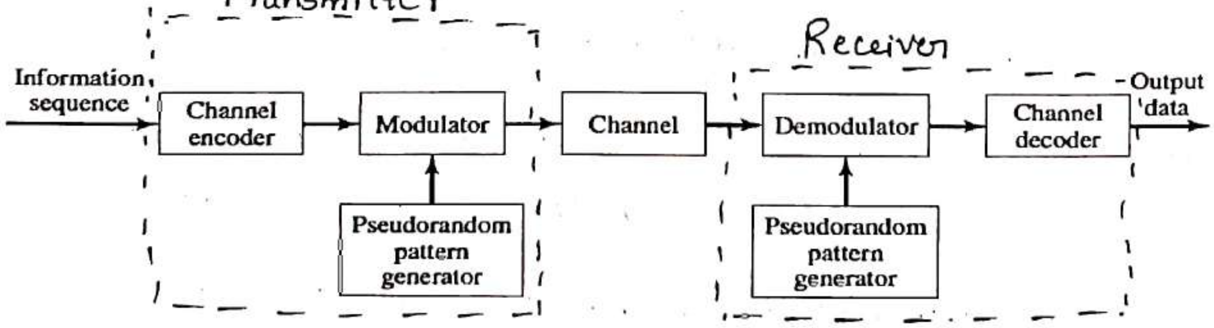


Figure a. Model of spread-spectrum digital communications system.

* The coded sequence is the given to the modulator. The Modulator gets pseudo-random (or) pseudo-noise (PN) sequence from the pseudo-random pattern generator.

* PN Sequence. which is binary -valued sequence; which is used to "Spread" the transmitted signal at the Modulat -or end. This Spreaded Signal is then transmitted over some channel.

Receiver:-

* at the receiver, the demodulator gets coded signal back from the spread spectrum signal.
* For this purpose the demodulator needs the same p-N sequence which was used at the transmitting end. this Sequence helpis to "despread" the received signal at the receiver end.
* Time synchronization of the PN sequence generated at the receiver with the PN sequence contained at the received signal is required in order to properly despread the received spread-spectrum signal.
* The channel decoder at the receiver then gets the binary information sequence back.
* Thus the Receiver can detect the transmitted spread spectrum signals only if it knows the p-N sequence. For any arbitrary receiver it is difficult to know the pseudo-noise sequence since it appears likenoise.

Introduction to DS-SS

[i.e DSSS for BaseBand Signal transmission] [i.e without Modulation techniques]

Terminologies

$b(t) \rightarrow$ Data Sequence (Narrowband)

$C(t) \rightarrow$ code sequence (Wide band) (PN sequence)

I_0 (or) N_0 - Noise powerdensity (W/Hz)

Spreading opr^n. $m(t) \rightarrow$ Spreaded Sequence.

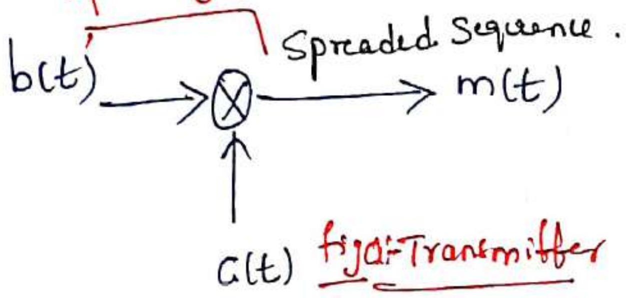


fig a: Transmitter

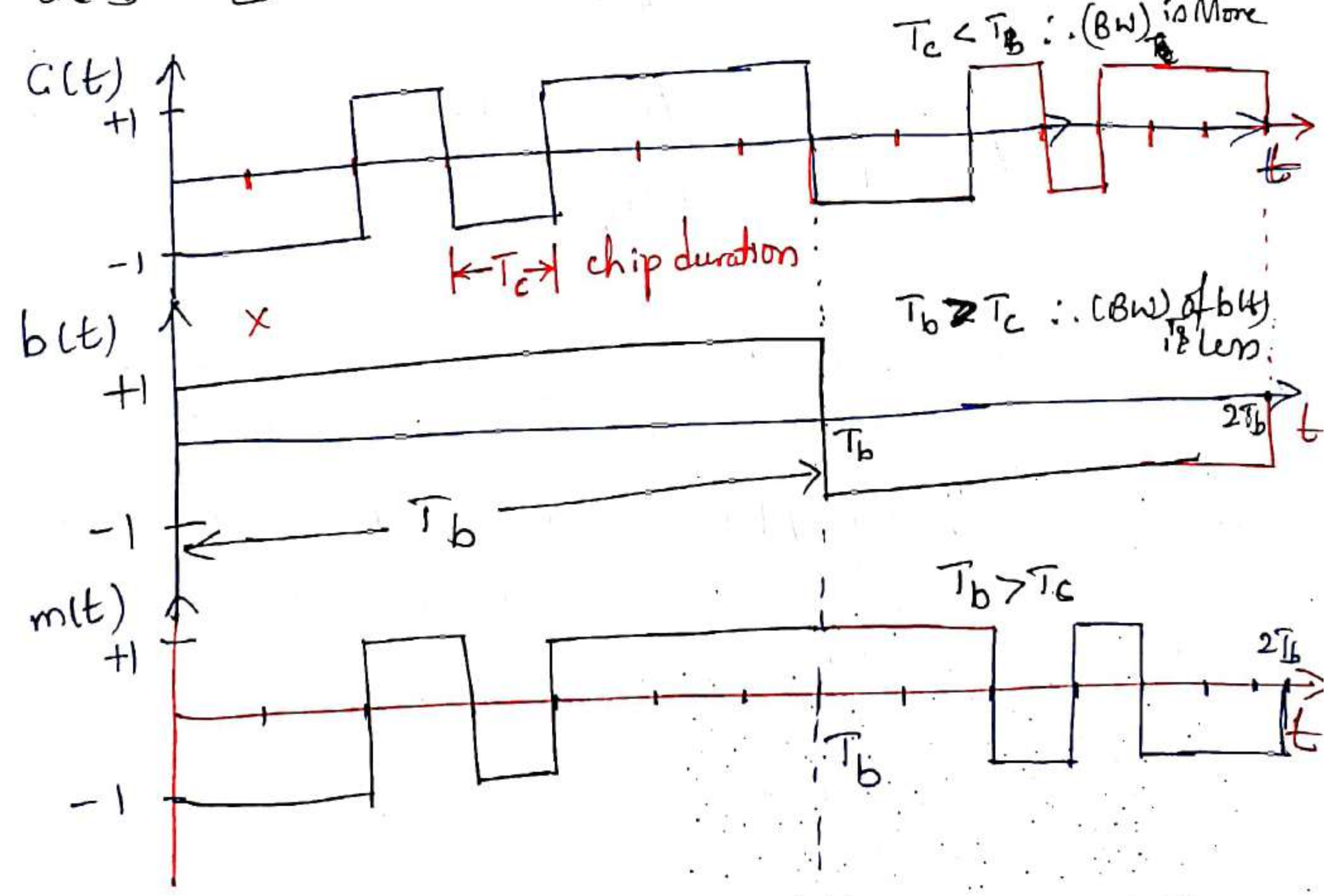
T_c - chip duration

T_b - bit duration

Note: $\boxed{BW = \frac{1}{T}}$ Hz

Consider a PN sequence

$C[n] = [0 \ 0 \ 1 \ 0 \ 1 \ 1 \ 1]$



Before spreading $BW = \frac{1}{T_b}$; $T_b \uparrow \Rightarrow BW \downarrow$

after spreading $BW' = \frac{1}{T_c}$; $T_c \downarrow \Rightarrow BW \uparrow$

Relationship b/w T_b and T_c

In general

$$T_c = \frac{T_b}{7} \Rightarrow \boxed{T_b = N \cdot T_c}$$

Processing gain (PG) :-

$$PG = \frac{T_b}{T_c} = N$$

$$N = \boxed{P \cdot G = \frac{1/T_c}{(1/T_b)} = \frac{\text{B.W required after spreading}}{\text{BW required before spreading}}}$$

$\Rightarrow$ B.W. required after spreading = N. (B.W) required Before Spreading.

By $N \uparrow$ => we can increas BW required after spreading.

i.e BW of spreaded signal is increased by increasing the value of N.

Eg:- Symbol rate $R_s = 1 kbs$

BW = 1 KHz (Before Spreading)

Symbol duration $T_s = \frac{1}{R_s} = 1 msec. = T_b$

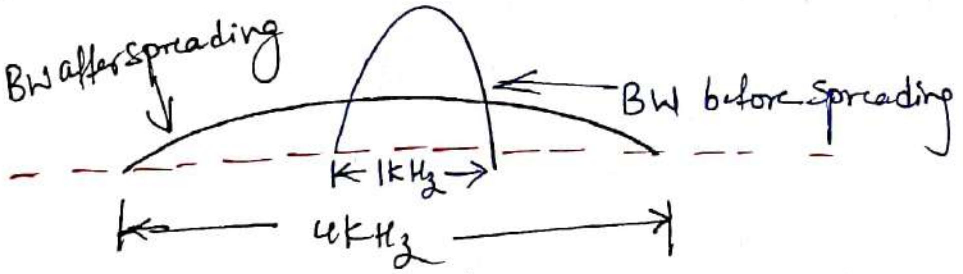
chip duration $T_c = \frac{T_s}{N} = \frac{1 msec}{4} = 0.25 msec.$

let $N = 4$

$$BW' = \frac{1}{T_c} = \frac{1}{0.25 \times 10^{-3}} = 4 KHz$$

B.W required after spreading.

In general $\boxed{\text{BW (after spreading)} = N \times \text{BW (Before spreading)}}$



Advantage of CDMA system / spread spectrum system, ?

i.e it can be used for Secured communication like Military applications.

In Military Application's, the Enemy pple they will introduce Some signal, which will look like the desired signal. i.e basically called Jamming signal, the person who will introducing the Jamming Signal is called Jammer.

Imp How CDMA / S.S given resistance for Jamming Signal ?

Mathematically (or) Effect of De-spreading Narrowband Signal

Spreaded Signal $m(t) = b(t) \cdot c(t)$

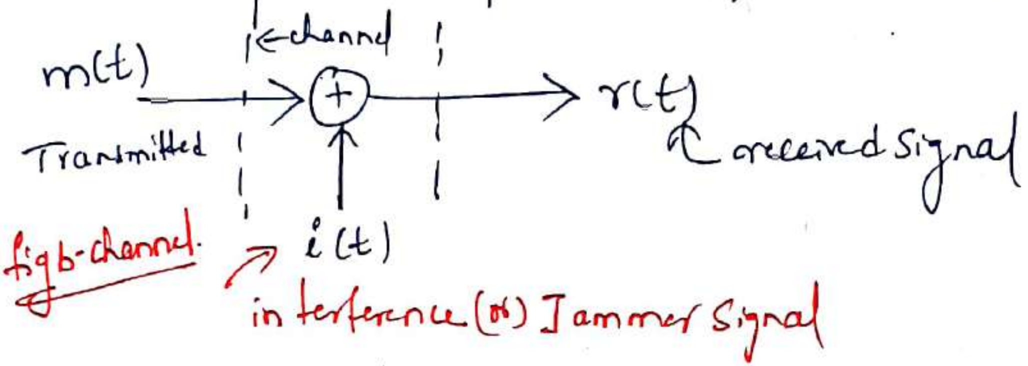


fig b - channel.

Received Signal $r(t) = m(t) + i(t)$

$$r(t) = b(t) \cdot c(t) + i(t)$$

at Receiver :- the Received Signal is Multiplied by Same Spreaded Code / P.N code.

i.e

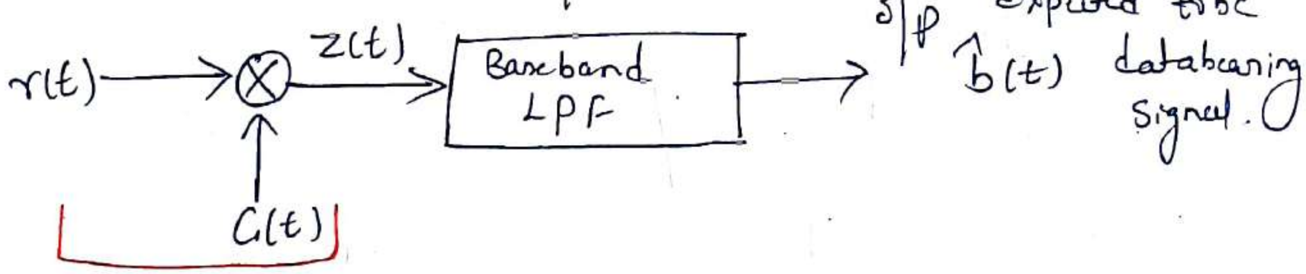


fig c (Receiver) Despreading operⁿ.

$$z(t) = r(t) \cdot c(t)$$

$$z(t) = [b(t) \cdot c(t) + i(t)]\, c(t)$$

$Z(t) = b(t)\, c^2(t) + i(t)\, c(t)$

∵ $c^2(t) = 1$

$c^2(t) = 1$

∴ $Z(t) = b(t) + i(t)\, c(t)$ Eliminate

despreaded term. ↓ LPF i/p of

Spreading term (ie interference term in get despreaded)

Frequency domain Analysis of b(t).

b(t)

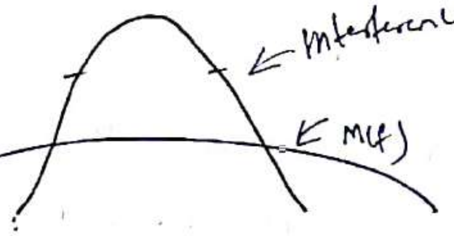
← Jammer doesnot know This is not the transmitted Signal [he will produce the Signal which will similar to B(t)]

m(t)

after spreading of this Transmitted Signal.

i(t)

Received Signal r(t)



R(f) ie M(f) & I(f) are added.

at Receiver:-

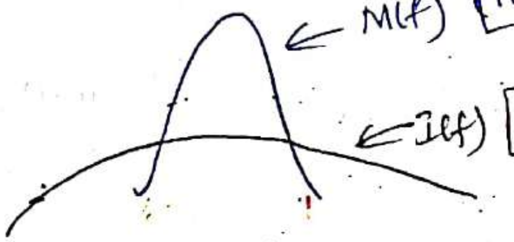
$Z(t) = r(t) \cdot c(t)$

$= [b(t)c(t) + i(t)]c(t)$

$= b(t) + \underline{i(t)c(t)}$

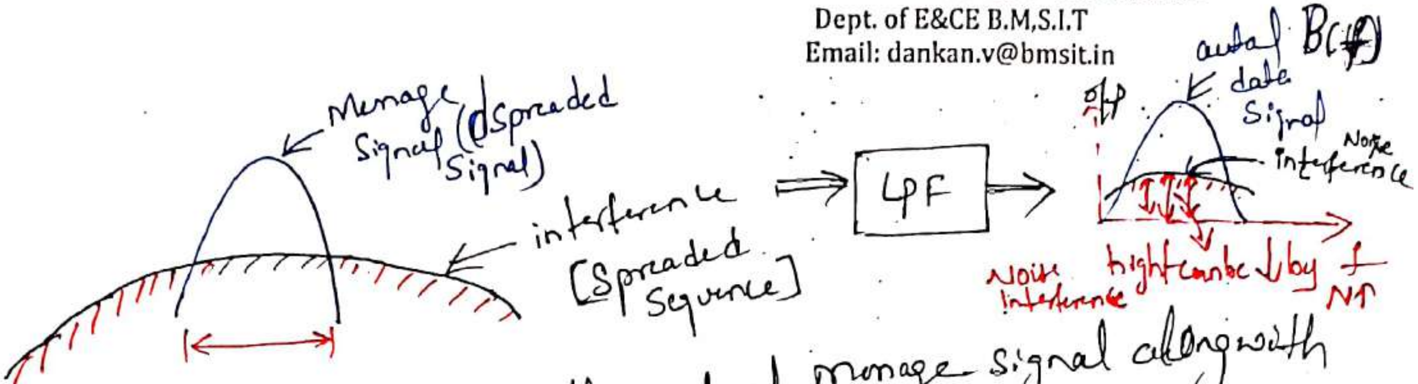
actual Sequence is despreaded

interference is spreaded



At the receiver end, the actual message/data Signal is despreaded and interference Signal is spreaded.

ⓔ



* o/p the LPF is the actual message signal alongwith some portion of the interference will also occurred but that can be Minimized by increasing N value

ie $N\uparrow \Rightarrow$ Spreading $\uparrow \Rightarrow$ Noise interference decreases [ie Noise power density decreases].

Note:- Summary [~~Effect~~ ~~of despreading~~ Narrow band Signal].

i ~~*~~ So, Spread Spectrum Communication not only provides Security it also decreases the Noise power density (i.e Noise interferences).

ii. By increasing the Value of N, Noise power density can be very much decreased. ie $N\uparrow \Rightarrow$ Spreading $\uparrow \Rightarrow N_o\downarrow$ @ $I_o\downarrow$
I_o @ N_o - Noise power density W/H_z

iii * the ability of CDMA / Spread Spectrum Communication is "Resistance to Jamming".

Spread Spectrum technique

Direct Sequence Spread spectrum (DS-SS)

Frequency hopping Spread Spectrum (FHSS)

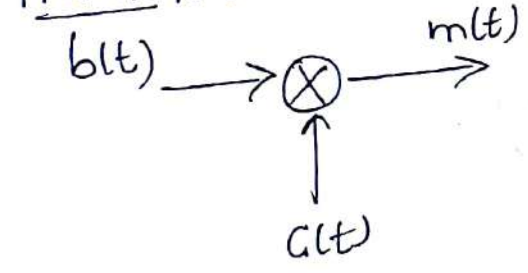
- Slow F.H
- Fast F.H.

here
* Mod^{n} used is BPSK (Binary phase shift keying)

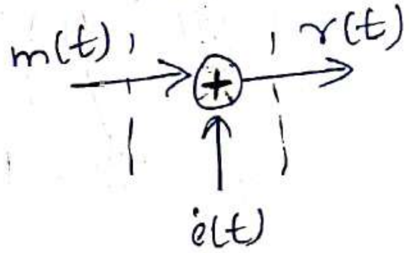
* Mod^{n} used is M-ary Frequency shift keying (MFSK)

i. DSSS with BPSK as Mod^{n} technique.

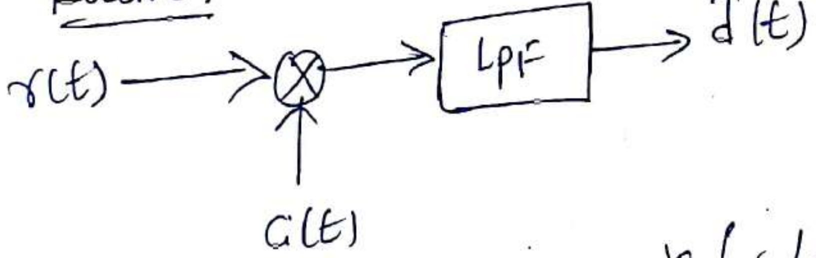
Transmitter



channel



Receiver



✓ This block diagram is valid for base band Signal's [i.e for shorter distance communication.

* for Long distance transmission use Mod^{n} technique is required.

fig: DSSS for Baseband Signal transmission for shorter distance. [i.e without Mod^{n} used]

9

* For Long distance Communication (Ex:- Satellite channel) the spreaded Signal m(t) should be Modulated by RF Modulation. [here preferred RF Mod^n is BPSK].

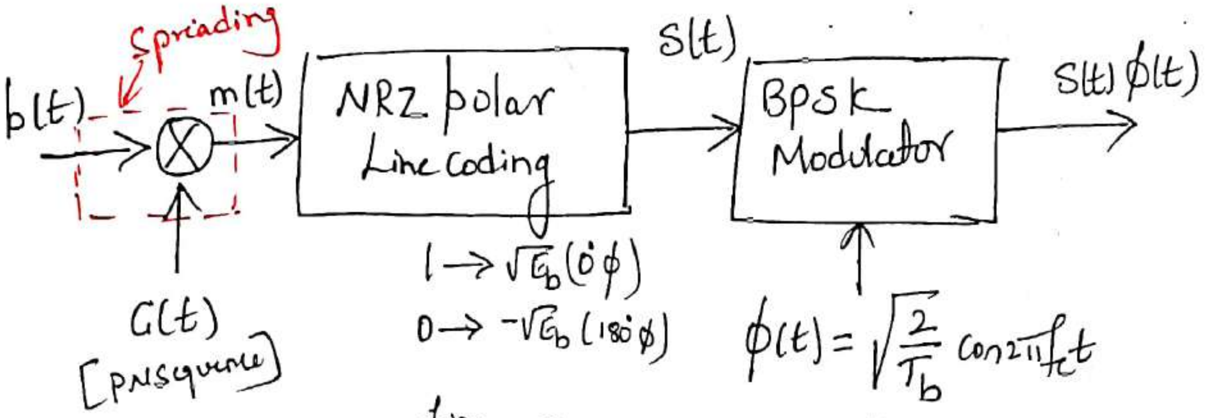


fig a. DSSS-Transmitter

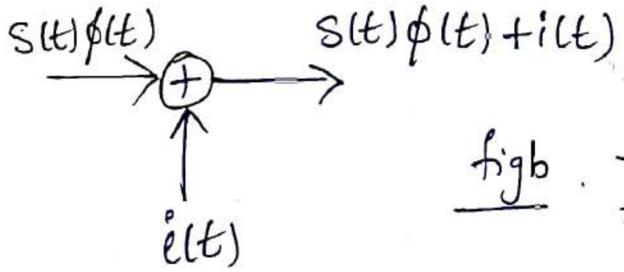


fig b. Channel

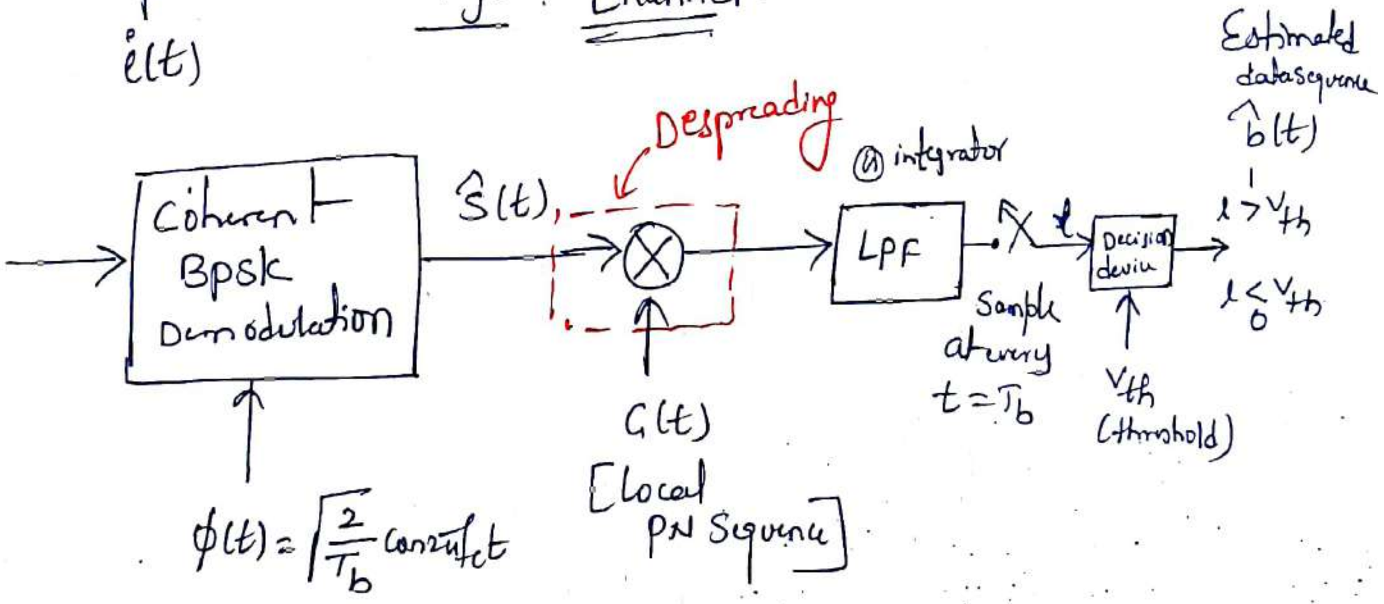


fig c. DSSS-Receiver

Transmitting Bpsk signal without spreading.

Signal to Noise ratio $\underset{\text{without spreading}}{(SNR_I)} = \frac{P}{J} = \frac{\text{Signal power}}{\text{Noise/Jamming (or) interference power}}$.

J - Jamming signal power (or) interference power.

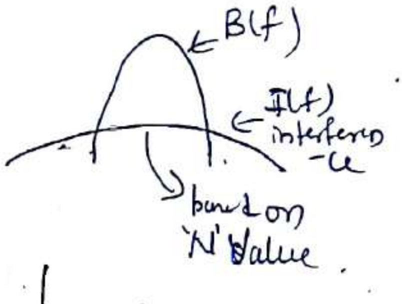
P - Signal $\overset{\text{avg}}{\wedge}$ power

Average power $P = \frac{E_b}{T_b}$

$$\underset{\text{Without spreading}}{(SNR)_I} = \frac{E_b}{J\,T_b} \longleftarrow ⓐ$$

After spreading, then Bpsk Mod^n

$$\underset{\text{(after spreading)}}{(SNR)_o} = \frac{P}{J}$$



Jamming signal power (or) Interference power after spreading is J/N

$$\therefore \; (SNR)_o = \frac{P}{(J/N)}$$

where N - length of PN sequence (or) Processing gain

P - avg signal power.

$$(SNR)_0 = \frac{E_b}{(T_b J/N)}$$

$$(SNR)_0 = \frac{E_b \cdot N}{T_b \cdot J} = N \cdot \left(\frac{E_b}{T_b \cdot J}\right) \leftarrow ⓑ$$

By Comparing eqn ⓐ & ⓑ after spreading

Processing Gain (PG) ⇒ Gain achieved in Signal to Noise ratio due to Spreading

$$PG = \frac{(SNR)_0}{(SNR)_I} = \frac{\text{Signal to noise ratio before spreading}}{\text{Signal to noise ratio after spreading}}$$

$$= \frac{\frac{E_b}{JT_b} \cdot N}{E_b/JT_b} = N$$

$$\boxed{P \cdot G = N}$$

To achieve More gain in Signal to Noise ratio just increase the Value of N. ie length of PN Sequence.

from eqn ⓐ & ⓑ

$$\underset{\text{after Spreading}}{(SNR)_0} = N \cdot \underset{\text{Before Spreading}}{(SNR)_I}$$

(j)

XI) Probability Error of DSSS :—

Prob. of Bit Error rate of BPSK Modulation under AWGN channel is

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \longleftarrow ①$$

W.k.t

Signal to Noise ratio ⇒ $(SNR)_0 = \frac{E_b}{JT_b} \cdot N$

$$N = \frac{T_b}{T_c}$$

$$(SNR)_0 = \frac{E_b}{J\cancel{T_b}} \cdot \frac{\cancel{T_b}}{T_c} = \frac{E_b}{J \cdot T_c}$$

Note:-
old BPSK S/N Before spreading $\left(\frac{E_b}{N_0}\right)$ ⇒ Signal to Nois ratio

$\left(\frac{E_b}{N_0}\right)_{new} = \frac{E_b}{JT_c}$ replace — New SNR after spreading

i.e probability Error of BPSK System after using Spread Spectrum [ie Prob. Error of DSSS System]

$$\boxed{P_{e\,(DSSS)\,BPSK} = \frac{1}{2}\, erfc\left(\sqrt{\frac{E_b}{JT_c}}\right)} \longleftarrow ②$$

→ Bit Error rate Expression of DSSS-BPSK Modulated Signal.

Comparing eqⁿ ① & eqⁿ ②

$$\frac{E_b}{N_0} = \frac{E_b}{JT_c} = \frac{P \cdot T_b}{J \cdot T_c}$$

$P = E_b / T_b$
$\therefore E_b = P \cdot T_b$

$$\frac{J}{P} = \frac{(T_b / T_c)}{(E_b / N_0)}$$; where $\frac{T_b}{T_c} = P.G$

$$\frac{J}{P} = \frac{P.G}{(E_b/N_0)}$$

$$\boxed{\left[10\log_{10}\left(\frac{J}{P}\right)\right]_{dB} = 10\log_{10}(PG) - 10\log_{10}\left(\frac{E_b}{N_0}\right)}$$

$$\boxed{\left(\frac{J}{P}\right)_{dB} \text{ (or) } (J\cdot M)_{dB} = (PG)_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)}$$

J.M - Jamming Margin. $= \frac{J}{P} = \frac{\text{Interference power}}{\text{Signal power}}$.

Significance of J.M = ?

Problem:- given $T_b = 4.095$ msec i. Find P.G

$T_c = 1\mu$sec ii. LFSR Length ? ie m = ?

i. $P.G = N = \frac{T_b}{T_c} = \frac{4.095\times10^{-3}}{1\times10^{-6}}$

$$\boxed{PG = N = 4095}$$

ii. LFSR Length ie no of shift registers

$N = 2^m - 1$

$2^m - 1 = 4095$

$2^m = 4096$

$\Rightarrow \boxed{m = 12}$

iii. For Satisfactory reception, we may assume that avg. Pe should not Exceed 10^{-5}. find J.M

$$P_{e_{(BPSK)}} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

To achieve $P_e \simeq 0.387 \times 10^{-5}$, we need a SNR of i.e $\boxed{\left(\frac{E_b}{N_0}\right) = 10}$

$$(JM)_{dB} = 10\log_{10}(PG) - 10\log_{10}\left(\frac{E_b}{N_0}\right)$$

$$= 10\log_{10}(4095) - 10\log_{10}(10)$$

$$\underline{\underline{(JM)_{dB} = 26.1\ dB}}$$

Interference (J)

$$JM = \frac{PG}{(E_b/N_0)} = \frac{4095}{10} = 409.5 = \frac{J}{P} = \frac{\text{Noise Power}}{\text{Signal power}}$$

Inferences:-
Information bits at the receiver o/p can be detected realibly when the noise (or) interference at the Receiver input is up to 409.5 times the Received signal power.

$P_e = 10^{-5}$ [ie out of 1,00000 bit one bit can be Error].

* $\Rightarrow$

m

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Explaination:

* In a Normal / Conventional Receivers, the received Signal power must be More than the Noise power, in order to achieve desired bit error rate of 10^{-5} [ie out of 1 lakh bits one bit can be a Error].

* But In DSSS - Receiver, Even if the noise power (or) Interference power is 409.5 times More than the received Signal power, we can achieve the desired bit Error rate of 10^{-5}. This is the greatest advantage of DSSS.

$$\text{i.e}\quad J.M = \frac{J}{P} = 409.5 = \frac{\text{Noise/interference power}}{\text{Received Signal power}}.$$

$$\Rightarrow \text{Noise (or) Interference power} = 409.5\,[\text{Received Signal power}]$$

Note: DSSS is the
→ powerful advantage against Jamming (interference), which is realized through the Clever use of Spread Spectrum.

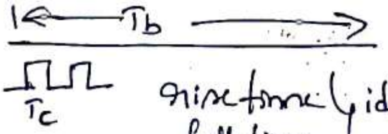
* Jamming Margins is a Measure immunity against Jamming.

Limitation of DSSS :-

$$JM = \frac{N}{(E_b/N_0)}\,; \quad JM\uparrow \Rightarrow N\uparrow \text{ and } N = \frac{T_b}{T_c} = \frac{\text{fixed}}{\text{Varied}}$$

to $N\uparrow \Rightarrow T_c\downarrow$

Can't increase N value after certain maximum value.



rise time } ideally can be Zero
fall time } but not practically.

i.e There is a N↑ results Limitations of rise / fall time. this is the draw-back of DSSS and this can be overcome by FSSS.

n

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5.2 Direct Sequence Spread Spectrum Systems

- Explain the generation and demodulation of direct sequence spread spectrum signals with necessary equation and block diagram. **(07 Marks) June-July 2018.**
- Explain the working of Direct Sequence Spread Spectrum transmitter and receiver with neat diagram, waveform and expression. **(08 Marks) Dec 2018-Jan 2019.**
- Explain the generation and demodulation of DS spread spectrum signal. **(06 Marks) June-July 2019.**
- Explain the generation of direct sequence spread spectrum with relevant waveforms and spectrums. **(07 Marks) Dec 2019- Jan 2020.**

Soln:- Transmitter:-

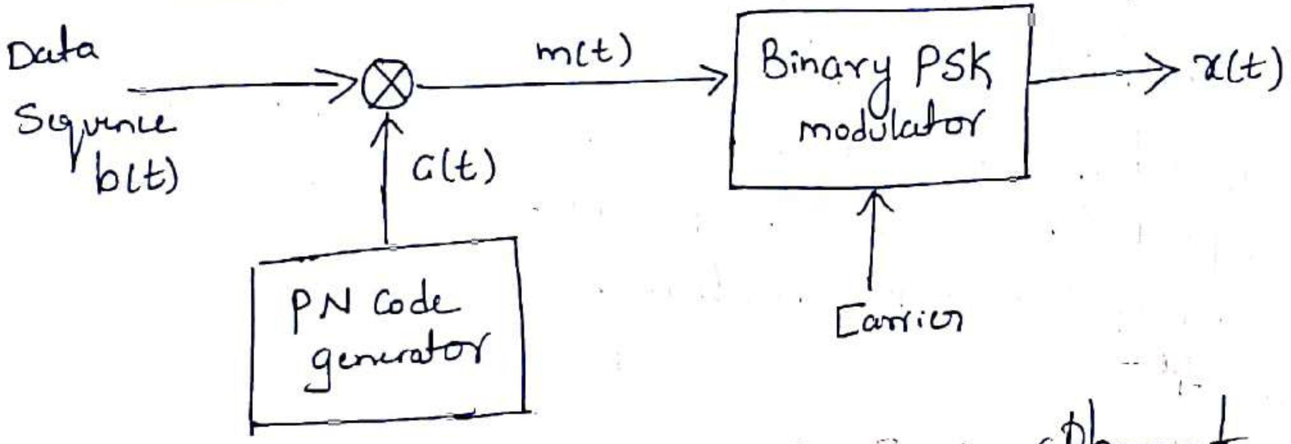


fig a. Direct-sequence Spread coherent phase shift keying. Transmitter.

The transmitter includes two stages of Modulation

* In 1st stage, the data sequence b(t) is modulated with the code Sequence C(t). So the spread Signal is

$$\boxed{m(t) = b(t) \cdot C(t)}$$

* In 2nd stage, the Spread Signal m(t) is Modulated with the binary PSK Modulator.

3

Table:- Truth table of phase Modulation $\theta(t)$, Radians

		Polarity of Data Sequence b(t) at Time t	
		+	−
Polarity of PN Sequence c(t) at time t	+	0	π
	−	π	0

* When the polarity of b(t) and c(t) are same, the product $b(t)\cdot c(t) = 1$, hence the phase of the BPSK signal is $(2\pi f_c t)$.

* Ily when b(t) and c(t) are of different polarities the product $b(t)\cdot c(t) = -1$, hence the phase of the BPSK Signal $(2\pi f_c t + \pi)$ radians.

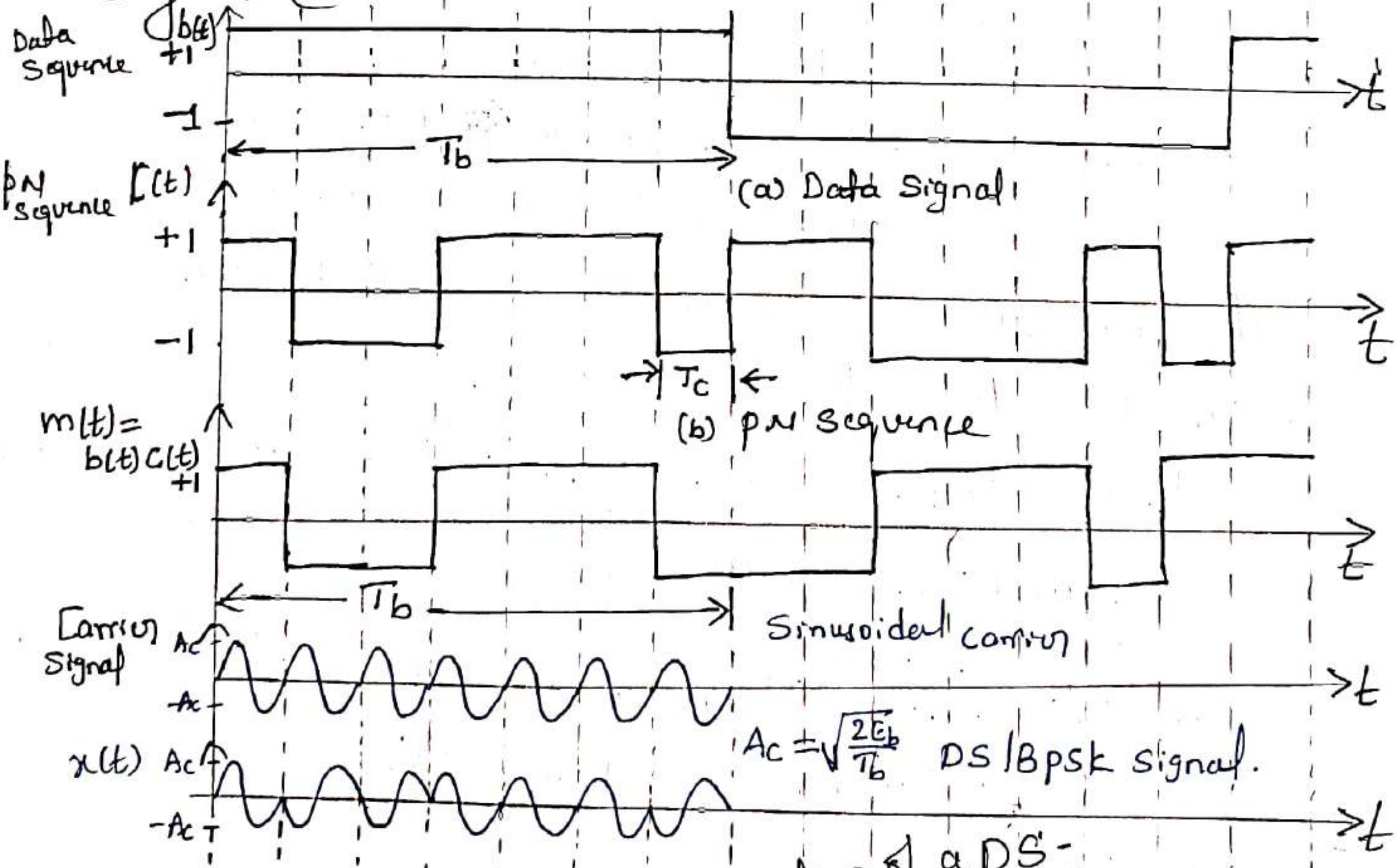
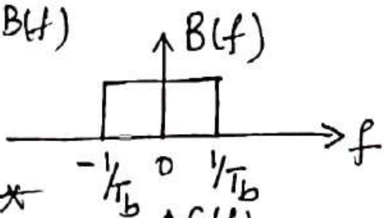


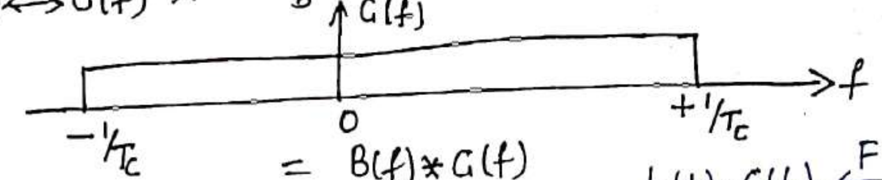
fig: Generation of a DS-Spread Spectrum Signal.

$b(t) \xleftrightarrow{FT} B(f)$



(a) data signal b(t) in FD (spectrum)

$c(t) \xleftrightarrow{F.T} C(f)$ *



$= B(f) * G(f)$

$b(t) \cdot G(t) \xleftrightarrow{F.T} B(f) * G(f)$

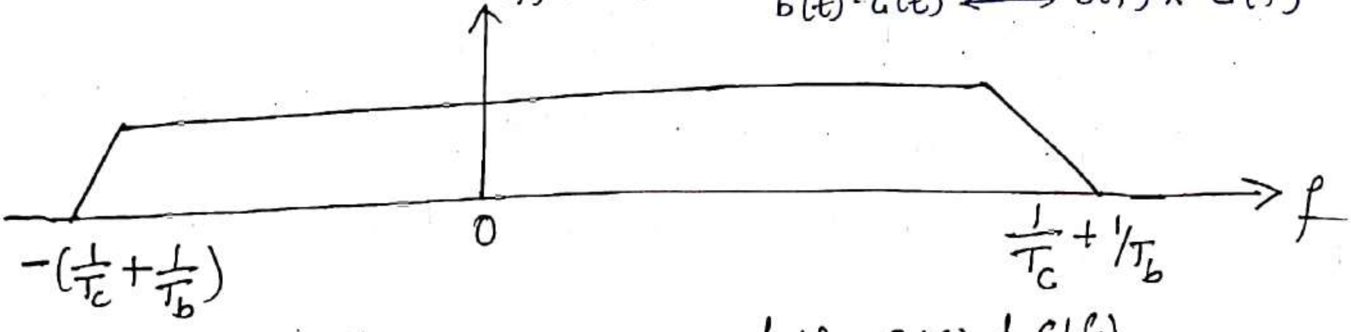


fig:- Convolution of Spectra of the B(f) & G(f).

where T_c is called the "chip interval". The reciprocal of T_c i.e $\left(\frac{1}{T_c}\right)$ is called chip rate and corresponds (approximately) to the Bandwidth W of the transmitted signal (or) spreaded signal i.e

$$\underbrace{(BW)}_{\text{Spreaded Signal}} \simeq \frac{1}{T_c} \text{ Hz}$$

* The ratio of the bit interval T_b to the chip interval T_c is usually selected to be an integer number.

i.e $L_c = \frac{T_b}{T_c}$

'L_c' is the number of chips of the PN code sequence (per) / information bit.

(or) L_c represents the number of possible 180° phase transistions in the transmitted signal during the bit interval T_b.

Receiver:-

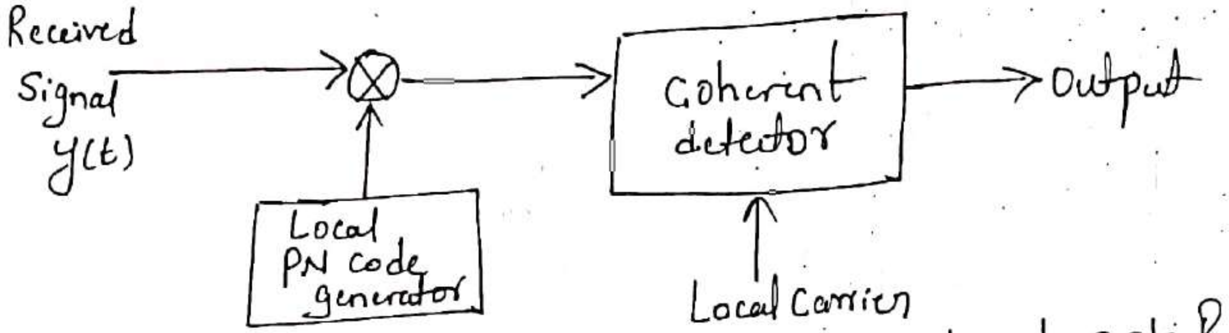


fig6. Direct Sequence spread coherent psk Receiver

The receiver Consist of two stages of demodulation

- * In 1st Stage demodulation, the received signal y(t) and a Locally generated replica of the PN Sequence are applied to a product/multiplier.
- * In 2nd stage, m(t) is despread by multiplying it by c(t). i.e it consists of a coherent detector, the output which provides an estimate of the original data sequence.

Model for Analysis:-

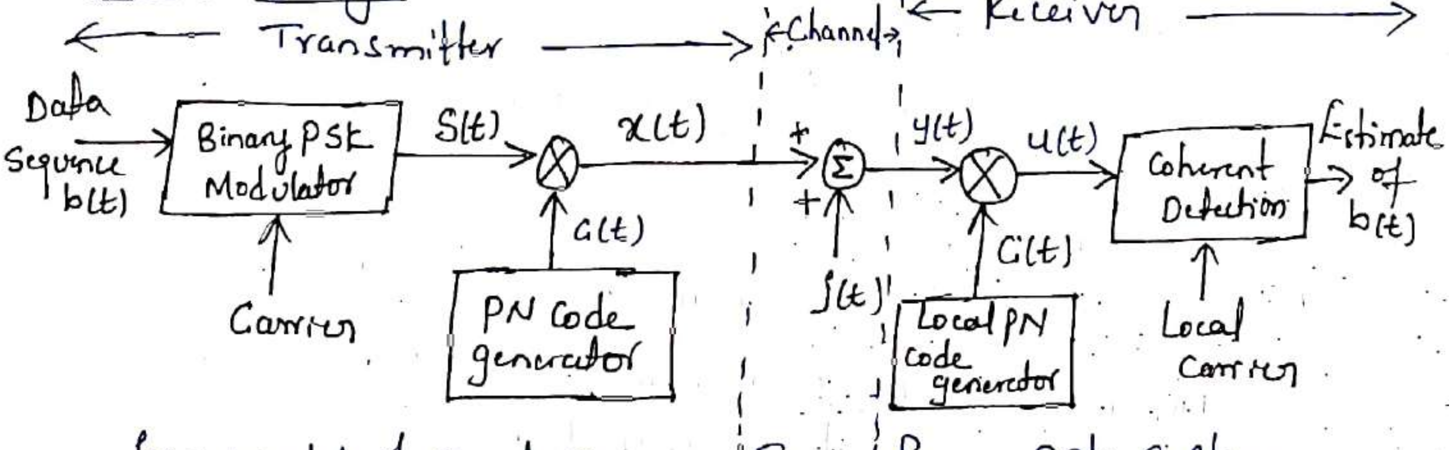


fig5. Model of Direct Sequence Spread Binary PSK System.

* The Spread Spectrum i.e $m(t) = b(t)\,c(t)$ can also be performed prior to phase modulation.

* For analysis purpose, the spectrum spreading and the BPSK are interchanged because both are Linear operation.

* In this model, it is assumed that the interference $j(t)$ Limits performance, so that the effect of channel noisy may be ignored.

from fig c. channel output is given by

$$y(t) = x(t) + j(t)$$

but $x(t) = s(t)\,c(t)$.

$$\Rightarrow \boxed{y(t) = s(t)\,c(t) + j(t)} \longrightarrow ①$$

where, $s(t)$ is the binary psk signal and $c(t)$ is the PN Sequence.

* In the Receiver, the received Signal $y(t)$ is 1st multiplied by the PN Sequence $c(t)$

Thus $\boxed{u(t) = y(t)\,c(t)} \leftarrow ②$

7

using eqⁿ ② in eqⁿ ①

$$u(t) = [c(t) \cdot s(t) + j(t)]\, c(t)$$

$$= c^2(t) \cdot s(t) + j(t)\, c(t)$$

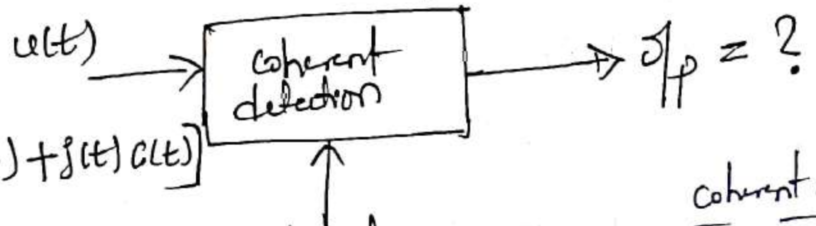
∵ $c^2(t) = 1$.

$$\boxed{u(t) = s(t) + \underline{j(t)\, c(t)}}$$

← spreading code affects only j(t).

* This u(t) is fed to coherent detector to detect the information signal b(t).

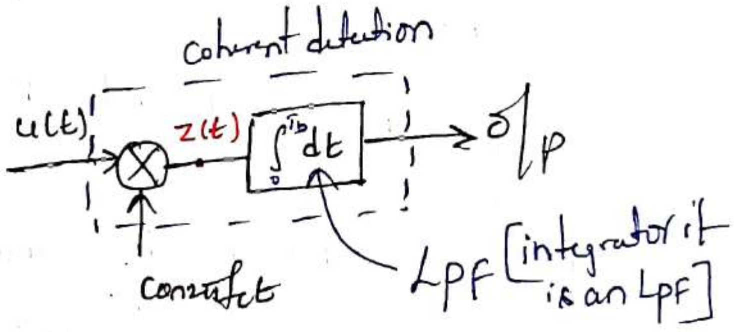
ie



$= [s(t) + j(t)\, c(t)]$

and

$s(t) = b(t) \cdot [\cos 2\pi f_c t]$



$z(t) = [u(t) \cdot \cos 2\pi f_c t]$

$$\Rightarrow z(t) = b(t) \cos 2\pi f_c t \cdot \cos 2\pi f_c t = b(t) \cos^2 2\pi f_c t$$

$$z(t) = b(t)\left[\frac{1 + \cos 4\pi f_c t}{2}\right]$$

$$= \frac{b(t)}{2} + \underbrace{\overset{\times\ \text{HFC}}{\frac{b(t)}{2} \cos 4\pi f_c t}}_{\text{Eliminated}} \longrightarrow \boxed{\text{LPF}} \longrightarrow \underset{\text{Estimated o/p}}{\frac{b(t)}{2}} ✓$$

Performance of DS-SS System :-

The performance of DS-SS system canbe evaluated on the basis of processing gain and probability of Error.

Processing Gain :- (PG)

Processing gain (PG) is defined as the ratio of the Bandwidth of spreaded message signal to the Bandwidth of the unspreaded data signal.

$$\text{i.e} \quad PG = \frac{\text{BW of Spreaded Signal}}{\text{BW of Unspreaded Signal}}$$

Where BW of data signal (unspreaded) $= R_b = \frac{1}{T_b}$ Hz.

BW of Spreaded Signal $W_c = \frac{1}{T_c}$

$$\therefore \; PG = \frac{1/T_c}{1/T_b} = \frac{T_b}{T_c} \quad ; \quad \text{Since } T_b > T_c$$

$$\boxed{PG = \frac{T_b}{T_c}} \in \text{integer no.} \longleftarrow ①$$

W.K.T One bit period T_b of data signal is equal to 'L_c' bits periods of PN code signal (ie $L_c T_c$)

$$\text{i.e} \quad \boxed{T_b = L_c \cdot T_c} \longleftarrow ②$$

from eq^n ① and eq^n ②

$$PG = \frac{L_c \cdot \cancel{T_c}}{\cancel{T_c}} = L_c$$

$$\boxed{PG = L_c} \leftarrow ③$$

5.2.2 Probability of Error for DS-BPSK System

$$P_e \simeq \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{P_N \cdot T_c}}\right) \leftarrow ④$$

where P_N is the average interference power (or) avg. Noise power.

Jamming Margin (or) Anti-Jam characteristics :-

w.k.t for coherent psk, probability of Error

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \leftarrow ⑤$$

N_0 - PSD of Noise

Comparing eq^n ④ and eq^n ⑤

$$\boxed{P_N = \frac{N_0}{T_c} = N_0 \cdot W} \Leftarrow \boxed{N_0 = P_N \cdot T_c} = P_N \cdot T_c \leftarrow ⑥$$

Since the bit Energy $E_b = P_R \cdot T_b \leftarrow ⑦$ $= P_R \cdot T_b$; Joules

where P_R - average signal power.

T_b - is the bit duration.

We may Express the bit energy to noise density ratio as

using eq^n ⑥ & ⑦ $\Rightarrow \frac{E_b}{N_0} = \frac{P_R \cdot T_b}{J \cdot T_c} = \frac{P}{J}\left(\frac{T_b}{T_c}\right) = \frac{P}{J}(L_c) = \frac{P}{J}(P.G)$

$$\frac{P_N}{P_R} \Rightarrow = \boxed{\frac{P_N}{P_R} = \frac{P.G}{(E_b/N_0)}} = \text{Jamming Margin.}$$

The ratio of average interference power to average signal power is called <u>Jamming Margin</u> (i.e P_N/P_R). $= \frac{P_N}{P_R}$

$$(\text{Jamming Margin})_{dB} = 10\log_{10}\left[PG/(E_b/N_0)\right]$$

$$\boxed{(JM)_{dB} = 10\log_{10}(PG) - 10\log_{10}(E_b/N_0)}$$

$$\boxed{(J.M)_{dB} = 10\log_{10}\left(\frac{T_b}{T_c}\right) - 10\log_{10}(E_b/N_0)}$$

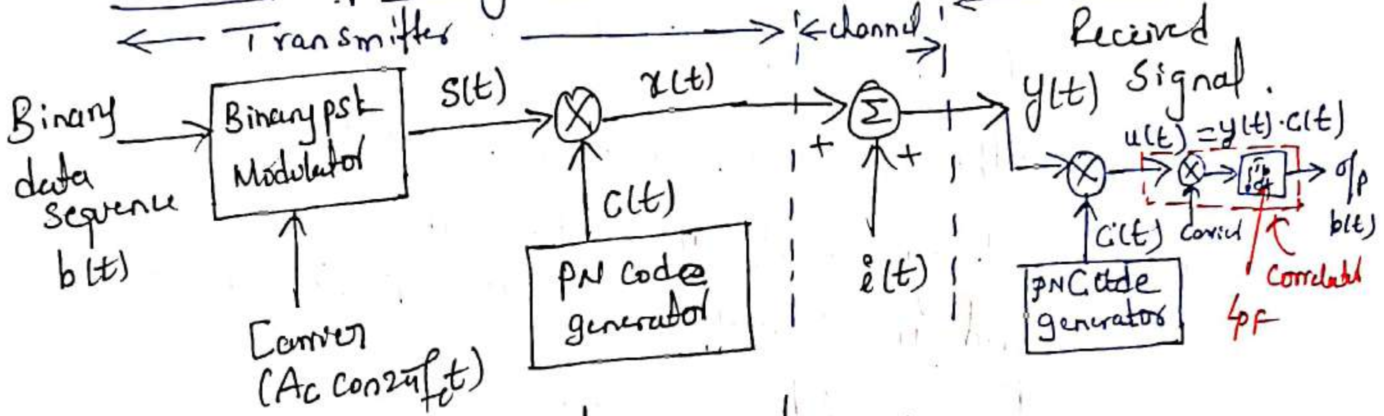
Note:- i.e the ratio $\left(\frac{P_R}{P_N}\right) << 1.$

5.2.1 Effect of De-spreading on a narrowband Interference

Explain the effect of dispreading on a Narrow band interference in direct sequence spread spectrum systems. A direct sequence spread spectrum signal is designed to have the power ration P_R/P_N at the intended receiver is 10^{-2}. If the desired $\frac{E_b}{N_0} = 10$ for acceptable performance, determine the minimum value of processing gain. **(06marks) June-July 2018.**

Soln:-

i Effect of Despreading on a Narrowband Interference :-



The received signal is given by

$$y(t) = x(t) + i(t)$$

$$= s(t)\,c(t) + i(t)$$

$$y(t) = b(t)\,A_c\cos(2\pi f_c t)\cdot c(t) + i(t) \leftarrow ①$$

where $i(t)$ denotes the interference.

The despreading operation at the receiver yields $u(t)$

i.e $u(t) = y(t)\,c(t)$

$$= b(t)\,A_c\cos(2\pi f_c t)\,\overset{\nearrow 1}{c^2(t)} + i(t)\,c(t) \qquad \Big|\; \because c^2(t) = 1$$

$$u(t) = A_c\,b(t)\cos 2\pi f_c t + \underline{\underline{i(t)\,c(t)}} \leftarrow ②$$

↖ wide spectrum.

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→ "The effect of multiplying the interference i(t) with c(t), is to spread the bandwidth of i(t) to W Hz.

ie bcz c(t) BW is $\frac{1}{T_c} = W$ Hz (Higher BW)

Let us consider a sinusoidal interfering signal of the form

$$i(t) = A_I \cos(2\pi f_I t) \leftarrow ③$$

where f_I is the frequency within the Bandwidth of the transmitted signal. its multiplication with c(t) results in a wideband interference with power spectral density

$$I_0 = \frac{P_I}{W} \text{ watts}/Hz . \leftarrow ④$$

where $P_I = \frac{A_I^2}{2}$ is the avg. power of the interference.

* Since the desired signal is demodulated by a matched filter (correlator) LPF that has a bandwidth $R_b = \frac{1}{T_b}$, the total power in the interference at the output of the demodulator

is $\Rightarrow I_0 W = \frac{P_I \cdot R_b}{W} = \frac{P_I}{(W|R_b)} = \frac{P_I}{(T_b|T_c)} = \frac{P_I}{L_c} \leftarrow ⑤$

∴ the power in the interfering signal is reduced by an amount equal to the bandwidth expansion factor (W|Rb).

The factor (W|Rb) = Tb|Tc = Lc is called the "processing gain" of the spread spectrum system. "The reduction in interference power is the basic reason for using spread spectrum signals to transmit digital information over channels with interference.

12

ii) Problem

soln: given power ratio $\left(\frac{P_R}{P_N}\right) = 10^{-2}$

P_R - avg. received signal power

P_N - avg. noise power

and $\frac{E_b}{N_0} = 10.$

minim value of PG = ?

W.k.t $\left(\frac{P_N}{P_R}\right) = \frac{P.G}{(E_b/N_0)}$

ie Jamming Margin

$\Rightarrow P.G = \left(\frac{P_N}{P_R}\right) \cdot (E_b/N_0)$

Since given $\frac{P_R}{P_N} = 10^{-2}$

$\therefore P_N/P_R = 10^{+2}$

$\therefore P.G = 10^2 \times 10 = 100 \times 10$

$$\boxed{P.G = L_c = 1000}$$

The necessary processing Gain is $\underline{L_c = 1000}$.

5.2.2 Probability of error (statement only)

In a DS/BPSK system, the feedback shift register used to generate the PN sequence has length m=19. The system is required to have an average probability of symbol error due to externally generated interfering signals that does not exceed 10^{-5}. Calculate the following system parameters in decibels: i. Processing gain ii. Antijam margin
(Assume $Q(4.25)=10^{-5}$ or $erfc(3) = 2x10^{-5}$) **(04 Marks) Dec 2018-Jan 2019 / (04 Marks) June-July 2019.**

Soln:- $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{P_N T_c}}\right)$

given PN sequence Length m = 19.

i. $P.G = L_c = 2^m - 1 = 2^{19} - 1 = 524287.$

$$(PG)_{dB} = 10\log_{10}(P.G) = 10\log_{10}[524287)$$

$$\boxed{(PG)_{dB} = 57.1956}\ dB$$

ii. Jamming Margin (or) Anti Jam Margin

$$J.M = \frac{P_N}{P_R} = \frac{PG}{(E_b/N_0)} \longleftarrow ①$$

W.K. $P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \longleftarrow ⓐ$

and given $Q(4.25) = 10^{-5} \longleftarrow ⓑ$

Comparing ⓐ and ⓑ eq. $\Rightarrow \left(\sqrt{\frac{2E_b}{N_0}}\right) = 4.25$

$$\frac{2E_b}{N_0} = (4.25)^2$$

$$\frac{2E_b}{N_0} = 18.0625$$

$$\Rightarrow \frac{E_b}{N_0} = \frac{18.0625}{2} = 9.03125 \leftarrow ⓒ$$

using eq^n ⓒ in eq^n ①

$$\text{i.e} \quad JM = \frac{P.G}{(E_b/N_0)} = \frac{524287}{9.03125}$$

$$J.M = 58052.53979$$

$$(J.M)_{dB} = 10\log_{10}(J.m) = 10\log_{10}(58052.53979)$$

$$\boxed{(J.M)_{dB} = \underline{\underline{47.6382}}}\ dB.$$

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5.3 Some applications of DS Spread Spectrum Signals

List and briefly explain any 3appliations of direct sequence spread spectrum. **(05 Marks) Dec 2018-Jan 2019.**

DSSS is used in a number of areas where its properties have enabled it to provide some unique advantages over other techniques.

Some of the applications of DSSS are :-

i) Low detectability Signal transmission.

ii) Code division Multiple Access.

iii) Communication over channel with Multipath.

iv) Global Navigation Sattellite System. (GNSS)

v) Convert Communications.

*a. GNSS :- Satellite based navigation Systems use DSSS as this gives a signal gain by spreading the signal out over a wide bandwidth. it also Enables different satellites to use the same channel without mutual interference.

①

*** b. Convert Communications :-**

DSSS was first used to provide Secure and convert communications. The signals were initially difficult to detect as they sounded like broadband noise and often would have been mistaken for that. also to access the data it is necessary to know the code used to generate the signal.

*** c. CDMA Cellphone technology :-** The DSSS technique was used to provide a multiple access scheme that was used for 3G cellular technology. Each mobile used used a different access code (or) Spreading code and this enabled multiple users to access the basestation on the same frequency.

*** d. Communication Over channels with Multipath :-**

→ Multipath propogation, an inherent feature of a mobile communications channel, results in a received signal that is dispersed in time.

→ DS Spread spectrum is a particularly effective way to generate a wide band signal for resolving multipath signal components.

→ By Separating multipath components we may also reduce the effects of fading.

Write a note on low detectability signal transmission as an application of direct sequence spread spectrum. **(03 Marks) June-July 2018.**

Soln:-

Low - detectability Signal Transmission :-

* In this application, the signal spectral density is intentionally kept small with respect to the channel noise and receiver noise so that presence of the Signal is not detected easily.
* if the DS spread spectrum signal occupies a bandwidth W and the power spectral density of the additive noise is N_0 W/Hz, the average noise power in the bandwidth W is $P_N = W N_0$.
* The average received Signal power at the intended receiver is P_R. the signal is transmitted at a power level such that $(P_R/P_N) << 1$.
* The intended receiver can recover the weak information - bearing signal from the background noise with the aid of the processing gain and the coding gain.
* However, any other receiver which has no knowledge of the PN code sequence is unable to take advantage of the processing gain and the coding gain.

①

Write a note on code division multiple access as an application of direct sequence spread spectrum. **(03 Marks) June-July 2018.**

CDMA with DSSS :-

* In this application, many users transmit their signals on the same channel bandwidth.
* Each transmitter receiver pair has a distinct P-N sequence. Thus signals of a particular transmitter are received by its intended receiver only. Even if many users are transmitting at the same time. This method is also called Spread Spectrum Multiple access (SSMA).
* The signals from other users appear as additive interference which are rejected by the spread spectrum decoder.
* The Level of interference depends upon the number of users transmitting at any time.
* The Main advantage of CDMA is that the number of users sharing the same channel can be increased (or) decreased very easily.
* Large number of users can transmit on the same channel if their messages are for short periods of time. for this method it is desirable that the PN sequences be Mutually orthogonal.

Write a note on application of spread spectrum in wireless LAN's. **(04 Marks) June-July 2019 / (03 Marks) Dec 2019- Jan 2020.**

Soln:- WLAN :- (wirelen Local Area Networks) widely use Spread Spectrum Communications. IEEE 802.11 is a standard that is developed for mobile communication and widely implemented throughout the world.

* The standard defines three types of physical Layer Communications. These are

i. Infrared (IR) Communication.
ii. Direct Sequence Spread Spectrum Communications.
iii. Frequency Hopping Spread Spectrum Communications.

* Spread Spectrum is Currently the most widely used transmission technique for wirelen LANs. it was initially developed by military Security applications.

* Wirelen LAN Security :-

i. prevents 'Eavesdropping' of Wirelen Link
ii. Prevents 'hacking' into Wirelen LAN's.

* Table. shows the advantage of using Spread Spectrum in Wirelen LAN's.

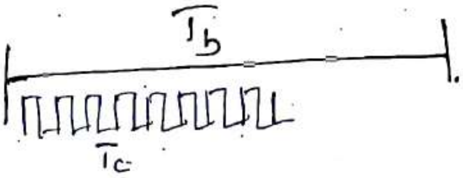
Wireless transmission technique :-

	Spread-Spectrum
Frequency	— 902 MHz to 928 MHz 2.4 GHz to 2.43 GHz
Maximum coverage	105 to 800 feet. (or) 50,000 square feet..
Line of Sight required	NO
Transmitted power	Less than 1 W.
License required	NO

Frequency Hopping Spread Spectrum

Drawback of DSSS

$$PG = \frac{T_b}{T_c} = N$$



- ✓ To increase PG, we need to accomodate more no. of chips within the one bit duration.
- ✓ T_c is the polar waveform, ⊓⊔. theoretically it may have Zero rise time, and Zero fall time. but practically

⊓⊔ $t_r = t_f = 0$ (ideal)

(practical)

Beyond certain level, we cannot reduce T_c duration. bcz of rise time & fall time issues.

∴ we will go for Other technique called Frequency Hop-spread Spectrum.

Group of k binary bits → set of 2^k carrier frequencies
↳ M-ary FSK Signal.

Eg:- $k=2$

$00 - f_1$
$01 - f_2$
$10 - f_3$
$11 - f_4$
} MFSK Signal.
↳ Carrier frequency.

$k = 3$ bit
$000 - f_1$
$001 - f_2$
⋮
$111 - f_8$
} 8-ary FSK.

The MFSK signal finally Modulates a Carrier which hops randomly.

"Spread spectrum in which the carrier hops randomly from one frequency to another is called "Frequency Hop(FH) Spread spectrum"."

∴ "By choosing a large number of randomly hoping discrete carrier frequencies, it is possible to have a Modulated Signal of wide bandwidth, which in turn results in a Large processing gain.

Note:-

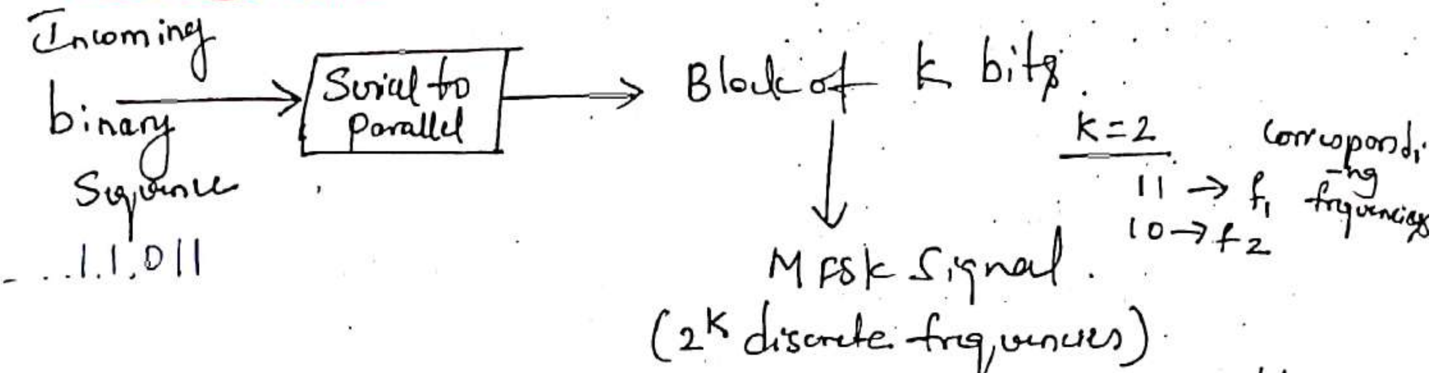
i. DSSS → Spectrum spreading is instantaneous

ii. FHSS → Spectrum spreading is Sequential.

→ 1st stage :- Frequency Modulator (M-~~F~~SK)
2nd stage :- Frequency Mixer (frequency adder)

$$f_1 \rightarrow \otimes \rightarrow f_1+f_2$$
$$\uparrow f_2 \qquad f_1-f_2$$

FH/MFSK Transmitter:-



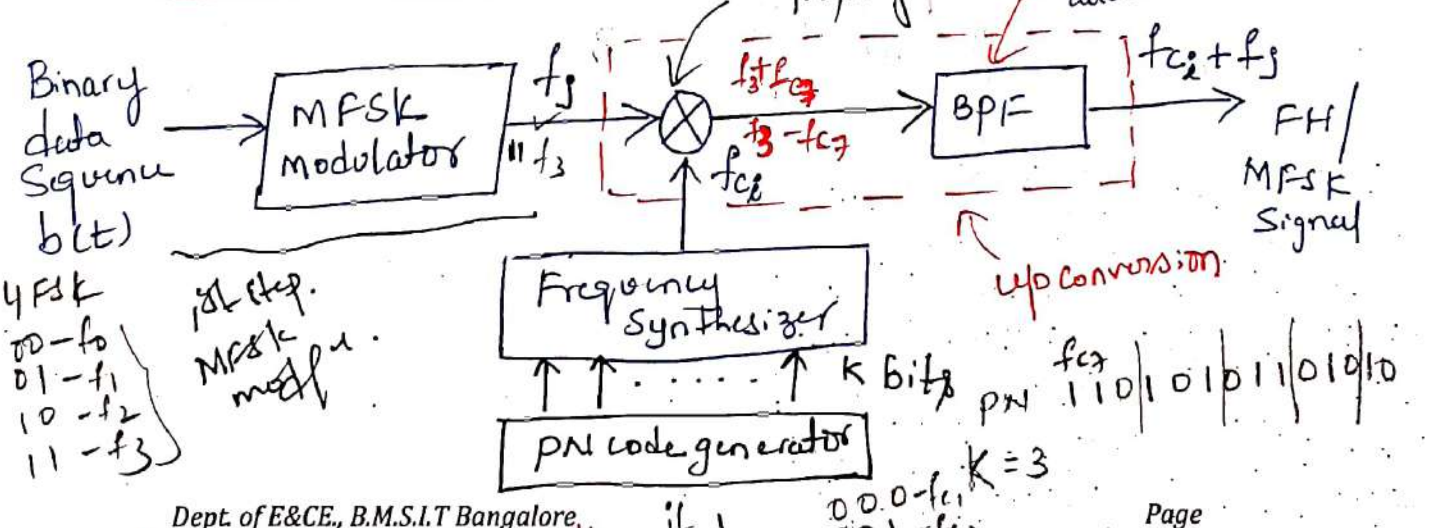
The output of MFSK Modulator is the Mixed with the output of a frequency Synthesizer.

The frequency Synthesizer's output is one of 2^k values, where k is the number of bits of the PN sequence generator output.

As a Result, the carrier frequency hops over 2^k distinct values.

The BPF pass the Sum frequency component for transmission and rejects the difference frequency.

FH/MFSK Transmitter:-



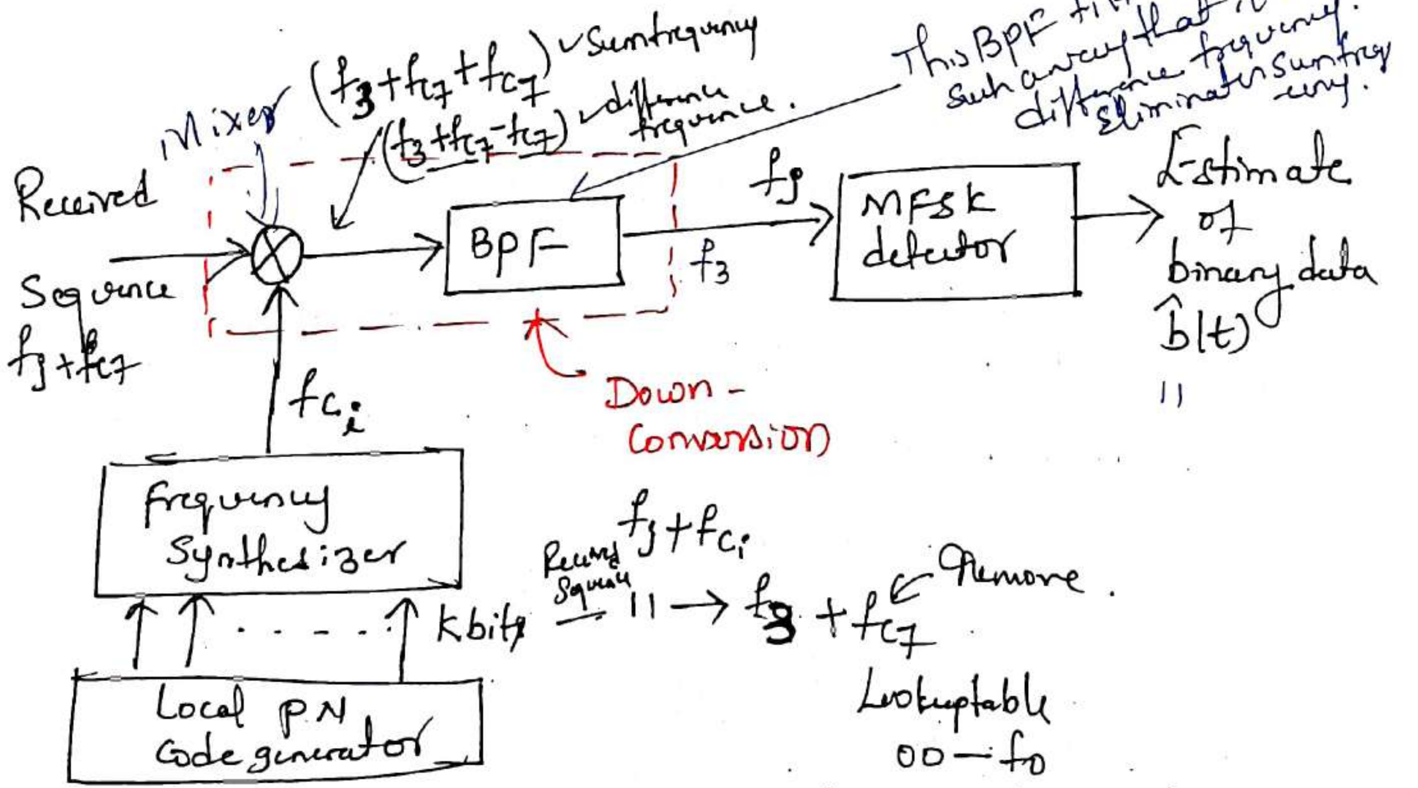
③ fig4:- FH/MFSK Transmitter

if Segment size is 3bit

000 – f_{c1}
001 – f_{c2}
010 – f_{c3}
111 – f_{c8}

FH/MFSK Receiver:-

- In the first stage, mixing operation (down conversion) removes the frequency hopping.
- The Mixer Inputs are the received signal and the output of a Local frequency synthesizer that is in synchronous with that of the transmitter.
- The output of the Mixer is passed through a BPF which selects the difference frequency component from the Mixer.
- The o/p of the BPF is the MFSK signal, which is demodulated using non-coherent MFSK detector.



Lookuptable
00 — f_0
01 — f_1
10 → f_2
11 — f_3.

fig 5:- FH/MFSK Receiver

(4)

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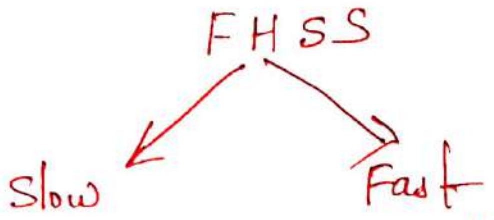


i. Slow Frequency Hopping Spread Spectrum

Notations
- T_h (or) T_c – chip duration (or) hopping duration.
- T_s – Symbol duration
- T_b – Bit duration

- R_s - Symbol rate
- R_b - bit rate
- R_c - chiprate (R_h) (or) hopping rate.

i. Slow FHSS :-

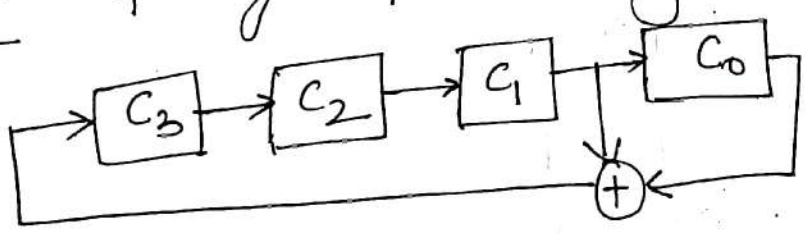
$R_s = n \cdot R_h$ (ie hopping rate is less than the Symbol rate).

$\Rightarrow \frac{1}{R_s} = \frac{1}{n \cdot R_h}$ [ie in one hope we transmit multiple Symbols]

$$\boxed{nT_s = T_c}$$

Several Symbols are transmitted in each frequency hop.

Eg:- 4-stage LFSR. are given



given

Initial state 1 0 0 0.

No. of bits per MFSK Symbol $K = 2$.

No. of MFSK tones $M = 2^K = 4$.

Length of PN Segment per hop k=3.

total no. of frequency hops $2^k = 2^3 = 8$.

	$= C_1 \oplus C_0$ C_3	C_2	C_1	C_0
initial state	1	0	0	0
1.	0	1	0	0
2	0	0	1	0
3	1	0	0	1
4	1	1	0	0
5	0	1	1	0
6	1	0	1	1
7.	0	1	0	1
8	1	0	1	0
9	1	1	0	1
10	1	1	1	0
11	1	1	1	1
12	0	1	1	1
13	0	0	1	1
14	0	0	0	1
15	1	0	0	0

o/p of S.R 'C0' is the PN sequence

← PN Sequence.

← repeated (initial state)

P.N sequence

$C = 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 1$ bits

Question's

i. Period of PN Sequence ?

Ans:- $m=4$

$$N = 2^4 - 1 = 2^4 - 1 = 15.$$

ii. PN sequence of one periodic Length ?

Ans:- 0 0 0 1 0 0 1 1 0 1 0 1 1 1 1.

iii. anum Carrier hops to a new frequency after transmitting two MFSK symbols (or) 4 information bits. → i.e ($T_c = 2T_s$)

Binary data sequence

1 0 0 0 1 1 0 1 0 0 0 1 1 1 1 1 1 0 0 1

Sketch hopping frequency v/s time ?

Ans:-

Dibit	MSFK tone in Hz
0 0	f_0
0 1	f_1
1 0	f_2
1 1	f_3

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PN Sequence -segment k = (3bit) given	Hopping carrier frequency in Hz
000	f_{c0}
001	f_{c_1}
010	f_{c_2}
011	f_{c_3}
100	f_{c_4}
101	f_{c_5}
110	f_{c_6}
111	f_{c_7}

Binary Sequence (dibit)

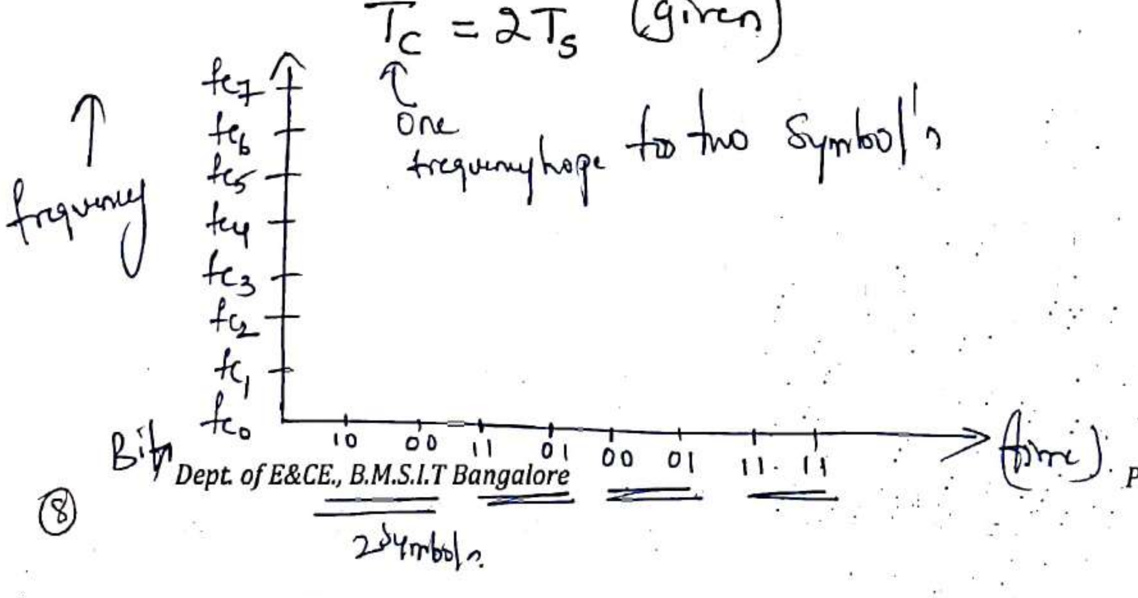
10	00	11	01	00	01	11	11	10	01
f_2	f_0	f_3	f_1	f_0	f_1	f_3	f_3	f_2	f_1

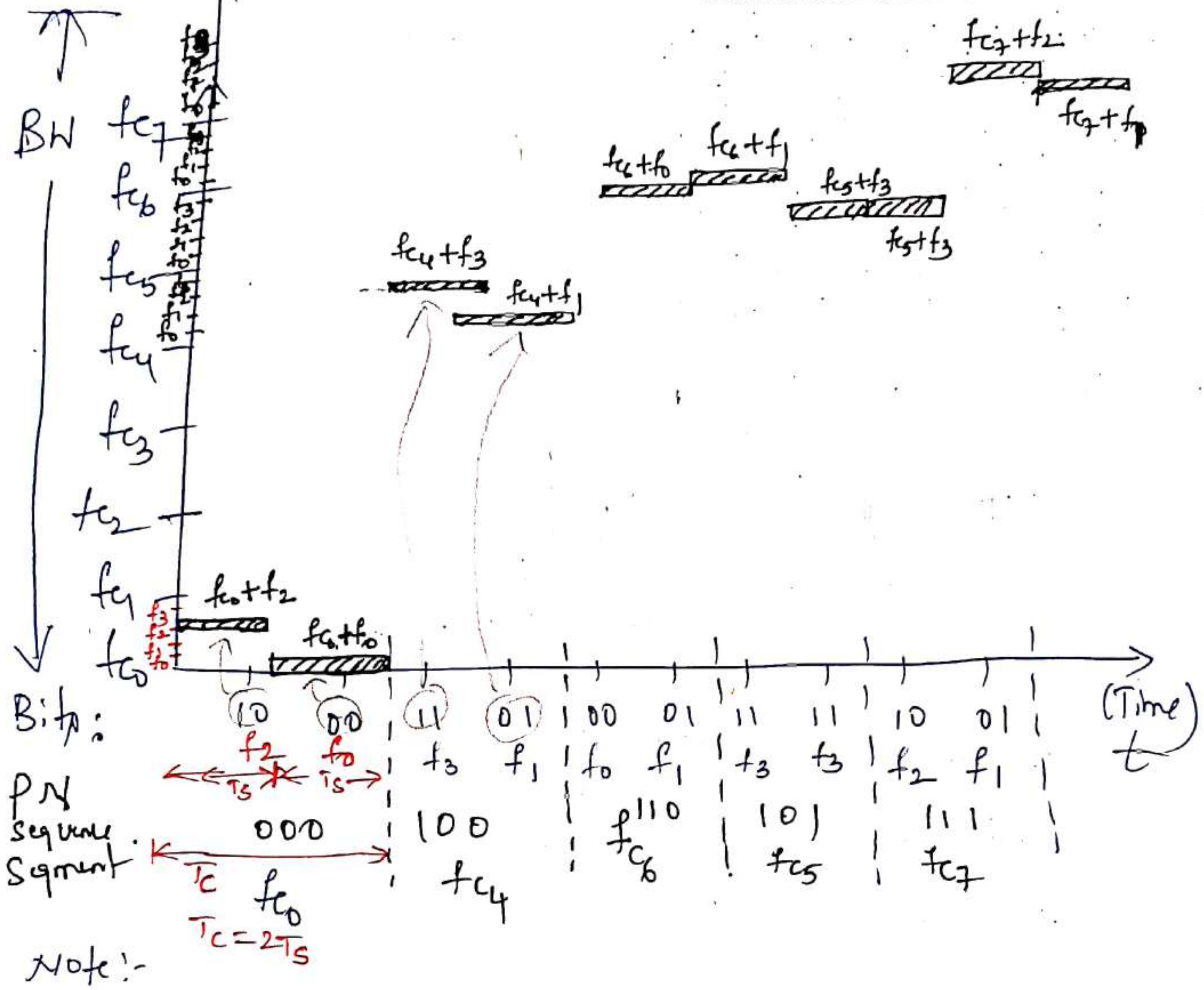
PN sequence :- (k = 3bit) ← repeated PN sequence is

000	100	110	101	111	000	100	110	101	111
f_{c0}	f_{c4}	f_{c6}	f_{c5}	f_{c7}	f_{c0}	f_{c4}	f_{c6}	f_{c5}	f_{c7}

$T_C = 2T_s$ (given)

One frequency hope to two Symbol's





Note:-

* Carrier frequency is not fixed and it is hopping

at Receiver:- (Same PN Sequence)

000 100

← Tc → | ← f_{c4} →

f_{c0}

Received Signal is undergo down convert.

$$f_{c0}+f_2-f_{c0}=f_2 \longrightarrow 10$$

$$f_{c0}+f_0-f_{c0}=f_0 \longrightarrow 00$$ llly.

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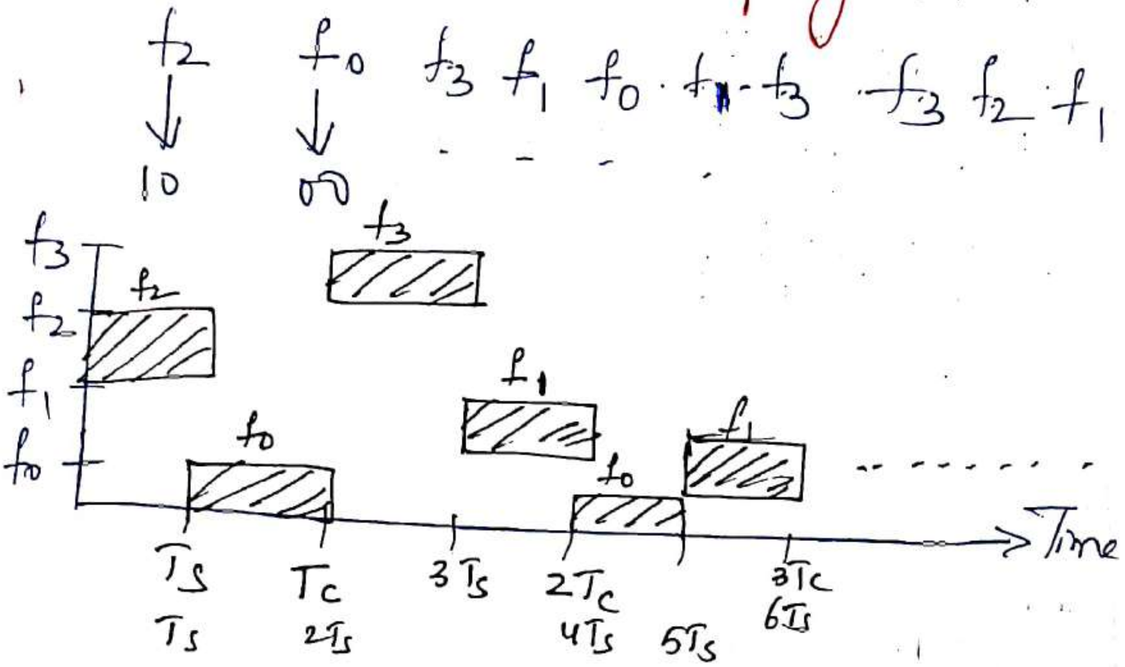
iv. Sketch the Dechopped frequency v/s time.



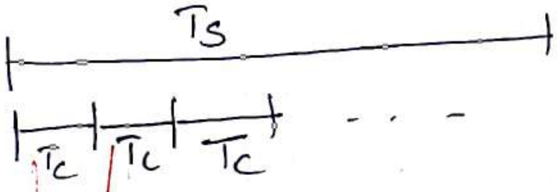
Fast Frequency Hopping Spread Spectrum (Fast FHSS).

[ie hopping rate is higher than Symbol rate].

$\Rightarrow R_H = n \cdot R_s$

$$\frac{1}{R_H} = \frac{1}{n \cdot R_s}$$

$$\underset{\text{chipduration}}{nT_C} = \underset{\text{symbolduration.}}{T_S}$$



ie Same symbol is transmitted in multiple hops.
f_1, f_2 . . . .

advantage is ⇒ Frequency diversity.

Eg:- Fast FHSS/MFSK.

Given data
no. of bits per MFSK Symbol $k=2$.

$M = 2^k = 2^2 = 4$. (ie 4 possible MFSK Signals).

Dibit	msktone
00	f_0
01	f_1
10	f_2
11	f_3

PN sequence Segment Length $k=3$

no of frequency hops $= 2^k = 2^3 = 8$ possible frequencies.

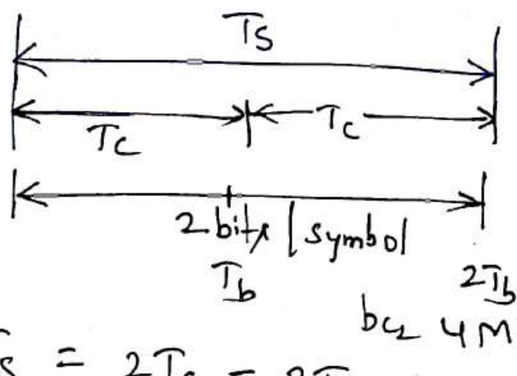
PN Sequence Segment	Hopping carrier frequency (in Hz)
000	f_{c0}
001	f_{c1}
010	f_{c2}
011	f_{c3}
100	f_{c4}
101	f_{c5}
110	f_{c6}
111	f_{c7}

8 different hopping frequencies

Question's

1. Determine the relationship b/w bit rate and chip rate.

2 hops per MFSK symbol. (given)



1 MFSK Symbol is txtted in two hops.

$\boxed{T_S = 2T_C}$

ie $\boxed{T_S = 2T_b}$

bcz 4 MFSK. (Each symbol transmitted by two bits).

$\Rightarrow T_S = 2T_C = 2T_b$

$\Rightarrow \boxed{T_C = T_b} \Rightarrow R_b = R_c$

Bitrate = chiprate or hopping rate.

eg. given

Binary sequence

01101100 . . . .

PN sequence

1111 0001 0011 010 . . . . .

Sketch the variation of frequency of the transmitted sequence with time.

soln:- binary sequence 01 | 10 | 11 | 00

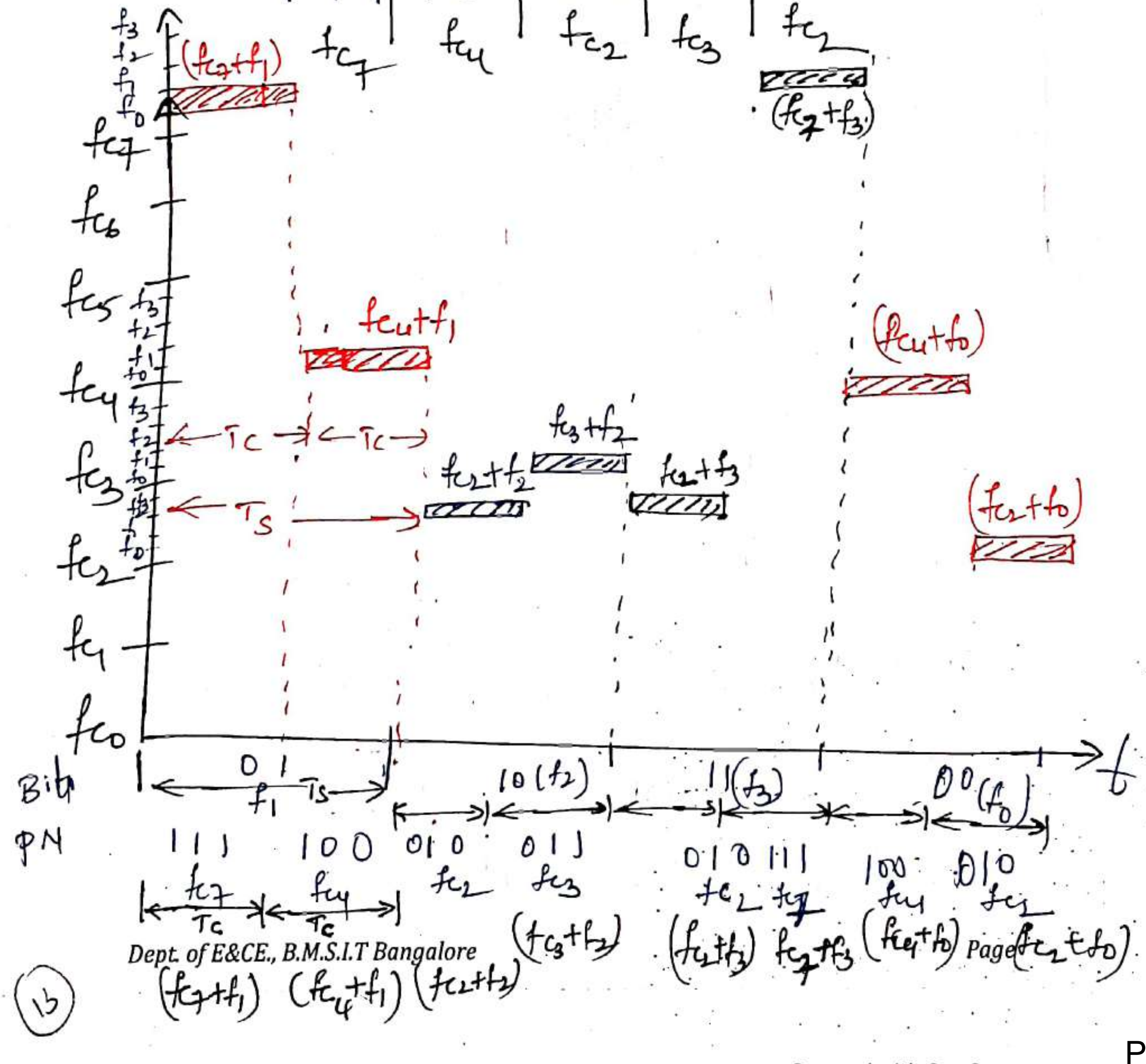
Ts f1 f2 f3 f0

one symbol is transmitted two different carrier frequencies.

PN sequence

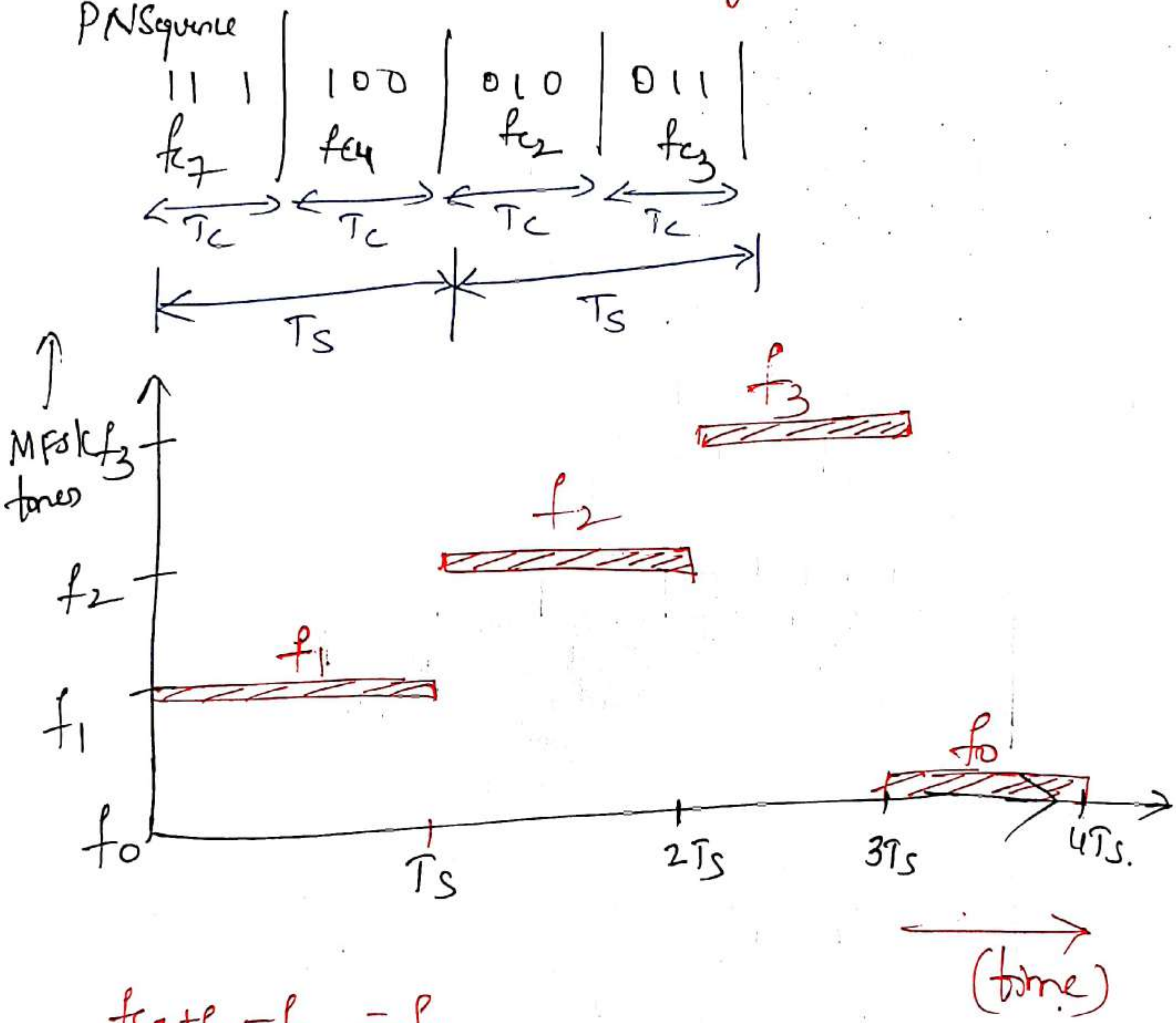
111 | 100 | 010 | 011 | 010

fc7 fc4 fc2 fc3 fc2



iii: Sketch the dehopped MFSK signal with time:

same.



$f_{c7}+f_1-f_{c7}=f_1$

$f_{c4}+f_1-f_{c4}=f_1$

$$\text{Processing Gain (PG)} = \frac{\text{B.W of RF Signal after Spreading}}{\text{B.W of Modulate Signal (Before spread)}}.$$

k bit's 2^k $\Rightarrow$ $BW_{RF} = 2^k \cdot BW_{mod}$

$\therefore$ $\boxed{P.G = 2^k}$

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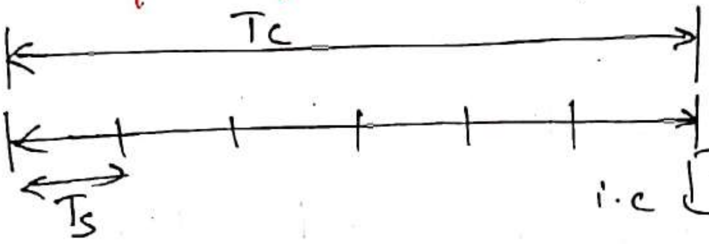
Example1:- Slow FH/MFSK

No.of bits per MFSK symbol = 4.

No.of MFSK symbols per hop = 6.

PG = ?

Soln:-



i.e $\boxed{T_c = 6\,T_s}$

Each symbol ⇒ 4 bits/symbol.

i.e $\boxed{T_s = 4\,T_b}$

$$P.G = \frac{T_c}{T_b} = \frac{6T_s}{T_b} = \frac{6\times 4\cancel{T_b}}{\cancel{T_b}} = \underline{\underline{24}}.$$

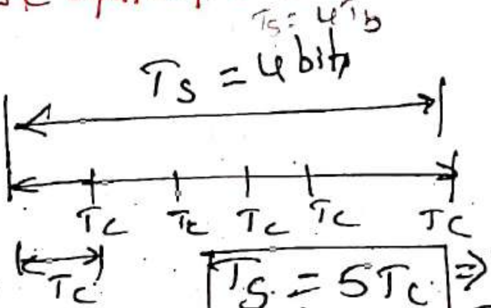
$$\boxed{P.G = 24}$$

Example2 Fast FH/MFSK.

No.of bits per MFSK symbol = 4.

No.of hops per MFSK symbol = 5.

PG = ?



Soln:-

In Each symbol ⇒ 4 bits Transmitted

$\boxed{T_s = 5T_c} \Rightarrow T_c = \frac{T_s}{5}$

$$P.G = \frac{T_s}{T_c} = \frac{T_s}{T_s/k} = \frac{T_s}{T_s}\times k = 5 \qquad \boxed{T_c = T_s/k}$$

$= 5$ hops $\times$ 4 bits/hop $= 20 = PG$.

Example 3. DSSS

Given $(J_m)_{dB} > 26\,dB.$

$\left(\frac{E_b}{N_0}\right) = 10.$

find No. of stages i.e m ?

Soln:- $(J_m)_{dB} = 10\log_{10}(PG) - 10\log_{10}(E_b/N_0).$

$26 + 10\log_{10} 10 = 10\log_{10}(PG)$

$26 + 10 = 10\log_{10} PG \Rightarrow PG = 10^{(36/10)}$

$\Rightarrow PG = 10^{3.6} = \underline{3981.071} \simeq 3981$

$PG = N = 2^m - 1 = 3981$

$2^m = 3981 + 1 \Rightarrow 2^m = 3982$

$\Rightarrow m = \log_2 3982 = 11.9592 \simeq 12 \in Z+.$

$\therefore \boxed{m = 12}$ no of ff's.

$PG = N = 2^m - 1 = 2^{12} - 1 = 4095.$

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5.5 Frequency Hopped Spread Spectrum

- With a neat block diagram, explain the frequency hopped spread spectrum. **(07 Marks) June-July 2018.**
- With a neat block diagram, explain frequency Hopped spread spectrum technique. Explain the terms chip rate, Jamming Margin and Processing gain. **(08 Marks) Dec 2018-Jan 2019.**
- Explain with neat block diagram FH spread- Spectrum system. **(06 Marks) June-July 2019**
- With necessary block diagram, explain the transmitter and receiver of frequency ho spread spectrum. **(08 Marks) Dec 2019- Jan 2020.**

Soln:- The Spread Spectrum in which the carrier hops randomly from one frequency to another is called "frequency Hop" Spread spectrum technique.

The modulation used is MFSK (M-ary FSK).

∴ The Combination of FH and M-ary FSK is referred to as FH/MFSK.

FH/MFSK Transmitter:-

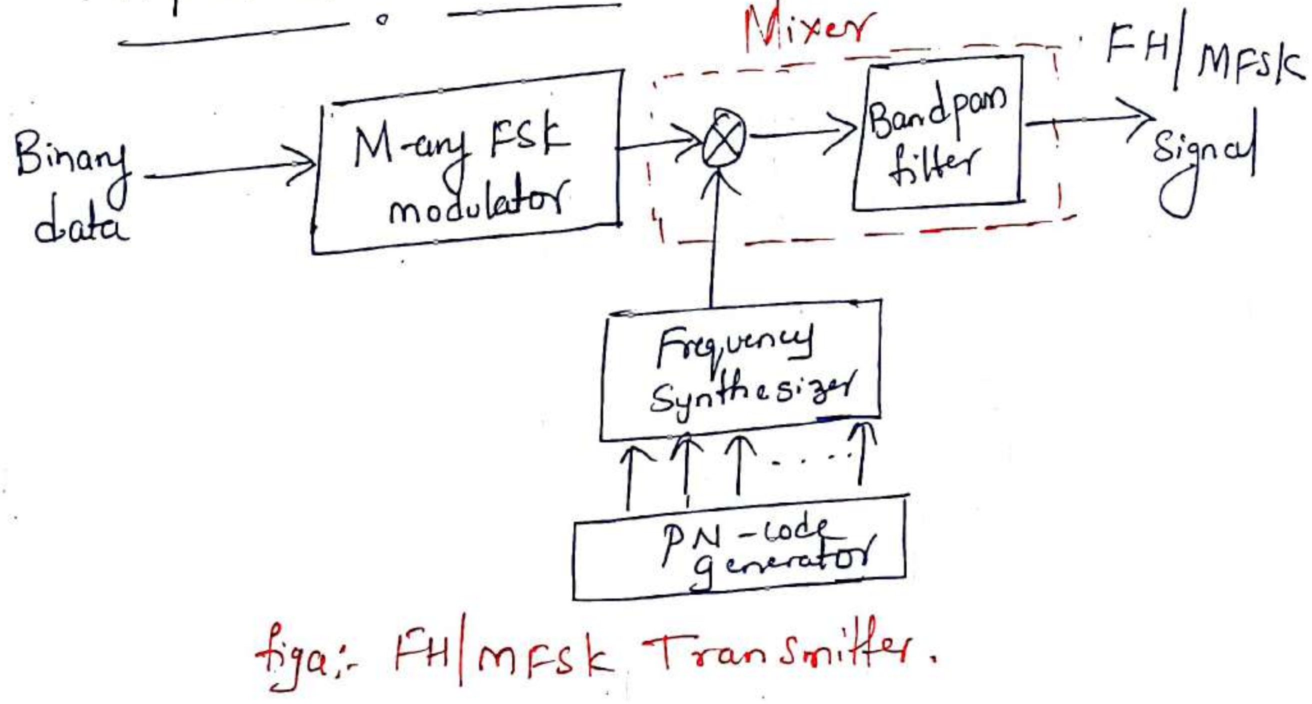


fig a:- FH/MFSK Transmitter.

* Fig(a) shows the block diagram of an FH/MFSK transmitter, which involves frequency modulation followed by Mixer.

* 1st the incoming binary data is applied to M-ary FSK System. The resulting modulated wave and the output from a digital frequency Synthesizer are then applied to a Mixer that consists of a Multiplier followed by a Band pass filter.

* The BPF is designed to select the Sum frequency Component resulting from the Multiplication process.

* P.N Sequence will drive the frequency synthesizer, which Enables carrier frequency over 2^k distinct values.

* On Single hop, Bandwidth of the transmitted signal is Same as that resulting from the use of Conventional MFSK format.

* For complete range of 2^k frequency hops, FH/MFSK Signal occupies a much Larger Bandwidth.

FH/MFSK Receiver:-

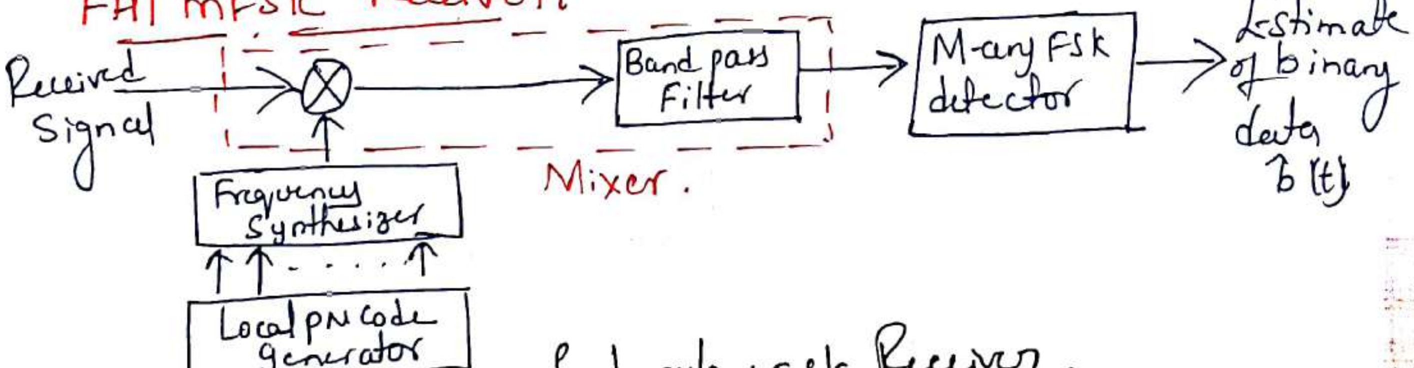


fig b: FH/MFSK Receiver.

②.

* Fig b. shows block diagram of FH/MFSK receiver.

*The frequency hoping is 1st removed by mixing the received signal with the output of a Local frequency Synthesizer that is Synchronously controlled in the Same manner as that in the transmitter.

* The output of Mixer is then passed through a BPF which selects the difference frequency component from the Mixer.

* The output of a BPF is MFSK Signal which is demodulated using non-coherent MFSK detector to get original data.

* In FH System an FH tone of shortest duration is referred as "chip".

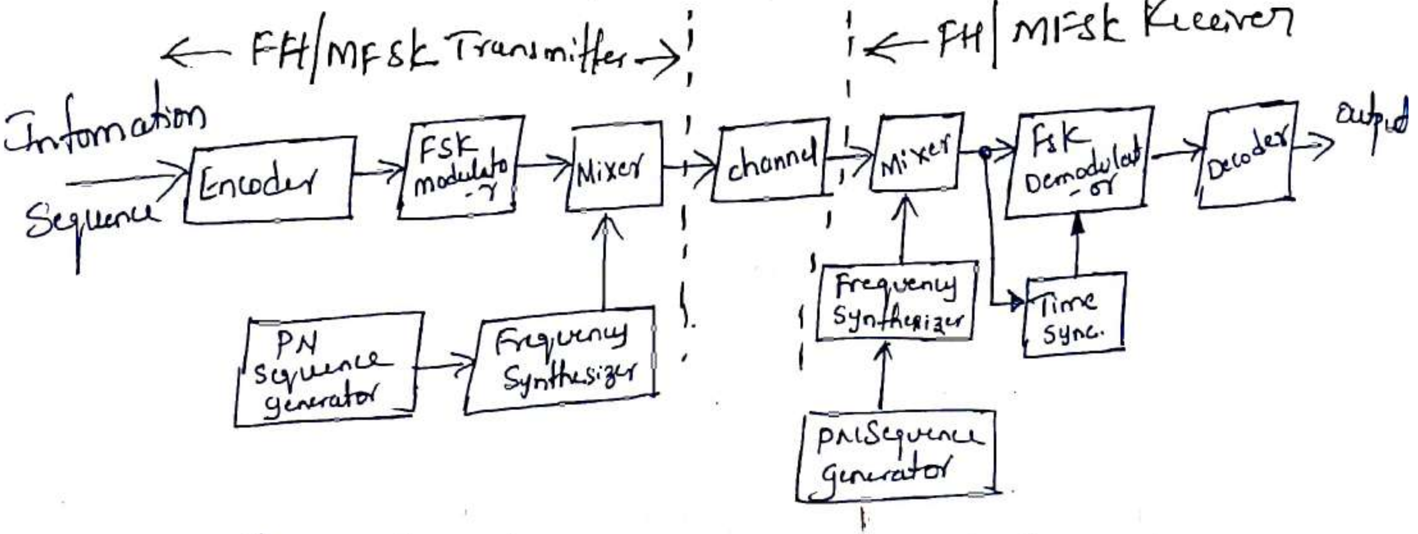


fig c:- Block diagram of an FH Spread-Spectrum System.

Chip rate (R_c)

In FH/MFSK the indvidual frequency of smallest duration is called 'chip'. In slow frequency hopping multiple symbols are transmitted per hop.

chip rate 'R_c' is equal to symbol rate 'R_s' for slow frequency hopping.

$$\boxed{R_c = R_s} = \frac{R_b}{k}$$

Here R_b is the input bits rate and k - no. of bits per symbol.

Processing Gain (PG) :-

$$PG = \frac{\text{B.W of Spreaded Signal}}{\text{B.W of unspreaded signal}} = \frac{\text{B.W of FH Signal}}{\text{B.W of baseband Signal}}$$

$$PG = \frac{2^k \cdot \cancel{f_s}}{\cancel{f_s}} = 2^k$$

let f_s - be the symbol frequency.
w.k.t 2^k frequency hops generated bcz of k-bits of PN sequence.

$$\boxed{PG = 2^k}$$

Jamming Margin :-

$$\boxed{(\text{Jamming Margin})_{dB} = 10\log_{10}(PG) - 10\log_{10}(E_b/N_0)}$$

A slow frequency Hopped/MFSK system has the following parameters,

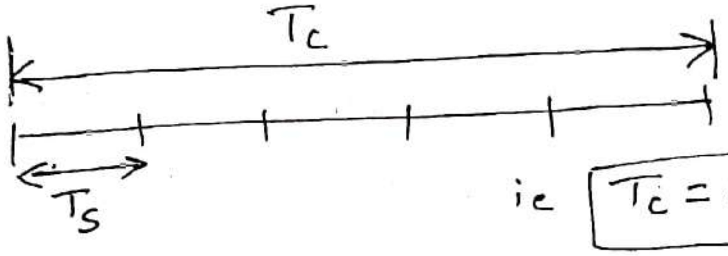
- i. The number of bits/MFSK symbols=4.
- ii. The number of MFSK symbols per hop=5.
- iii. Calculate the processing gain of the system in decibles. **(03 Marks) Dec 2018-Jan 2019.**

Soln: Given slow FH/MFSK sytem.

No. of bits per MFSK symbols = 4.

No. of MFSK symbols per hop = 5.

P.G = ?



ie $\boxed{T_c = 5T_s}$

Each symbol 4 bits are transmitted

i.e 4 bits/symbol

$$\boxed{T_s = 4T_b}$$

$$P.G = \frac{T_c}{T_b} = \frac{5 \cdot T_s}{T_b} = \frac{5 \times 4 \cancel{T_b}}{\cancel{T_b}} = 20$$

$$\boxed{P.G = 20}$$

$$(PG)_{dB} = 10 \log_{10} (PG) = 10 \log_{10} 20 = 13.010$$

$$(PG)_{dB} = 13.010 \, dB$$

Problems on PN Sequence Generation:

A PN sequence is generated using a feedback shift register of length 4 (i.e.4 stage). Find the generated output sequence if the initial contents of the shift register are 1000. If the chip rate is 10^7 chips/sec, calculate the chip and PN sequence duration, and period of output sequence. Draw its schematic arrangement.

Sol. : **i) To obtain the PN sequence :**

There are 4 stages of feedback shift registers. From Table the feedback taps on (4, 1) gives maximum length sequence. This means outputs of stage 4 and stage 1 are mod-2 added and given to input of stage 1. This is shown in Fig.

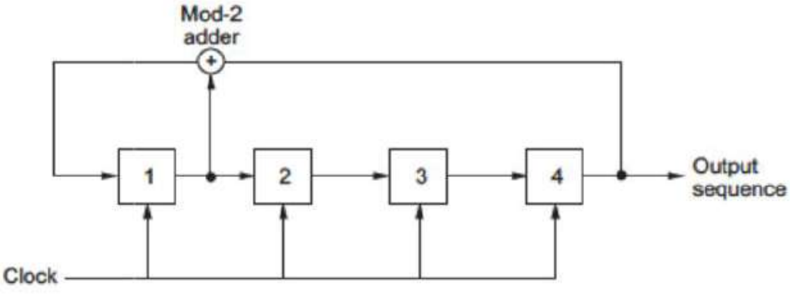


Fig. **A four-stage shift register to generate PN-sequence**

Table shows the PN-sequence generated.

Shift no.	State of shift register				Mod-2 adder output	PN - sequence
	S_1	S_2	S_3	S_4	$S_1 \oplus S_4$	S_4
0	1	0	0	0	1	0
1	1	1	0	0	1	0
2	1	1	1	0	1	0
3	1	1	1	1	0	1
4	0	1	1	1	1	1
5	1	0	1	1	0	1
6	0	1	0	1	1	1
7	1	0	1	0	1	0
8	1	1	0	1	0	1
9	0	1	1	0	0	0
10	0	0	1	1	1	1
11	1	0	0	1	0	1
12	0	1	0	0	0	0
13	0	0	1	0	0	0
14	0	0	0	1	1	1
15	1	0	0	0	1	0

Table **Generation of PN-sequence for m = 4**

Thus the generated PN sequence is,

0 0 0 1 1 1 1 0 1 0 1 1 0 0 1 ......

ii) To obtain chip duration

The chip rate is $R_c = 10^7$ chips/sec. Hence the chip duration is

$$T_c = \frac{1}{R_c} = \frac{1}{10^7} \text{ sec} = 0.1\ \mu\text{sec.}$$

iii) To obtain length of PN sequence

The length of the PN sequence is,

$$N = 2^m - 1$$
$$= 2^4 - 1 = 15 \text{ digits.}$$

iv) To obtain period of output sequence

Hence period of the output sequence is,

$$T_b = NT_c$$
$$= 15 \times 0.1\mu sec$$
$$= 1.5\ \mu sec$$

For a linear feedback shift register with three stages (m=3), evaluate the maximum length PN sequence for feedback taps = (3, 1). Draw the schematic arrangement and verify all the properties of PN sequence in generated output. Sketch the sequence, its autocorrelation function and PSD function if chip rate happens to be 10 MHz.

Test all three properties of ML sequence after generating PN sequence for a 3 stage feedback shift register. (Assume 100 as initial state) **VTU : June-17, Marks 10**

Sol. : i) Schematic arrangement :

There will be 3 stages in the shift register. The feedback taps will be taken from outputs of first and third stage. Fig. shows the scheme to generate PN sequence.

ii) To obtain PN sequence :

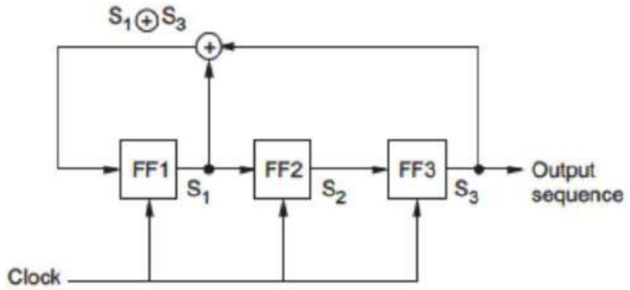


Fig. A 3-stage feedback shift register with (3, 1) taps

Let us assume that the initial contents of the shift register are $s_1 = 1$, $s_2 = 0$ and $s_3 = 0$. Table shows the generated sequence.

Sr. No.	State of shift register $s_1\ s_2\ s_3$	Mod-2 adder output $s_1 \oplus s_3$	PN - sequence s_3
1	1 0 0	$1 \oplus 0 = 1$	0
2	1 1 0	$1 \oplus 0 = 1$	0
3	1 1 1	$1 \oplus 1 = 0$	1
4	0 1 1	1	1
5	1 0 1	0	1
6	0 1 0	0	0
7	0 0 1	1	1
8	1 0 0	1	0 ← Sequence repeats here

Table Generation of PN sequence

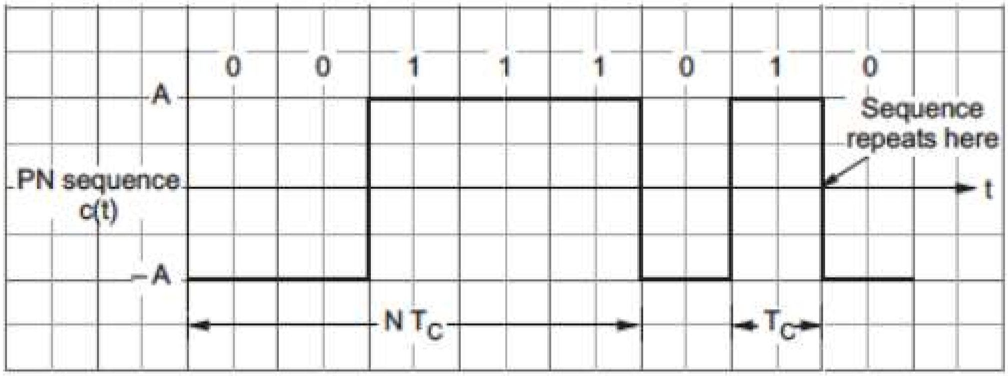


Fig. Sketch of the PN sequence of length N = 7

Thus the generated pseudo random sequence is,

0 0 1 1 1 0 1 ......

Length of this sequence is $2^m - 1 = 2^3 - 1 = 7$.

Fig. 5.3.10 shows sketch of the sequence in NRZ form.

iii) To verify the properties of maximum length sequence :

The generated PN sequence is,

$$C_n = \{0\ 0\ 1\ 1\ 1\ 0\ 1\}$$

1. **Balance property :** In each period number of 1's is always one more than number of 0's. In the above sequence, observe that there are three 0's and four 1's. This satisfies balance property.
2. **Run property :** When there are 'm' stages in the shift register, then generated sequence contains 2^{m-1} runs. Here $m=3$. Hence there will be $2^{3-1} = 4$ runs. These are given below :

$$C_n = \{\underbrace{0\ 0}_{Run1}\ \underbrace{1\ 1\ 1}_{Run2}\ \underbrace{0}_{Run3}\ \underbrace{1}_{Run4}\}$$

Thus there are total 4 runs,

Run 1 = {0 0}

Run 2 = {1 1 1}

Run 3 = { 0 }

Run 4 = { 1 }

i) Here two runs (i.e. half of total 4 runs) are of length 1. These runs are run-3 and run-4.

ii) One run (i.e. one fourth of total 4 runs) is of length 2. This run is run-1. Thus run property is satisfied.

3. **Correlation property :** An autocorrelation of the sequence is periodic and binary valued. This is shown next.

iv) Autocorrelation of the PN sequence :

We know that autocorrelation of the PN sequence is given as

$$R_c(\tau) = \begin{cases} 1 - \dfrac{N+1}{NT_c}|\tau| & for\ \ |\tau| < T_c \\ -\dfrac{1}{N} & elsewhere \end{cases}$$

Here length, $N = 7$ and chip rate $R_c = 10$ MHz. Hence $T_c = \dfrac{1}{R_c} = \dfrac{1}{10 \times 10^6} = 1 \times 10^{-7}$

$$R_c(\tau) = \begin{cases} 1 - \dfrac{7+1}{7 \times 1 \times 10^{-7}}|\tau| & for\ \ |\tau| < 1 \times 10^{-7} \\ -\dfrac{1}{7} & elsewhere \end{cases}$$

$$= \begin{cases} 1 - 11428 \times 10^6 (|\tau|) & for\ \ |\tau| < 1 \times 10^{-7} \\ -\dfrac{1}{7} & elsewhere \end{cases}$$

An autocorrelation function based on above equation is sketch in Fig.

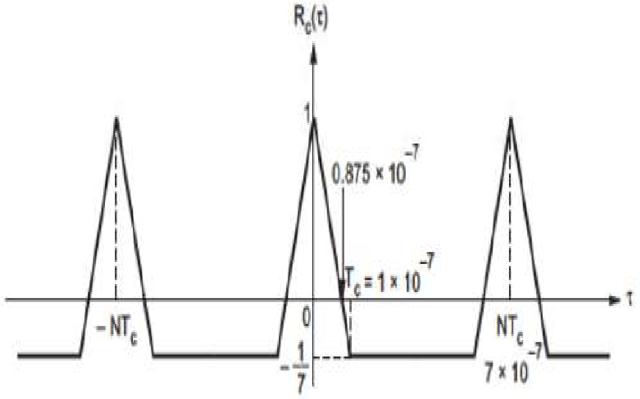


Fig. Autocorrelation of the PN sequence with $N = 7$ and $R_c = 10$ MHz

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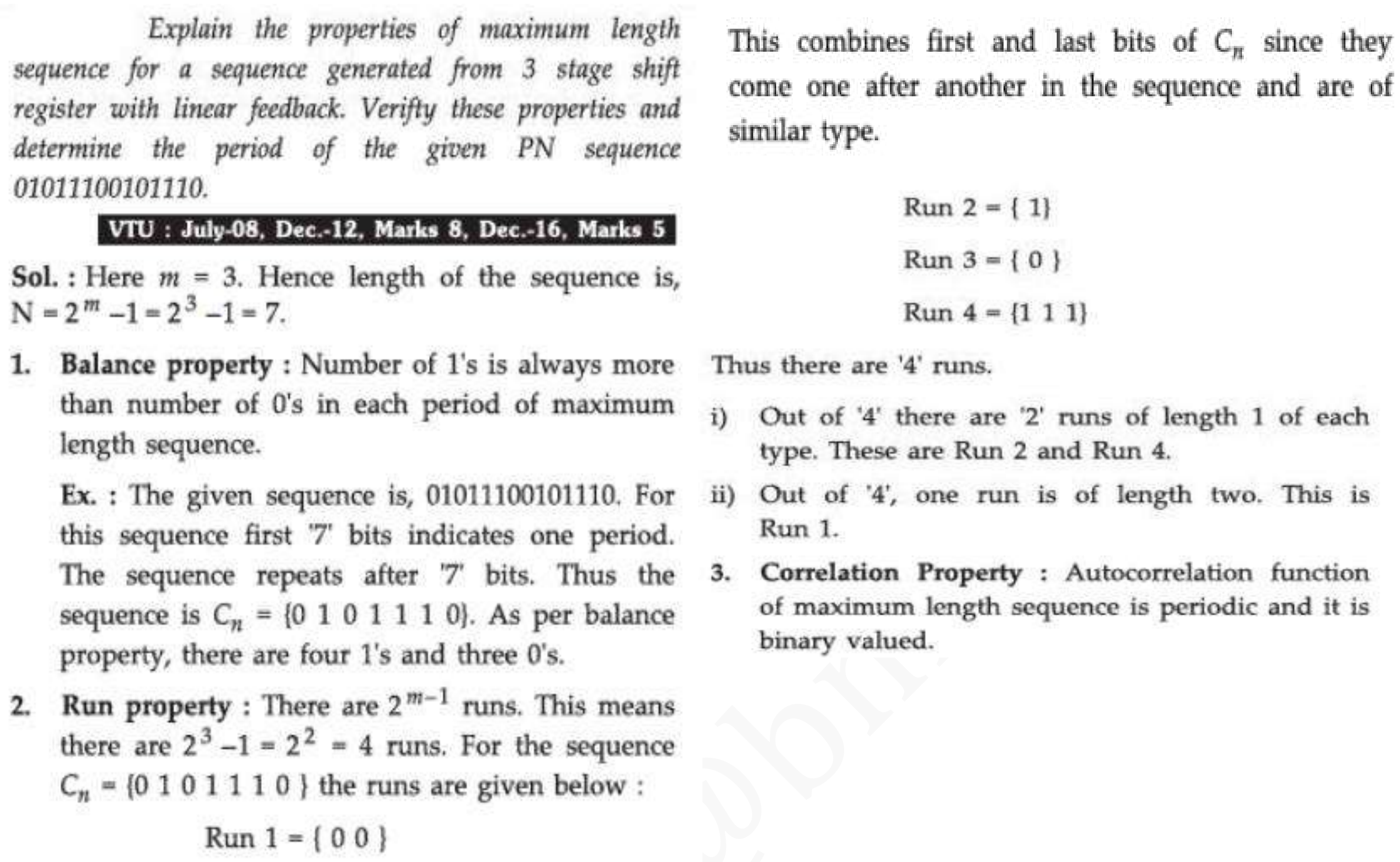


Explain the properties of maximum length sequence for a sequence generated from 3 stage shift register with linear feedback. Verifty these properties and determine the period of the given PN sequence 01011100101110.

VTU : July-08, Dec.-12, Marks 8, Dec.-16, Marks 5

Sol. : Here $m = 3$. Hence length of the sequence is, $N = 2^m - 1 = 2^3 - 1 = 7$.

1. **Balance property :** Number of 1's is always more than number of 0's in each period of maximum length sequence.

   Ex. : The given sequence is, 01011100101110. For this sequence first '7' bits indicates one period. The sequence repeats after '7' bits. Thus the sequence is C_n = {0 1 0 1 1 1 0}. As per balance property, there are four 1's and three 0's.

2. **Run property :** There are 2^{m-1} runs. This means there are $2^3 - 1 = 2^2 = 4$ runs. For the sequence C_n = {0 1 0 1 1 1 0 } the runs are given below :

   Run 1 = { 0 0 }

   This combines first and last bits of C_n since they come one after another in the sequence and are of similar type.

   Run 2 = { 1}

   Run 3 = { 0 }

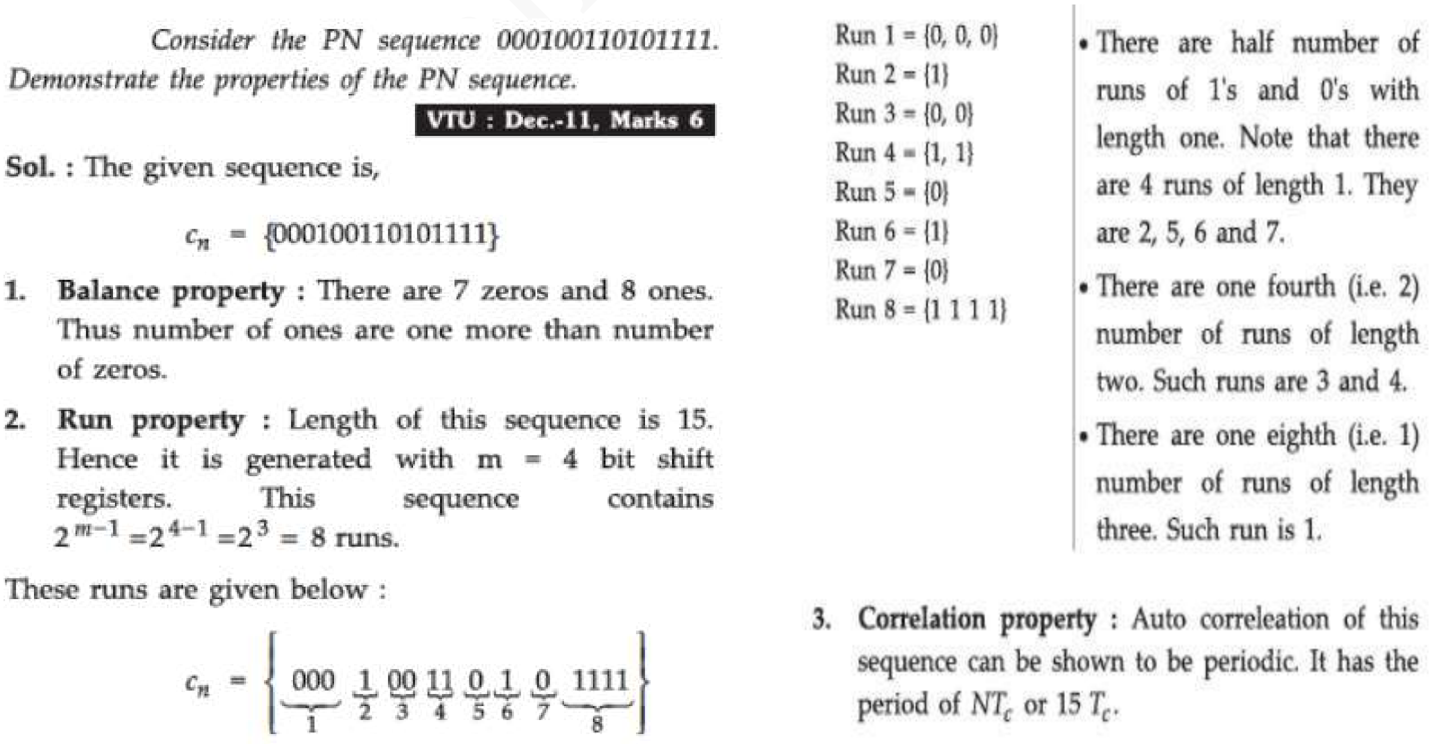
   Run 4 = {1 1 1}

   Thus there are '4' runs.

   i) Out of '4' there are '2' runs of length 1 of each type. These are Run 2 and Run 4.

   ii) Out of '4', one run is of length two. This is Run 1.

3. **Correlation Property :** Autocorrelation function of maximum length sequence is periodic and it is binary valued.



Consider the PN sequence 000100110101111. Demonstrate the properties of the PN sequence.

VTU : Dec.-11, Marks 6

Sol. : The given sequence is,

$$c_n = \{000100110101111\}$$

1. **Balance property :** There are 7 zeros and 8 ones. Thus number of ones are one more than number of zeros.

2. **Run property :** Length of this sequence is 15. Hence it is generated with m = 4 bit shift registers. This sequence contains $2^{m-1} = 2^{4-1} = 2^3 = 8$ runs.

These runs are given below :

$$c_n = \left\{ \underbrace{000}_{1}\ \underbrace{1}_{2}\ \underbrace{00}_{3}\ \underbrace{11}_{4}\ \underbrace{0}_{5}\ \underbrace{1}_{6}\ \underbrace{0}_{7}\ \underbrace{1111}_{8} \right\}$$

Run 1 = {0, 0, 0}
Run 2 = {1}
Run 3 = {0, 0}
Run 4 = {1, 1}
Run 5 = {0}
Run 6 = {1}
Run 7 = {0}
Run 8 = {1 1 1 1}

- There are half number of runs of 1's and 0's with length one. Note that there are 4 runs of length 1. They are 2, 5, 6 and 7.
- There are one fourth (i.e. 2) number of runs of length two. Such runs are 3 and 4.
- There are one eighth (i.e. 1) number of runs of length three. Such run is 1.

3. **Correlation property :** Auto correleation of this sequence can be shown to be periodic. It has the period of NT_c or 15 T_c.

Problems on Direct Spread Spectrum

The direct sequence spread spectrum communication system has following parameters.

Data sequence bit duration, $T_b = 4.095$ *ms*

PN Chip duration, $T_c = 1$ *μs*

$\frac{E_b}{N_0} = 10$ *for average probability of error less than* 10^{-5}.

Calculate processing gain and jamming margin.

VTU : Dec.-15, Marks 6

Sol. : Given data : Sometimes the one bit period of pseudo-noise sequence (i.e. PN sequence) is also called as one '*Chip*'. Here one chip duration is T_c i.e. $T_c = 1\ \mu\ s$ and $T_b = 4.095$ ms.

Processing Gain

Processing gain is given as,

$$PG = \frac{T_b}{T_c} = \frac{4.095 \times 10^{-3}}{1 \times 10^{-6}} = 4095$$

Since $PG = N$, the length of the bit sequence is 4095.

Jamming margin

the jamming margin is given as,

$$\text{Jamming Margin} = \frac{J}{P_s} = \frac{PG}{E_b/N_0} = \frac{4095}{10}$$

$$= 409.5$$

Comment on result : This shows that information bits at the receiver output can be detected with the probability of error less than 10^{-5} even when noise interference is upto 409.5 times the received signal power.

The jamming margin can also be calculated in dB i.e.

$$(\text{Jamming Margin})_{dB} = (PG)_{dB} - 10\log_{10}\left(\frac{E_b}{N_0}\right)$$

$$= 10\log_{10}(4095) - 10\log_{10}(10)$$

$$= 36.1 - 10 = 26.1 \text{ dB}$$

Explain the significance of jamming margin in DS-SS system. A BPSK-DSSS system, using coherent detection, is used to transmit data at 250 bps and system has to work in a hostile jamming environment with minimum error performance of one error in 20,000 bits. Determine the minimum chipping rate, if the jamming signal is 300 times stronger than the received signal ? Assume Q(3.9) = 0.00005.

Sol. : Step 1 : To obtain E_b / N_0

The probability of error of BPSK is given as,

$$P(e) = \frac{1}{2} erfc\sqrt{\frac{E_b}{N_0}}$$

From equation (5.5.26),

$$\frac{E_b}{N_0} = \frac{P_s}{J} \cdot \frac{T_b}{T_c}$$

Since jamming signal is 300 times stronger than received signal, $\frac{J}{P_s} = 300$. And $T_b = \frac{1}{250}$. Putting these values in those equation,

$$\frac{E_b}{N_0} = \frac{1}{300} \cdot \frac{1/250}{T_c} = \frac{1}{75000\,T_c}$$

We know that,

$$erfc(u) = 2\,Q\,(\sqrt{2}\ u)$$

$$\therefore \quad P_e = \frac{1}{2} \cdot 2Q\left[\sqrt{2} \cdot \sqrt{\frac{E_b}{N_0}}\right]$$

$$= Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

One error in 20,000 bits represents error probability of $P_e = \frac{1}{20{,}000} = 0.00005$. Hence above equation will be,

$$0.00005 = Q\left(\sqrt{2\frac{E_b}{N_0}}\right)$$

We are given that Q(3.9) = 0.00005. Hence,

$$\sqrt{2\frac{E_b}{N_0}} = 3.9$$

$$\text{or} \quad 2\frac{E_b}{N_0} = 15.21$$

Step 2 : To obtain chip rate.

Putting value of $\frac{E_b}{N_0}$,

$$2 \cdot \frac{1}{75000\,T_c} = 15.21$$

$$\therefore \quad T_c = \frac{1}{570375}$$

$$\text{Chipping rate} = \frac{1}{T_c} = 570375 \text{ bps}$$

In a DS/BPSK system, the feedback shift register used to generate the PN sequence has length m = 19. The syestem is required to have a probability of error due to extermally generated interfering signals that does not exceed 10^{-5}. Calculate the processing gain and antijam margin in decibels. Use erf (3) = 0.99998.

VTU : July-13, Marks 5

Sol. : Here $m = 19$, $P_e = 10^{-5}$.

i) Processing gain :

$$PG = N = 2^m - 1 = 2^{19} - 1 = 524287 = 10 \log_{10} 524287 = 57.2 \text{ dB}$$

ii) Antijam margin :

For BPSK,

$$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

$$2 \times P_e = erfc \sqrt{\frac{E_b}{N_0}}$$

$$2 \times 10^{-5} = erfc \sqrt{\frac{E_b}{N_0}}$$

$$1 - 2 \times 10^{-5} = 1 - erfc \sqrt{\frac{E_b}{N_0}}$$

$$0.99998 = erf \sqrt{\frac{E_b}{N_0}}, \quad \text{since } erf(x) = 1 - erfc(x)$$

Here it is given that $erf(3) = 0.99998$. Hence,

$$\sqrt{\frac{E_b}{N_0}} = 3 \quad \Rightarrow \quad \frac{E_b}{N_0} = 9$$

Antijam margin is given as,

$$(\text{Jamming margin}) \text{ dB} = (\text{Processing gain})_{\text{dB}} - 10 \log_{10} \left(\frac{E_b}{N_o} \right)$$

$$= 57.2 \text{ dB} - 10 \log_{10} 9 = 47.65 \text{ dB.}$$

A spread spectrum communication system has the following parameters :
Information bit duration = T_b = 4 milli secs,
PN chip duration = T_c = 2 micro secs. Find the bit rate of the binary data, PN sequence length, bandwidth of the PN sequence and processing gain of the system. **VTU : May-11, Marks 5**

Sol. : **Given data**

$$T_b = 4\times10^{-3} \text{ sec}$$

$$T_c = 2\times10^{-6} \text{ sec}$$

i) Bit rate of binary signal $f_b = \frac{1}{T_b} = \frac{1}{4\times10^{-3}} = 250$ bits/sec

ii) PN sequence length

Length, $N = \frac{T_b}{T_c} = \frac{4\times10^{-3}}{2\times10^{-6}} = 2{,}000$

iii) Bandwidth of the PN sequence

Bandwidth of the PN sequence is basically bandwidth of the spreaded signal. i.e.,

$$\text{BW} = \frac{1}{T_c} = \frac{1}{2\times10^{-6}} = 500 \text{ kHz}$$

iv) Processing gain

$$PG = N = 2000.$$

Problems on FH/MFSK

A slow FH/MFSK system has the following parameters,
The number is bits/MFSK symbol = 4
The number of MFSK symbols per hop = 5
Calculate the processing gain of the system in decibels.

VTU : Aug.-05, Feb.-06, 08

Sol. : Let f_s be symbol frequency. There are 4-bit per MFSK symbol. Hence bandwidth of unspreaded signal will be $\frac{f_s}{4}$. Similarly, there are 5 MFSK symbols per hop. Hence bandwidth of the spreaded signal will be 5 f_s. Hence processing gain will be,

$$\text{Processing gain } (PG) = \frac{\text{BW of spreaded signal}}{\text{BW of unspreaded signal}}$$

$$= \frac{5\,f_s}{f_s/4} = 5\times4 = 20$$

$$PG \text{ in dB} = 10\log_{10} PG$$

$$= 10\log_{10} 20$$

$$= 13 \text{ dB}$$

A slow FH/MFSK system has the following parameters :
The number of bits per MFSK symbol = 4
The number of MFSK symbols per hop = 6
Calculate the processing gain of the system.

VTU : Aug.-06, June-16, Marks 6

Sol. : Let f_s be the symbol frequency. There are 4-bits per MFSK symbol. Hence bandwidth of unspreaded signal will be $\frac{f_s}{4}$. Similarly there are 6 MFSK symbols per hop. Hence bandwidth of spreaded signal will be 6 f_s.

$$\therefore \text{ Procesing Gain (PG)} = \frac{\text{BW of spreaded signal}}{\text{BW of unspreaded signal}}$$

$$= \frac{6\,f_s}{f_s/4} = 6\times4 = 24$$

$$= 10\log_{10} 24 = 13.8 \text{ dB.}$$

Advantages and Disadvantages of DS-SS System

Advantages of direct sequence symbols

1. This system has best noise and antijam performance.
2. Unrecognized receivers find it most difficult to detect direct sequence signals.
3. It has best discrimination against multipath signals.

Disadvantages of direct sequence systems

1. It requires wideband channel with small phase distortion.
2. It has long acquisition time.
3. The pseudo-noise generator should generate sequence at high rates.
4. This system is distance relative.

Advantages and Disadvantages of FH-SS System

Advantages of frequency hopping system

1. These systems bandwidth (spreads) are very large.
2. They can be programmed to avoid some portions of the spectrum.
3. They have relatively short acquisition time.
4. The distance effect is less.

Disadvantages of frequency hopping systems

1. Those systems need complex frequency synthesizers.
2. They are not useful for range and range-rate measurement.
3. They need error correction.

Comparison between DS-SS and FH-SS

Table shows the comparison between direct sequence and frequency hopping spread spectrum systems.

Sr. No.	Parameter	Direct sequence spread spectrum	Frequency hop spread spectrum
1	Definition	PN sequence of large bandwidth is multiplied with narrowband data signal.	Data bits are transmitted in different frequency slots which are changed by PN sequence.
2	Spectrum of signal	Data sequence is spread over entire bandwidth of spread spectrum signal.	Data sequence is spread over small frequency slots of the spread spectrum signal.
3	Chip rate R_c	Chip rate is fixed. It is the rate at which bits of PN sequence occur. $R_c = \frac{1}{T_c}$	Chip rate is maximum of hop rate or symbol rate. $R_c = \max\left(R_h, R_s\right)$
4	Modulation technique	Normally uses BPSK modulation.	Normally uses M-ary FSK modulation.
5	Processing gain	$PG = \frac{T_b}{T_c} = N$	$PG = 2^t$, here t is the bits in PN sequence.
6	Probability of error	$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{JT_c}}$	$P_e = \frac{1}{2} e^{-\gamma_b R_c/2}$ Here $\gamma_b = \frac{E_b}{J_0}$
7	Effect of distance	This system is distance relative.	Effect of distance is less in this system.
8	Acquisition time	Acquisition time is long.	Acquisition time is short.

Applications of Spread Spectrum Modulation

VTU : Dec.-12

In the introduction we have seen how spread spectrum modulation is important for military application. Some more applications are listed in this section.

1. Antijamming capacity for military applications

The spread spectrum has the ability to resist the effect of intentional jamming. Previously this antijam capability was used in military applications. Some commercial applications also use spread spectrum because of its antijam capability.

2. Low proability of intercept

Lowprobability of intercept is an application of spread spectrum in military. In this case, the signal spectral density is kept small such that the presence of the signal is not detected easily.

3. Mobile communications

Spread spectrum is used in mobile communications. This is because the spread spectrum signal has the ability to resist the effects of multipath fading. Because of wide spectrum (i.e. bandwidth) of spread spectrum signals only small portion of the signals is in fade. The resistance to fading is important consideration in mobile communications.

4. Secured communication

The spread spectrum communications are 'secure'. Since pseudo-noise codes are used to generate spread spectrum signals, unwanted receivers cannot recognize the spread spectrum signals. The secrecy capability of spread spectrum is used in military as well as in many commercial applications.

5. Distance measurements

Spread spectrum communications are used in distance measurement. Broadband signals can be resolved in time more precisely than narrow band signals. Thus if broadband signals are transmitted, then the time delays can be measured more accurately and the distance measurement is accurate. Spread spectrum signals are used in radars and navigation systems for distance measurement.

6. Selective calling

Spread spectrum is also used in selective calling. In this, the central station communicates with number of different receiving points. This is accomplished pseudo-noise codes in spread spectrum signals.

7. CDMA communication

Spread spectrum is used in code division multiple access (CDMA) communication. In this system many users communicate at the same time and through the same channel. Each user is allotted a particular code. The signals are separated at the receiver because of these codes. Those codes are allotted by pseudo-noise sequence. The important advantage of CDMA is that the message of some other user is not decoded. And number of users can communicate simultaneously through the same channel.

5.6 CDMA based on IS-95

- With the help of CDMA it is possible to transmit many DS spread spectrum signals so that they can occupy same channel bandwidth. Each channel has its own PN sequence.
- CDMA is used widey for voice communication. It is standardized as IS-95. It uses the frequency band of 800 MHz TO 1900 MHz. CDMA has the advantage that the frequency reuse factor is 1.
- The bandwidth of 1.25 MHz is used for transmission from base station to mobile, on forward link. On separate channel, the bandwidth of 1.25 MHz is used on reverse link.
- The DS spread spectrum signals with CDMA are transmitted on forward and reverse link with the chip rate of 1.2288×10^6 chips/sec.

IS-95 Forward link (Base Station to Mobile)

With a neat block diagram explain the CDMA system based on IS-95. **(08Marks) Dec 2019- Jan 2020.**

Soln: Fig. shows the block diagram of forward link.

- The speech encoder output is available at the rates of 9.6 kbps, 4.8 kbps, 2.4 kbps or 1.2 kbps. This data is encoded by convolutional encoder of rate 1/2 and constraint length of 9.
- The encoded data is then passed through a block interleaver. It overcomes the effects of burst errors. The output of interleaver is available at the rate of 19.2 kbps.
- The output of interleaver is scrambled by multiplication with long code. The long code generator has the chip rate of 1.2288 Mchips/sec. Its output is decimated to 19.2 kchips/sec.
- The Walsh sequence generator generates 64 bit long orthogonal Hadamard sequences. Each sequence is assigned to separate base station. There are 64 Hadamard sequences. Thus 64 channels are available for transmission.
- The Hadamard sequence multiplies with data sequences. Each encoded data bit is multiplied by the Hadamard sequence of length 64.
- The binary output sequence from multiplier is then multiplied with two PN sequences of length $N = 2^{15}$. It creates inphase and quadrature components.

- The I and Q components are passed through baseband shaping filters. The different base stations are identified by offsets of PN sequences.
- The I and Q DS spread signals are then modulated on two quadrature carriers and combined. This combined signal is then transmitted.
- The RAKE demodulator at the receiver is used to receive these signals. Viterbi soft devision decoder is used.

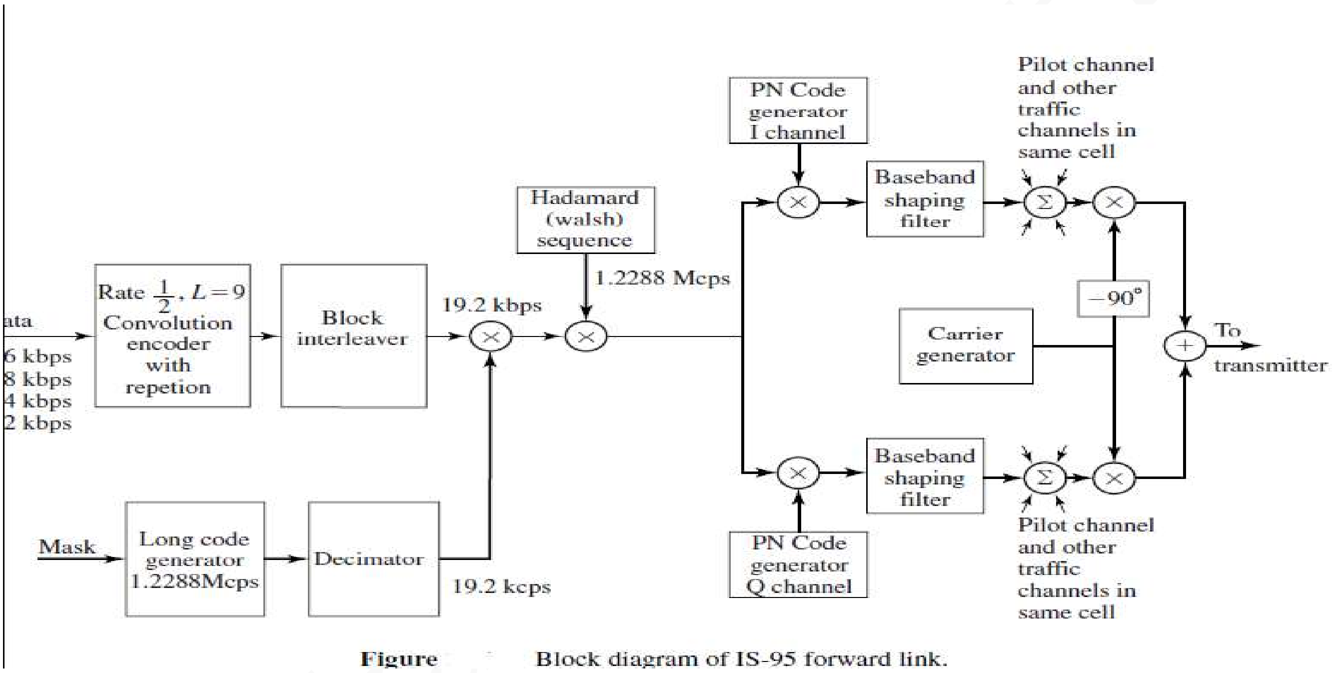


Figure Block diagram of IS-95 forward link.

IS-95 Reverse link (Mobile to Base Station)

With a neat block diagram, explain the IS-95 reverse link. (06 Marks) June-July 2019.

- Fig. shows the block diagram of reverse link. The mobile transmitters are battery operated. Hence power efficient transmission is used.
- The speech encoder output is available at the rates of 9.6 kbps, 4.8 kbps, 2.4 kbps or 1.2 kbps. This data is encoded a convolutional encoder of rate 1/2 and constraint length of 9.

- Every 20 msec frame, 576 encoded bits are passed through block interleaver. Output of block interleaver is available at the rate of 28.8 kbps.
- The output of block interleaver is modulated with 64 orthogonal signal sets using Hadamard sequences each of length 64.
- The output of modulator has the bit rate of 307.2 kbps. The modulated signal is then randomized with the help of long code generator. It provides code at the rate of 1.2288 M chips/sec. Due to randomization, consecutive signal bursts and interference effects are reduced.
- The signal is then spread by two separate PN sequences over 'I' and 'Q' channels. The PN generators run at the rate of 1.2288 M chips/sec. Thus there are four PN chips for every bit of Hadamard sequence from modulator.
- The 'I' and 'Q' signals are then filtered by baseband spectral shaping filters. These signals then modulate two quadrature carriers. The modulated signals are finally added. This is nothing but offset QPSK.
- Computationally efficient Hadamard transform is used to reduce the computational complexity at demodulation. Viterbi decoder is then used to decode the signal.

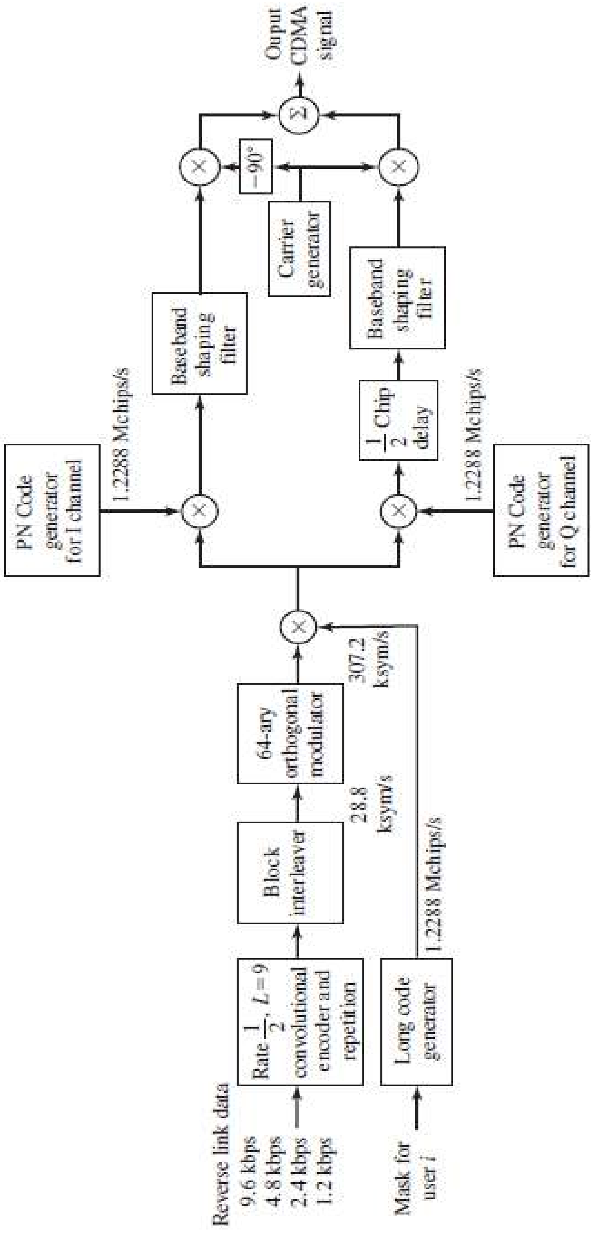


Figure 1 Block diagram of IS-95 reverse link.